$\sigma(e^+e^- \to$ hadrons) and the Heavy Quark Masses

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Introduction

SVZ sum rules

$\Upsilon(1S)$ system

Conclusions and outlook

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Introduction

- Desirable: precise knowledge of quark masses

  here: m_c, m_b (m_t: see talk by A. Penin)

- SVZ sum rule: low-order moments $\mathcal{M}_n^{\text{th}} = \frac{12\pi^2}{n!}\left(\frac{\text{d}}{\text{d}q^2}\right)^n \Pi(q^2)\Big|_{q^2=0}$

  (m_c, m_b) fixed-order calculation

  [Shifman et al.'79] precise of $R(s)$ necessary $R(s) = \frac{\sigma(e^+e^-\to\text{hadrons})}{\sigma_{\text{pt}}} = 12\pi\text{Im}\Pi$

- $\Upsilon(1S)$-system: lowest $(b\bar{b})$ bound state, non-relativistic system; α_s and v small $\longrightarrow$ potential NRQCD

- not: Υ sum rules

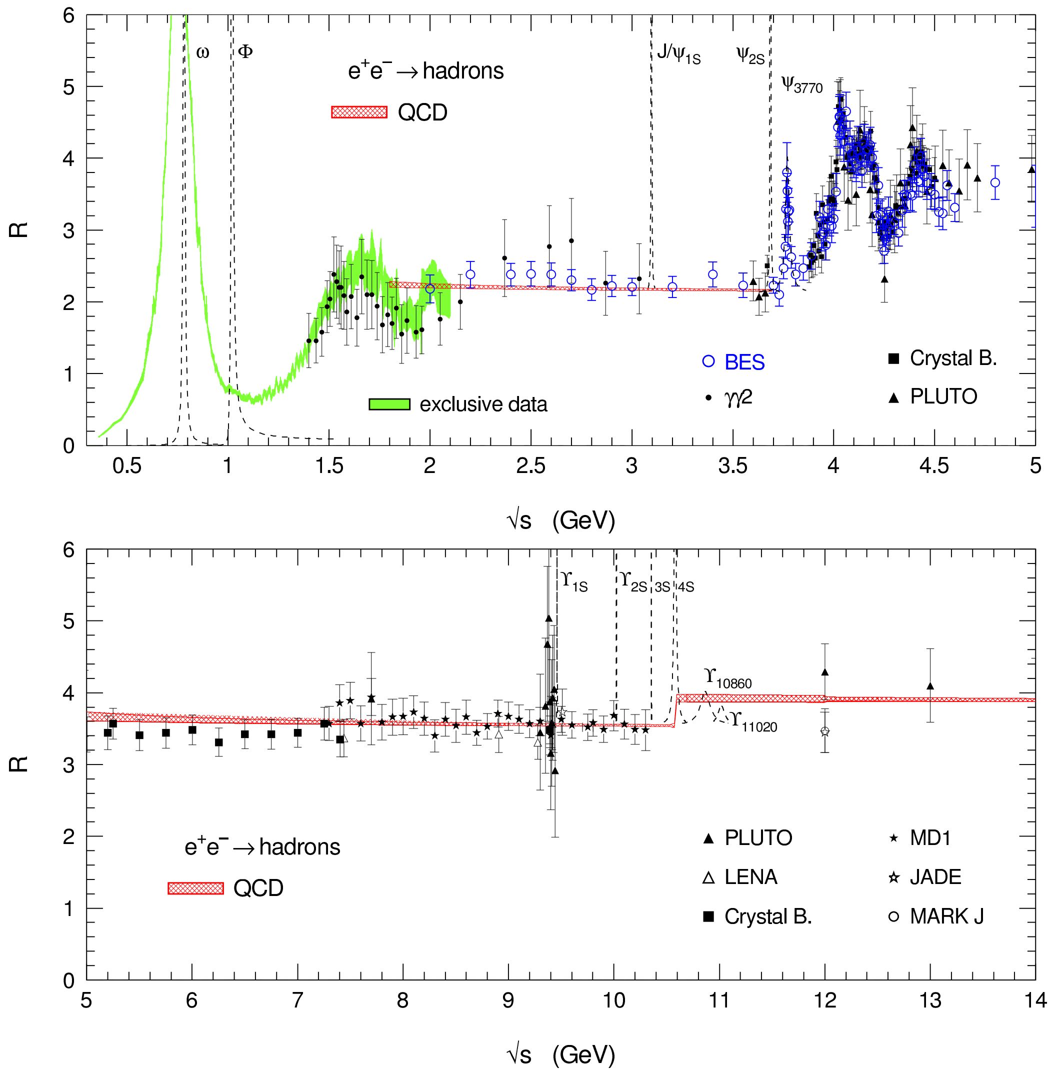
- pole mass: in general large perturbative corrections

  $\overline{\text{MS}}$ mass: in general small perturbative corrections

  ➪ use pole mass (or related one) at most in intermediate steps

  ➪ final result: $\overline{\text{MS}}$ mass

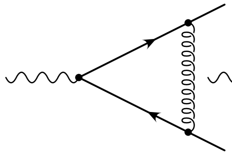
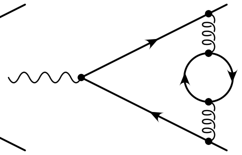
$R(s)$ — Experiment



- inclusive quantity
- experimentally accessible for $\sqrt{s} =$ $2m_\pi \ldots 3.6$ GeV$\ldots$ $\ldots 10.52$ GeV$\ldots M_Z \ldots$
- outside resonances; above threshold $\longrightarrow$ prediction from perturbative QCD possible

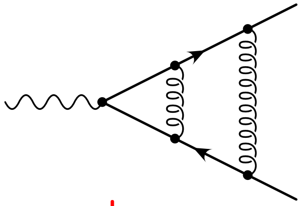
[Davier,Eidelman,Höcker,Zhang'02]

$R(s)$ — Theory

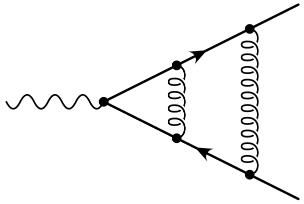
$\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2 n_l)$:
"direct" calculation

[Källen, Sabry'55; Hoang, Kühn, Teubner'95]



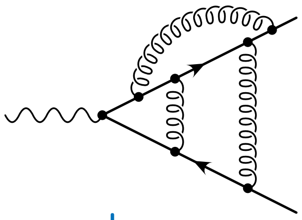
massless quarks:
integration-by-parts,
recurrence relations,
3-loop propagator-type integrals

[Chetyrkin, Kataev, Tkachov'79; Dine, Sapirstein'79; Celmaster, Gonsalves'80]



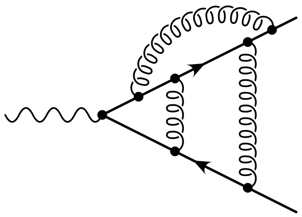
massive quarks: Padé approximation,
conformal mapping,
semi-numerical result

[Chetyrkin, Kühn, MS'95'96]



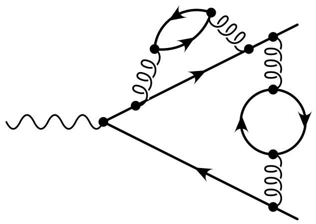
massless quarks: infra-red-rearrangement (IRR)

[Surguladze, Samuel'91; Gorishny, Kataev, Larin'91; Chetyrkin'97]



massive quarks,
$\alpha_s^3 m^2/s, \alpha_s^3 m^4/s^2$: OPE, RGE

[Chetyrkin, Kühn'90; Chetyrkin, Harlander, Kühn'00]

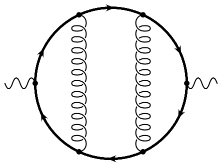


massless quarks, $\alpha_s^4 n_f^2$:
no recurrence relations

[Baikov, Chetyrkin, Kühn'01]

SVZ-SR: Determination of m_c

Low-order moments $\longrightarrow$ no resummation $\longrightarrow$ direct determination of $\overline{\text{MS}}$ mass



$\boxed{\text{Theory:}}$ $\mathcal{M}_n^{\text{th}} = \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)\Big|_{q^2=0} = \frac{9}{4}Q_c^2\left(\frac{1}{4m_c^2}\right)^n \bar{C}_n$

$\bar{C}_1, \ldots, \bar{C}_8$ known analytically up to order α_s^2 [Chetyrkin,Kühn,MS'95'96]

$\boxed{\text{Experiment:}}$ $\mathcal{M}_n^{\text{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s) = \mathcal{M}_n^{\text{exp,res}} + \mathcal{M}_n^{\text{exp,cc}} + \mathcal{M}_n^{\text{exp,cont}}$

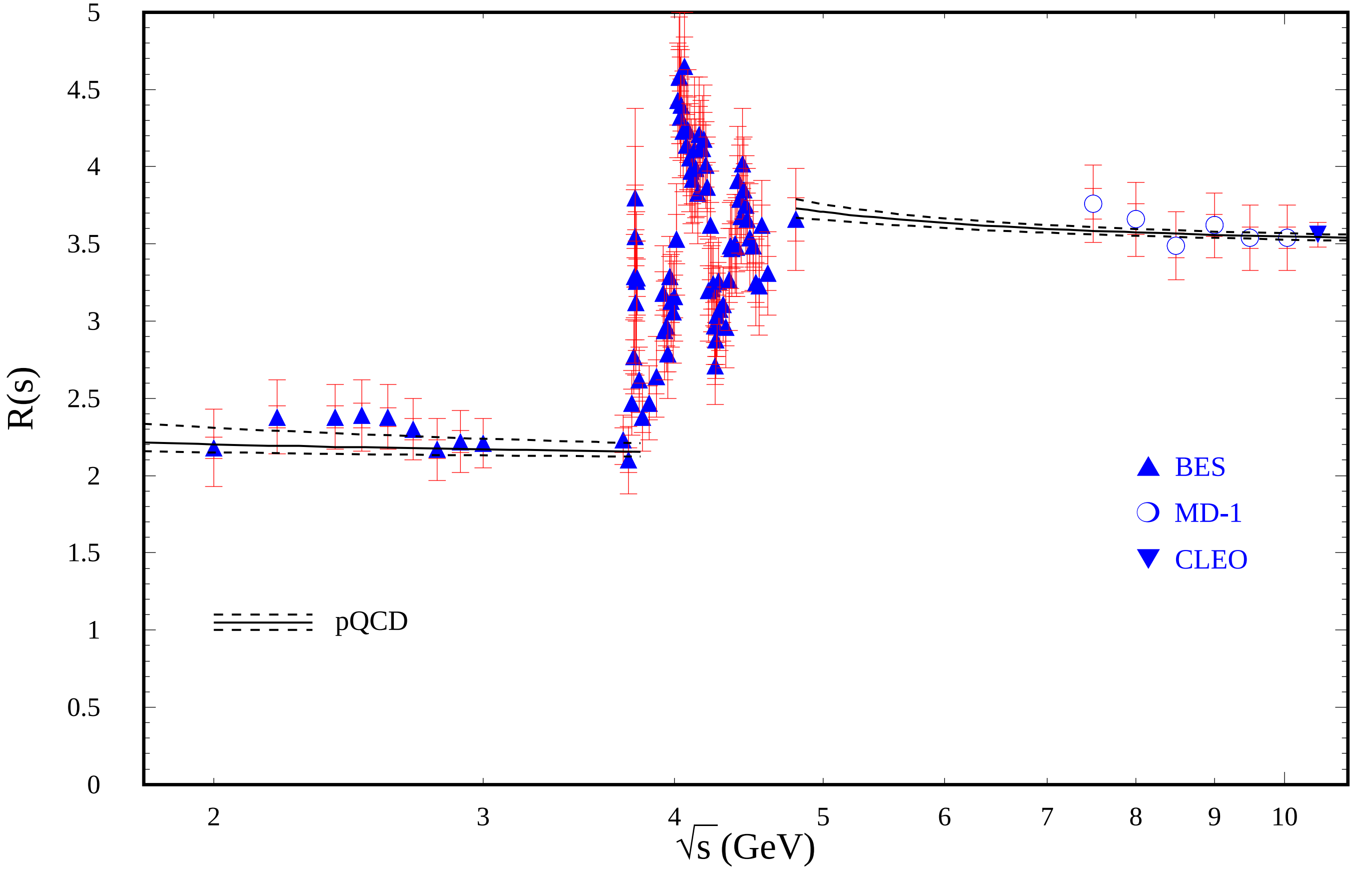
resonances (J/Ψ, Ψ')

below 4.8 GeV (BES)

above 4.8 GeV (theory)

n	$\mathcal{M}_n^{\text{exp,res}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{exp,cc}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{cont}}$ $\times 10^{(n-1)}$
1	0.1114(82)	0.0313(15)	0.0638(10)
2	0.1096(79)	0.0174(8)	0.0142(3)
3	0.1094(79)	0.0099(5)	0.0042(1)
4	0.1105(79)	0.0057(3)	0.0014(0)

$R(s)$ — Experiment vs Theory



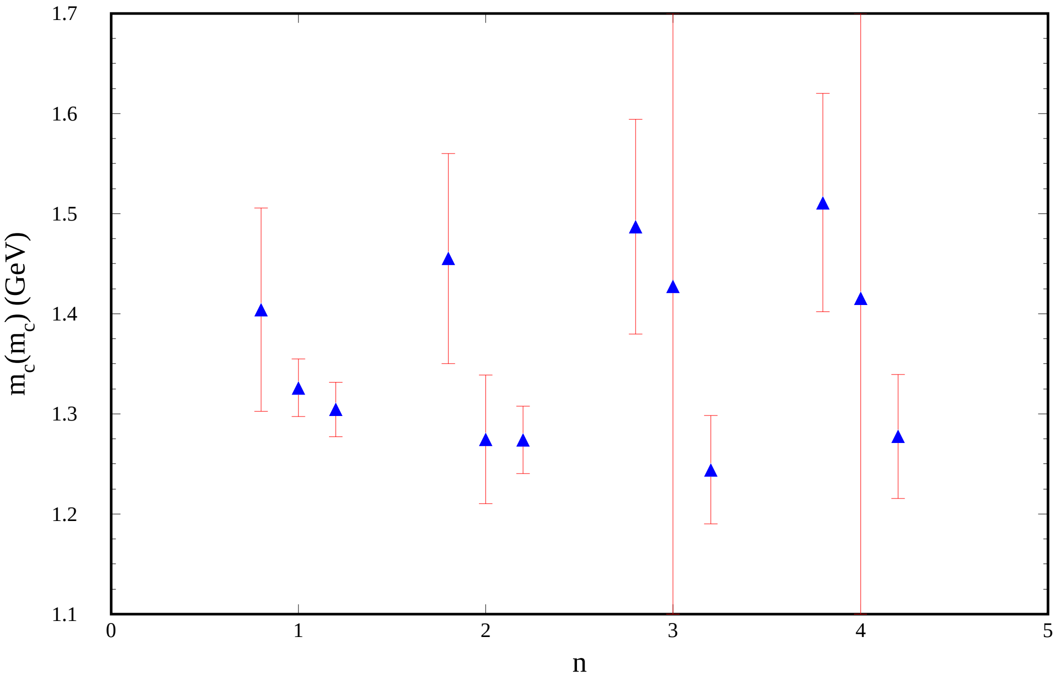
experiment	energy (GeV)	date	sys.error
BES	2 – 5	2001	4%
MD-1	7.2 – 10.34	1996	4%
CLEO	10.52	1998	2%

SVZ-SR: Determination of m_c

$$m_c(\mu) = \frac{1}{2}\left(\frac{\bar{C}_n(\log(m_c))}{\mathcal{M}_n^{\text{exp}}}\right)^{1/(2n)}$$

n	1	2	3	4
$m_c(3\text{ GeV})$	1.027(30)	0.994(37)	0.961(59)	0.997(67)
$m_c(m_c)$	1.304(27)	1.274(34)	1.244(54)	1.277(62)

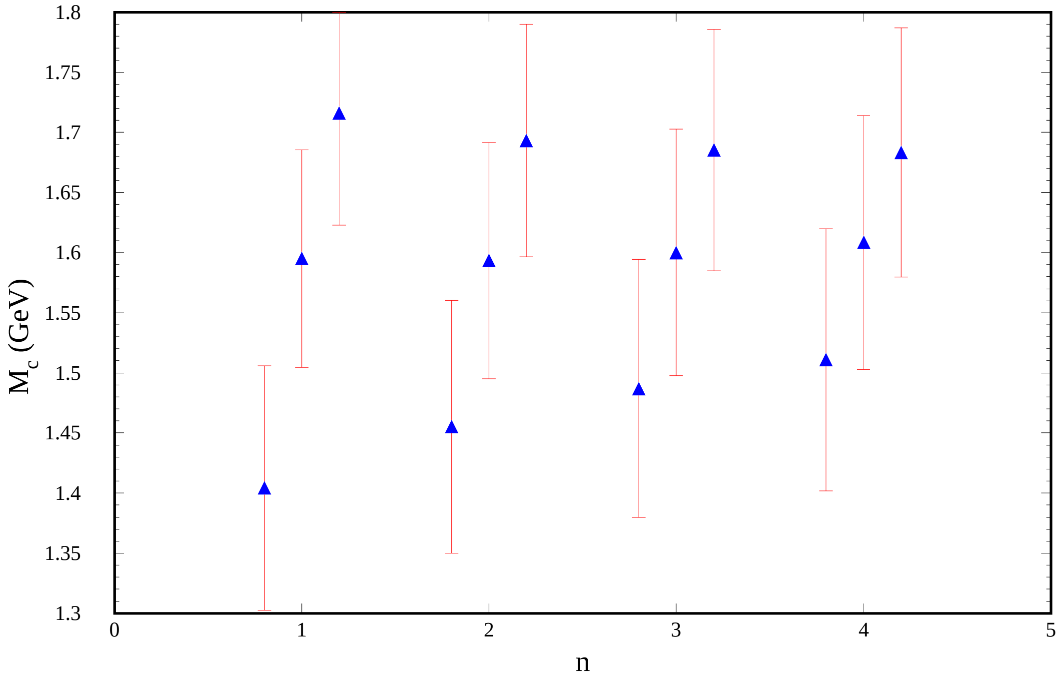
$m_c(m_c)$ for n=1,2,3 and 4 in LO, NLO and NNLO



SVZ-SR: Determination of m_c

For comparison: analysis using pole mass M_c

M_c for n=1,2,3 and 4 in LO, NLO and NNLO



SVZ-SR: Determination of m_c

use result for $n = 1$ $\boxed{m_c(m_c) = 1.304(27)\ \text{GeV}}$ [Kühn,MS'01]

reason:

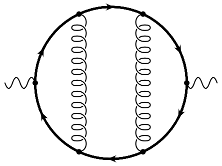
- condensates unimportant
- fixed-order result in α_s sufficient, no resummation in $1/v$ necessary
- dominant error from experiment ($\Gamma_{ee}(J/\Psi), \ldots$)
  small error from variation of μ and α_s

conversion to pole mass:

$$M_c^{(2-\text{loop})} = 1.514(34)\ \text{GeV}$$
$$M_c^{(3-\text{loop})} = 1.691(35)\ \text{GeV}$$

➪ dramatic shift! $\overline{\text{MS}}$ mass more stable

SVZ-SR: Determination of m_b



Theory: $\mathcal{M}_n^{\text{th}} = \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_b(q^2)\Big|_{q^2=0} = \frac{9}{4}Q_b^2\left(\frac{1}{4m_b^2}\right)^n \bar{C}_n$

$\bar{C}_1, \ldots, \bar{C}_8$ known analytically up to order α_s^2 [Chetyrkin,Kühn,MS'95'96]

Experiment: $\mathcal{M}_n^{\text{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}} R_b(s) = \mathcal{M}_n^{\text{exp,res}} + \mathcal{M}_n^{\text{exp,lin}} + \mathcal{M}_n^{\text{exp,cont}}$

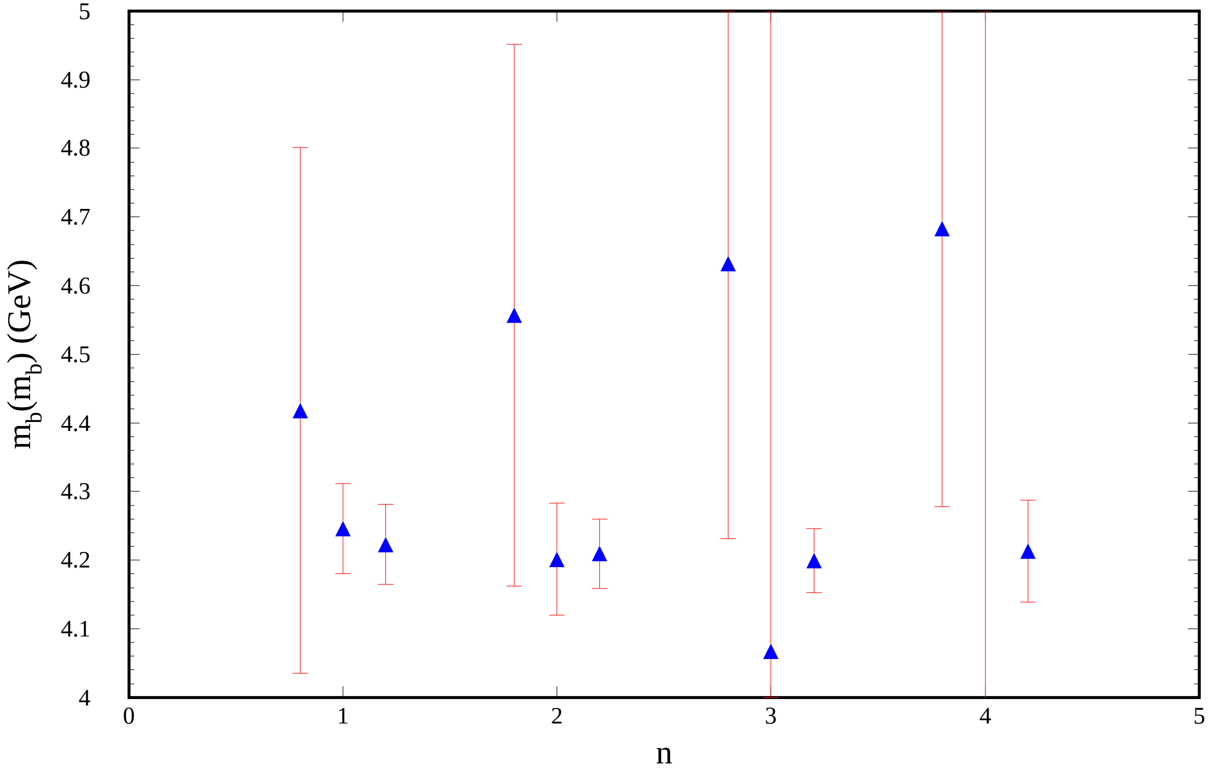
resonances ($\Upsilon(1S),\ldots,\Upsilon(6S)$)

$\mathcal{M}_n^{\text{exp,lin}}$: linear interpolation; small contribution; $\delta\mathcal{M}_n^{\text{exp,lin}} = \mathcal{M}_n^{\text{exp,lin}}$

continuum starts at 11.2 GeV

SVZ-SR: Determination of m_b

$m_b(m_b)$ for n=1,2,3 and 4 in LO, NLO and NNLO

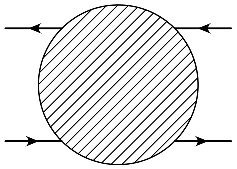


$$\boxed{m_b(m_b) = 4.191(51)\ \text{GeV}}$$

[Kühn,MS'01]

$$\begin{aligned} M_b^{(2-\text{loop})} &= 4.651(57)\ \text{GeV} \\ M_b^{(3-\text{loop})} &= 4.819(57)\ \text{GeV} \end{aligned}$$

Potential Non-relativistiv QCD



QCD $\underset{m}{\longrightarrow}$ NRQCD $\longrightarrow$ potential NRQCD

potential quarks: $(p_0, |\vec{p}|) \sim (mv^2, mv)$

ultra-soft gluons: $(p_0, |\vec{p}|) \sim (mv^2, mv^2)$

[Bodwin,Braaten,Lepage'95; Pineda,Soto'98]

- Hamiltonian known up to N^{3}LO [Kniehl,Penin,Smirnov,MS'01]
- use time-independent perturbation theory to solve Schrödinger equation around Coulomb solution
  $+$ retardation effects

$$E_n^{\text{p.t.}} = E_n^C + \delta E_n^{(1)} + \delta E_n^{(2)} + \delta E_n^{(3)}$$

$$E_n^C = -\frac{C_F^2 \alpha_s^2 m_b}{4n^2}$$

Determination of m_b: $\Upsilon(1S)$ system

$$\boxed{M_{\Upsilon(1S)} = 2m_b + E_1^{\text{p.t.}} + \delta^{\text{n.p.}} E_1}$$

$M_{\Upsilon(1S)} = 9.46030(26)$ GeV $\qquad \delta^{\text{n.p.}} E_1 = 60 \pm 30$ MeV

$E_1^{\text{p.t.}}$: perturbative contribution to energy level

$E_1^{\text{p.t.}}$ depends on m_b, α_s, μ
$L_\mu = \ln(\mu/(C_F \alpha_s m_b))$
β_0, β_1, β_2: QCD β function
C_A, C_F, T_F: colour factors
a_1, a_2, a_3: pert. corr. in V_{Coulomb}
S: Spin

$$\delta E_1^{(1)} = E_1^C \frac{\alpha_s}{\pi} \left[4\beta_0 L_\mu + 4\beta_0 + \frac{a_1}{2}\right]$$

$$\delta E_1^{(2)} = E_1^C \left(\frac{\alpha_s}{\pi}\right)^2 \Big[12\beta_0^2 L_\mu^2 + \left(16\beta_0^2 + 3a_1\beta_0 + 4\beta_1\right) L_\mu + \left(4 + \frac{2\pi^2}{3} + 8\zeta(3)\right)\beta_0^2$$
$$+2a_1\beta_0 + 4\beta_1 + \frac{a_1^2}{16} + \frac{a_2}{8} + \pi^2 C_A C_F + \left(\frac{21\pi^2}{16} - \frac{2\pi^2}{3} S(S+1)\right) C_F^2\Big]$$

[Pineda,Yndurain'98; Penin,Pivovarov'98; Melnikov,Yelkhovsky'99]

$$\delta E_1^{(3)} = \delta E_1^{(3)}\Big|_{\beta(\alpha_s)=0} + \delta E_1^{(3)}\Big|_{\beta(\alpha_s)}$$

$$\delta E_1^{(3)}\Big|_{\beta(\alpha_s)=0} = -E_1^C \frac{\alpha_s^3}{\pi}\Bigg\{-\frac{a_1a_2+a_3}{32\pi^2} + \left[-\frac{C_AC_F}{2} + \left(-\frac{19}{16} + \frac{S(S+1)}{2}\right)C_F^2\right]a_1 + \left[-\frac{1}{36} + \frac{\ln 2}{6} + \frac{L_{\alpha_s}}{6}\right]C_A^3$$
$$+\left[-\frac{49}{36} + \frac{4}{3}(\ln 2 + L_{\alpha_s})\right]C_A^2C_F + \left[-\frac{5}{72} + \frac{10}{3}\ln 2 + \frac{37}{6}L_{\alpha_s} + \left(\frac{85}{54} - \frac{7}{6}L_{\alpha_s}\right)S(S+1)\right]C_AC_F^2$$
$$+\left[\frac{50}{9} + \frac{8}{3}\ln 2 + 3L_{\alpha_s} - \frac{S(S+1)}{3}\right]C_F^3 + \left[-\frac{32}{15} + 2\ln 2 + (1-\ln 2)S(S+1)\right]C_F^2T_F$$
$$+\frac{49C_AC_FT_Fn_l}{36} + \left[\frac{11}{18} - \frac{10}{27}S(S+1)\right]C_F^2T_Fn_l + \frac{2}{3}C_F^3L_1^E\Bigg\}$$

[Kniehl,Penin,Smirnov,MS'01]

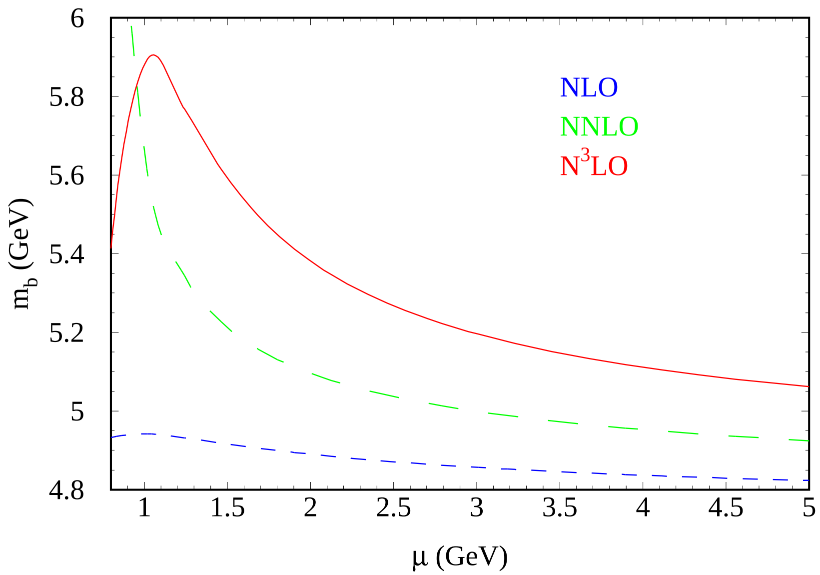
$$\delta E_1^{(3)}\Big|_{\beta(\alpha_s)} = E_n^C\left(\frac{\alpha_s}{\pi}\right)^3\Bigg\{32\beta_0^3L_\mu^3 + \left[40\beta_0^3 + 12a_1\beta_0^2 + 28\beta_1\beta_0\right]L_\mu^2 + \Bigg[\left(\frac{16\pi^2}{3} + 64\zeta(3)\right)\beta_0^3 + 10a_1\beta_0^2$$
$$+\left(40\beta_1 + \frac{a_1^2}{2} + a_2 + 8\pi^2C_AC_F + \left(\frac{21\pi^2}{2} - \frac{16\pi^2}{3}S(S+1)\right)C_F^2\right)\beta_0 + 3a_1\beta_1 + 4\beta_2\Bigg]L_\mu$$
$$+\left(-8 + 4\pi^2 + \frac{2\pi^4}{45} + 64\zeta(3) - 8\pi^2\zeta(3) + 96\zeta(5)\right)\beta_0^3 + \left(\frac{2\pi^2}{3} + 8\zeta(3)\right)a_1\beta_0^2$$
$$+\left(\left(8 + \frac{7\pi^2}{3} + 16\zeta(3)\right)\beta_1 - \frac{a_1^2}{8} + \frac{3}{4}a_2 + \left(6\pi^2 - \frac{2\pi^4}{3}\right)C_AC_F\right.$$
$$\left.+\left(8\pi^2 - \frac{4\pi^4}{3} + \left(-\frac{4\pi^2}{3} + \frac{4\pi^4}{9}\right)S(S+1)\right)C_F^2\right)\beta_0 + 2a_1\beta_1 + 4\beta_2\Bigg\}$$

[Penin,MS'02]

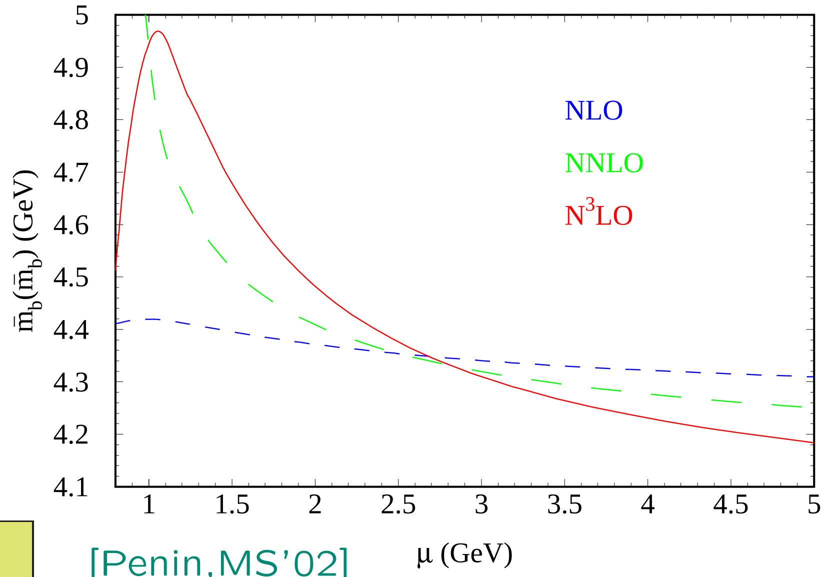
$L_{\alpha_s} = -\ln(C_F\alpha_s),\ L_\mu = \ln(\mu/(C_F\alpha_s m_b))$

Determination of m_b: $\Upsilon(1S)$ system

Solve $M_{\Upsilon(1S)} = 2m_b + E_1^{\text{p.t.}} + \delta^{\text{n.p.}}E_1$ for m_b (pole mass) at NLO, NNLO and N^{3}LO for $\alpha_s(M_Z) = 0.1185$ and fixed scale μ



pole mass $\longrightarrow$ $\overline{\text{MS}}$ mass: excellent convergence around $\mu \approx 2.7$ GeV, close to $\mu^{\text{soft}} = C_F m_b \alpha_s$



$\boxed{\overline{m}_b(\overline{m}_b) = 4.346 \pm 0.070 \text{ GeV}}$

[Penin,MS'02]

Comparison: m_c

Method	$m_c(m_c)$ (GeV)	
SR, NRQCD, NNLO	1.23(9)	[Eidemüller,Jamin'01]
Chauchy SR	1.37(9)	[Penarrocha,Schilcher'01]
Lattice, quenched	1.26(4)(12)	[Becirevic et al.'01]
Lattice, quenched, prel.	1.314(40)(20)(7)	[Rolf,Sint'01]
SVZ-SR, NNLO	1.304(27)	[Kühn,MS'01]
SR, NNLO, $q^2 \neq 0$, high moments, NO resummation	1.275(15)	[Ioffe,Zyablyuk'02]
SR, NRQCD, NNLO, $q^2 \neq 0$	1.19(11)	[Eidemüller'02]

Comparison: m_b

Method	$m_b(m_b)$ (GeV)	
Υ-SR, NNLO	4.21(11)	[Penin,Pivovarov'99]
Υ-SR, NNLO	4.20(10)	[Melnikov,Yelkhovsky'99]
Υ-SR, NNLO	4.25(8)	[Beneke,Signer'99]
Υ-SR, NNLO	4.17(5)	[Hoang'00]
Υ-SR, NNLO, $q^2 \neq 0$	4.24(10)	[Eidemüller'02]
SVZ-SR, NNLO	4.191(51)	[Kühn,MS'01]
$\Upsilon(1S)$, NNLO	4.210(90)(25)	[Pineda'01]
$\Upsilon(1S)$, NNNLO	4.346(70)	[Penin,MS'02]

Conclusions/Outlook

SVZ-SR [$m_c(m_c) = 1.304(27)$ GeV, $m_b(m_b) = 4.191(51)$ GeV]

- direct determination of $\overline{\text{MS}}$ mass
- NNLO
- very good agreement with lattice calculation
- error dominated by experiment

$\Upsilon(1S)$ system [$m_b(m_b) = 4.346(70)$ GeV]

- N^{3}LO
- relatively large non-perturbative contribution

Future: it would be nice to have a

- measurement of $R(s)$ around charm/bottom threshold region with 2% accuracy
- measurement of $\Gamma_{ee}(J/\Psi)$ and $\Gamma_{ee}(\Psi')$ with 2% accuracy

(... possibly done by CLEO (?) ...)

➪ δm_c and δm_b can significantly be reduced