Renormalization of the MSSM

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DESY

Hollik, Kraus, Roth, Rupp, Sibold, DS
[NPB 639 (02)]

- Symmetry identities of the MSSM:
  basic structure, problems, detailed structure

- Consequences:
  counterterm structure, renormalizability; determine symmetry-restoring c.t.s

- Applications:
  gauge dependence of $\tan\beta$,
  non-renormalization theorems

Motivation

Original motivation:

- Dim. Regularization susy-breaking
  $\longrightarrow$ susy-restoring c.t.s required

- Dim. Reduction math. inconsistent
  $\longrightarrow$ area of validity unclear

Renormalization: 1. check symmetries
2. calculate sym.-restoring c.t.s
3. add symmetric c.t.s

Deeper question: How to formulate the symmetries on the level of Green functions?

Solution: STI and WI of the MSSM

Additional use of STI, WI:

- Prove renormalizability, Applications, . . .

Basic structure of the symmetry identities

Basic aspects: SU(2)×U(1) gauge inv., Susy, symmetry breaking

Treatment analogous to: SM *[E. Kraus, 97]*
Susy Yang-Mills *[Maggiore, Piguet, Wolf, 96]*
[Hollik, Kraus, DS, 00]

- Symmetry content $\longrightarrow$ STI $S(\Gamma)=0$
  - Gauge inv., Susy, commutation relations, quantum corrections to transformations

- Fix Hypercharges such that $Q=T_3+\frac{Y}{2}$
  $\longrightarrow$ WI for $\mathrm{U}(1)_Y$:

$$\partial^\mu \frac{\delta\Gamma}{\delta V'^\mu} = -i\sum \frac{Y_i}{2}\ \varphi_i \frac{\delta\Gamma}{\varphi_i} + \mathrm{g.fix.}$$

- Symmetry breaking $\longrightarrow$ ext. fields

Susy mult.	$(a,\chi,f+f_0)\to(0,0,f_0)$
SU(2)×U(1)-mult.	$(\hat{\Phi}+\hat{\mathrm{v}})\to\hat{\mathrm{v}}$

Basic structure of the symmetry identities

In the SM, Susy Yang-Mills $\Rightarrow$

- symmetry-restoring c.t.s uniquely determined
- Remaining freedom: symmetric c.t.s
  $\leftrightarrow$ multiplicative renormalization
- IR-finiteness (*)
- Renormalizability

However, in the MSSM the detailed structure of the symmetry identities must be different from the one in the SM, SYM!
Complication of (*)

Problem

What is the problem?

Mixing $A^0 \leftrightarrow (G^0,\ A^\mu_{\mathrm{Long}},\ Z^\mu_{\mathrm{Long}})$

- R_ξ-gauge contains only G^0 and breaks SU(2)×U(1)-inv., U(1)$_Y$-WI

$$F^A = \partial^\mu A_\mu, \qquad F^Z = \partial^\mu Z_\mu + \xi M_Z G^0,$$

$\longrightarrow$ use external fields $(\hat{\Phi} + \hat{\mathrm{v}})$:

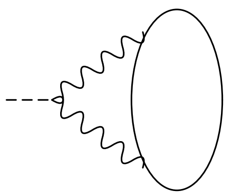
$$F^a = \partial^\mu V^a_\mu - 2\mathrm{Im}((\hat{\Phi} + \hat{\mathrm{v}})^{a\dagger}_i H_i)$$

- What is the multiplet structure of $(\hat{\Phi} + \hat{\mathrm{v}})$?

- Possible IR-divergences:

$$\mathcal{L}_{c.t.} = \hat{\Phi}_j A^\mu A_\mu + \ldots$$

$$W\Gamma = \ldots + \int \hat{\mathrm{v}}_i \frac{\delta\Gamma}{\delta\hat{\Phi}_j} \longrightarrow \quad \text{q=0}$$



$$\sim \int \frac{d^4k}{k^4}$$

Can $\hat{\Phi}$ lead to IR-div's in Γ, $W\Gamma$, $S(\Gamma)$?

Solution:

- $(\hat{\Phi}+\hat{\mathrm{v}})$: (adjoint)×(doublet representation)

$$\Rightarrow F^a = \partial^\mu V^a_\mu - 2\mathrm{Im}((\hat{\Phi}+\hat{\mathrm{v}})^{a\dagger}_i H_i)$$

compatible with U$(1)_Y$-WI and R_ξ-gauge

- Identify IR-dangerous components of $\hat{\Phi}$ (using IR-Powercounting) *[Lowenstein,75]*

  − prove that every dangerous term is excluded

- Use STI to control on-shell mixing ($p^2 = M_A^2$):

$$0 = \sim \underbrace{\Gamma_{A^0 H_i}}_{=0} + \sim \underbrace{\frac{\delta \mathcal{L}_{R_\xi}}{\delta A^0}}_{=0} + \sim \underbrace{\Gamma_{A^0 V^\mu}}_{=0}$$

$\Gamma_{A^0 A^0} = \Gamma_{A^0 G^0} = \Gamma_{A^0 A^\mu} = \Gamma_{A^0 Z^\mu} = 0$
simultaneously possible

Result:

Combining all elements yields

Detailed structure of the sym. identities
$\longrightarrow$ definition of the MSSM

- STI, WI:
  - Symmetry content on the quantum level
  - formulated for Green functions
- structure and IR-powercounting of $(\hat{\Phi} + \hat{\mathrm{v}})$, $(a, \chi, f + f_0)$ (and all other fields)
- Further symmetry identities:
  - Gauge-fixing conditions
  - Global symmetries: CP, R-Symmetry,...

Result:

Consequences of the symmetry identities

- MSSM defined via its symmetries
- symmetry-restoring c.t.s uniquely determined
- MSSM multiplicatively renormalizable, IR-finite
- On-Shell renormalization conditions can be satisfied

This provides the basis for

- Determination of symmetry-restoring c.t.s
- Applications

Symmetric counterterms and on-shell conditions

After adding symmetry-restoring c.t.s

Freedom: symmetric counterterms

Their structure can be determined

- they correspond to multiplicative renormalization:

$$\begin{aligned} g_i &\to g_i + \delta g_i \\ \phi &\to \sqrt{Z}\phi \end{aligned}$$

- they suffice to satisfy all on-shell conditions:

$$\begin{aligned} &\mathrm{Re}\Gamma_{A^0 A^0} = 0 && (p^2 = M^2_{A^0}) \\ &\mathrm{Re}\Gamma_{A^0 G^0} = 0 && (p^2 = M^2_{A^0}) \\ &\ldots \\ &(\stackrel{STI}{\Rightarrow} \Gamma_{A^0 A^\mu} = \Gamma_{A^0 Z^\mu} = 0) \end{aligned}$$

Consequences

Symmetry-restoring c.t.s

determined by two kinds of identities

$0 = \frac{\delta S(\Gamma)}{\delta \epsilon}$: Susy relation

$$\left\langle \epsilon\phi(s\overline{\psi})\right\rangle_{1PI} \Gamma_{\psi\overline{\psi}} + \left\langle \epsilon\psi(s\phi^{\dagger})\right\rangle_{1PI} \Gamma_{\phi\phi^{\dagger}} = 0$$

Literature: "Susy Ward Identities"

$0 = \frac{\delta S(\Gamma)}{\delta \epsilon \delta \overline{\epsilon}}$: Susy algebra

$$\left\langle \epsilon\phi(s\overline{\psi})\right\rangle_{1PI} \left\langle \overline{\epsilon}\overline{\psi}(s\phi)\right\rangle_{1PI} = -2p^{\mu}\gamma_{\mu} + \ldots$$

$\Rightarrow$ determine c.t.s such that all identities are satisfied $\longrightarrow$ unique results

$\Rightarrow$ 1-Loop results in SQED, SQCD: DRed preserves both kinds of identities

Applications

Apply STI for further studies

Tools: STI + Extensions: $\tilde{S}(\Gamma) = 0$

$s\xi \neq 0 \Rightarrow \partial_\xi \tan\beta$ calculable

$se \neq 0 \Rightarrow \delta e^{\mathrm{div}}$ calculable

Applications

Gauge dependence of $\tan\beta$

[A. Freitas, DS, 2002]

$$\tan\beta \;=\; \frac{v_2}{v_1} \quad \xrightarrow{R.T.} \quad \frac{\sqrt{z_2}(v_2+\delta v_2)}{\sqrt{z_1}(v_1+\delta v_1)}$$

$\delta\tan\beta$: gauge dependent

$\tan\beta$: gauge dep. depends on the scheme

Results: $\tan\beta$ gauge indep. if $\tilde{S}(\Gamma) = 0$

Dabelstein, $\overline{DR}$: $\partial_\xi \tan\beta \neq 0$

$\overline{DR}$: $\partial_\xi \tan\beta(R_\xi, 1\text{Loop}) = 0$

New schemes: $\partial_\xi \tan\beta = 0$

$\longrightarrow$ large num. instabilities

Conclusion: $\overline{DR}$-scheme suggested

Applications

Non-renormalization theorems

...absence of qu. div. and more...

New approach using extended STI

[Flume, Kraus 99, Kraus 01, Kraus, DS 02]

Origin

$$\begin{aligned} \mathcal{L}_{\text{susy}} &= \text{highest comp. of } (A, \Xi, \mathcal{L}_{\text{susy}}) \\ \Rightarrow \text{Diagram} &\overset{Q}{\leftrightarrow} \text{Diag. with lower d. of div.} \end{aligned}$$

Implementation

$$\begin{aligned} \text{Couplings } e, \ldots &\longrightarrow \text{susy mult. } (e, \chi, f), \ldots \\ \Rightarrow \mathcal{L}_{\text{susy}} &\longrightarrow \mathcal{L}_{\text{susy}}(e) - \chi \underbrace{\Xi}_{d=3.5} + f \underbrace{A}_{d=3} \end{aligned}$$

Applications

$\boxed{\begin{array}{l}\text{Supercoupling}\\(e,\chi,f)\end{array}} \rightarrow \boxed{\tilde{S}(\Gamma)=0} \rightarrow \boxed{\begin{array}{l}\text{New approach}\\\text{to NRTs}\end{array}}$

Advantages:

- Regularization-independent
- Works also in the WZ gauge
- Underlying algebraic origin apparent

Results (ex.: SQED)

- Only two independent divergences:
  $\delta Z_e^{(1)}$: only 1-Loop, $\delta Z_m^{(l)}$
- $\delta Z_{M_\lambda} = 2\delta Z_e$ (only 1-Loop)
  $\delta Z_b^{(l)} = (2l\ M_\lambda m/b + 1)\,\delta Z_m^{(l)}$
  $\delta Z_M^{(l)} = \frac{1}{2}l(l+1)\ \left(M_\lambda^2/b\right)\delta Z_m^{(l)}$

Relations for β functions in SQCD

- Quantization: Two anomalies

$$S(\Gamma) = r_{\mathsf{ax}}\Delta_{\mathsf{ax}} + r_{\mathsf{Susy}}\Delta_{\mathsf{Susy}}$$

Susy parameters:

$$\begin{aligned}\beta_{g^2} &= g^4(\beta^{(1)} + 8r_{\mathsf{ax}}\gamma)F(g^2)\\ \beta_m &= 2\gamma\end{aligned}$$

Soft parameters:

$$\begin{aligned}\beta_{M_\lambda} &= M_\lambda g^2 \partial_{g^2}\frac{\beta_{g^2}}{g^2}\\ \beta_M &= M\Big(\ldots + 4r_{\mathsf{ax}}g^2F(g^2)g^2\partial_{g^2}\gamma\Big)\\ \beta_b &= 2b(\gamma + 2\frac{M_\lambda m}{b}g^2\partial_{g^2}\gamma)\end{aligned}$$

- β_{g^2}, β_{M_λ} only independent at 1-loop

- $\Rightarrow$ β functions depend on: $\beta^{(1)}$,γ,r_{ax}, $F(g^2) = 1 + g^2 r_{\mathsf{Susy}} + \ldots$

Conclusions

Symmetries of the MSSM

STI, WI
in presence of $(\hat{\Phi}+\hat{\mathrm{v}})$, $(a,\chi,f+f_0)$

Consequences

MSSM IR-finite, mult. renormalizable

sym.-restoring c.t.s uniquely determined

Applications

$\partial_\xi \tan\beta \longrightarrow \overline{DR}$-scheme best

Non-renormalization theorems accessible

Symmetries of the MSSM

Symmetry content:

- SU(2)×U(1) gauge inv. , Susy
- $\{Q, \bar{Q}\} = 2\gamma^\mu P_\mu +$ gauge transf.
- Transformations receive quantum corrections

BRS formulation: Ghosts c_a, ϵ, ξ^μ,

$$\begin{aligned} s\varphi_i &= (c_a \delta_a^{\text{gauge}} + \epsilon Q - \xi^\mu P_\mu)\varphi_i, \\ s(\text{Ghost}) &= \sim \text{structure constants} \end{aligned}$$

$$\begin{aligned} s\mathcal{L} &= 0 && \Leftrightarrow \mathcal{L} \text{ BRS-invariant} \\ \mathcal{L}_{\text{fix}} &= s(\ldots) && \Leftrightarrow \mathcal{L}_{\text{fix}} \text{ unphysical} \end{aligned}$$

1. BRS Invariance on the quantum level:

STI $S(\Gamma) = 0$

$\Rightarrow$ structure constants; g. inv. , Susy of Γ;
unphysical nature of $\mathcal{L}_{\text{fix}} \longrightarrow$ unitarity

Symmetries of the MSSM

STI $\Leftrightarrow$ gauge group SU(2)$\times$U(1)

- SU(2)-couplings fixed by structure constants
- U(1)$_Y$-couplings not fixed

(e.g. δY_i compatible with STI)

2. WI for U(1)$_Y$:

$$\partial^\mu \frac{\delta\Gamma}{\delta V'^\mu} = -i \sum \frac{Y_i}{2} \; \varphi_i \frac{\delta\Gamma}{\varphi_i} + \text{g.fix.}$$

$\Rightarrow$ U(1)$_Y$-couplings fixed

Symmetries of the MSSM

Symmetry breaking :

via external fields with constant shifts

Susy mult.	$(a, \chi, f + f_0) \rightarrow (0, 0, f_0)$
SU(2)×U(1)-mult.	$(\hat{\Phi} + \hat{\mathrm{v}}) \rightarrow \hat{\mathrm{v}}$

3. Include ext. fields in STI, WI $\longrightarrow$

Susy : softly broken,

G. inv. : spontaneously broken

Symmetries of the MSSM

- Exact multiplet structure of $\hat{\Phi} + \hat{\mathrm{v}}$ determined by $\mathcal{L}_{\mathrm{fix}}$

- Differentiate explicit / spontaneous symmetry breaking

Explicit	Spontaneous breaking
$s(a, \chi, f + f_0)$ $= \epsilon Q(a, \chi, f + f_0)$	$s\hat{\Phi} = \hat{q},\ \ s\hat{q} = 0$
$S(\Gamma\|_{a=\chi=f=0}) = \epsilon f_0 \Delta$	$S(\Gamma\|_{\hat{\Phi}=\hat{q}=0}) = 0$
many new couplings	$\hat{\Phi}, \hat{q}$ only appear in $s(\ldots)$ $\Rightarrow$ no new phys. param.