Non-singular basis of master integrals

In 1991 A. Davydychev and later Z. Bern, L. Dixon and D. Kosower gave expressions for one-loop tensor integrals in terms of integrals with shifted dimension

J. Campbell, E.W.N. Glover, D.J. Miller (Nucl.Phys. B498 (1997) 397) investigated one-loop integrals in higher dimensions and descovered their good numerical stability

A simple formula expressing any n -point integrals in terms of integrals in higher dimensions was given in

J. Fleischer, F. Jegerlehner and O.V. Tarasov, Nucl.Phys. B566 (2000) 423 O.V. Tarasov, Phys.Rev.D54(1996) 6479

$$I_n^{(d)} = \frac{(d - \Sigma_{i=1}^n \nu_i + 1)G_{n-1}}{2\Delta_n} I_n^{(d+2)} - \sum_{k=1}^n \frac{(\partial_k \Delta_n)}{2\Delta_n} \mathbf{k}^- I_n^{(d)}$$

Numerical stability of integrals in higher dimensions can be seen from their integral representation

$$I_n^{(d)} = \Gamma\left(n - \frac{d}{2}\right) \int_0^1 dx_1 \ldots \int_0^1 dx_{n-1} \; J_n h_n^{(d/2-n)},$$

where

$$J_2 = 1, \qquad\qquad J_n = x_{n-2} x_{n-3}^2 \ldots x_1^{n-2}.$$

$$h_2 = p_{12}^2 x_1^2 + m_2^2 - x_1(p_{12}^2 - m_1^2 + m_2^2).$$

$$h_3 = -x_1 x_2 (1 - x_1) p_{13}^2 - x_1^2 x_2 (1 - x_2) p_{12}^2 - x_1 (1 - x_1)(1 - x_2) p_{23}^2$$
$$+ x_1 x_2 m_1^2 + x_1 (1 - x_2) m_2^2 + (1 - x_1) m_3^2,$$