Twist-3 effects for polarized virtual photon structure function g_2^γ

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1. Introduction

- **Spin-dependent nucleon structure function** $g_1^{p(n)}$
  - $\int_0^1 dx\,(g_1^p - g_1^n) = \frac{1}{6}\frac{G_A}{G_V}\left(1 - \frac{\alpha_s(Q^2)}{\pi} + \cdots\right)$

    (Bjorken sum rule)
  - **QCD axial anomaly** Kodaira (1980)
- **Spin-dependent nucleon structure function** $g_2^{p(n)}$
  - $\int_0^1 dx g_2^{p(n)} = 0$

    (Burkhardt-Cottingham (BC) sum rule)
  - **Twist-2 and twist-3 operators both contribute in the leading order of** $1/Q^2$
  - **Twist-2 part (Wandzura-Wilczek relation)**

$$g_2^{\mathrm{tw.2}} = g_2^{\mathrm{WW}}(x, Q^2) \equiv -g_1(x, Q^2) + \int_x^1 \frac{dy}{y} g_1(y, Q^2)$$

  - **Twist-3 part** $\overline{g}_2 = g_2 - g_2^{\mathrm{tw.2}}$

    The data show that $\overline{g}_2$ for nucleon appears to be small

    E143, E155
  - **Analysis of** $\overline{g}_2$ $\cdots$ **very complicated**

The number of participating twist-3 operators grows with spin (moment of $\overline{g}_2$) and these operators mix through renormalization.

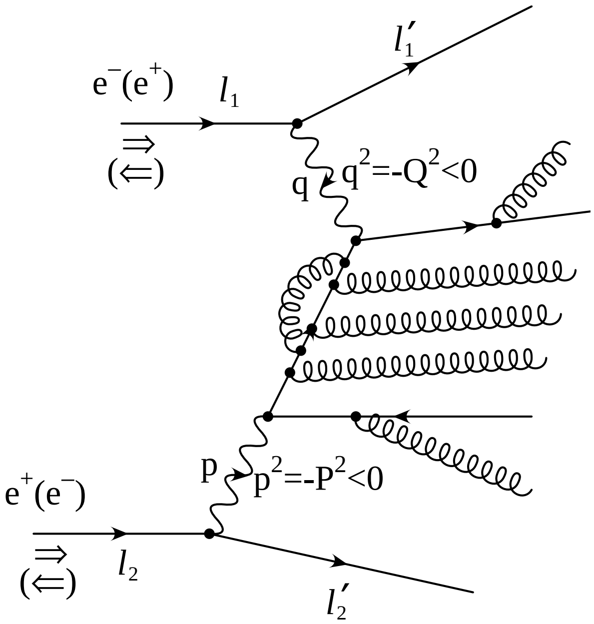
The analysis of the flavor nonsinglet of $\overline{g}_2$ becomes simple for large N_C Ali, Braun, Hiller (1991),

- **Polarized virtual photon structure functions**

g_1^γ and g_2^γ

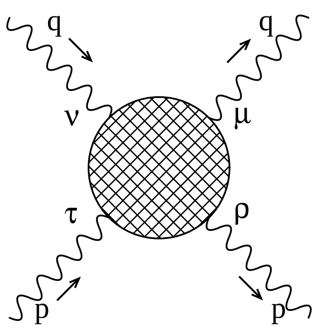
obtained from

- **the resolved photon process in the polarized version of of the DESY ep collider HERA**
- **polarized e^+e^- collision experiments**



The virtual photon-photon forward scattering

$$\gamma(q) + \gamma(p) \rightarrow \gamma(q) + \gamma(p)$$



The antisymetric part of the structure tensor

$$W^A_{\mu\nu\rho\tau} = \frac{1}{(p\cdot q)^2}[(I_-)_{\mu\nu\rho\tau}\ g_1^\gamma - (J_-)_{\mu\nu\rho\tau}\ g_2^\gamma]$$

with

$$(I_-)_{\mu\nu\rho\tau} \equiv p\cdot q\, \epsilon_{\mu\nu\lambda\sigma}\epsilon_{\rho\tau}{}^{\sigma\beta} q^\lambda p_\beta$$

$$(J_-)_{\mu\nu\rho\tau} \equiv \epsilon_{\mu\nu\lambda\sigma}\epsilon_{\rho\tau\alpha\beta} q^\lambda p^\sigma q^\alpha p^\beta - p\cdot q\, \epsilon_{\mu\nu\lambda\sigma}\epsilon_{\rho\tau}{}^{\sigma\beta} q^\lambda p_\beta$$

⊙ We analyse $g_2^\gamma(x, Q^2, P^2)$

– only exists for a virtual photon target $P^2 \neq 0$

– is studied in the kinematical region

$$\Lambda^2 \ll P^2 \ll Q^2$$

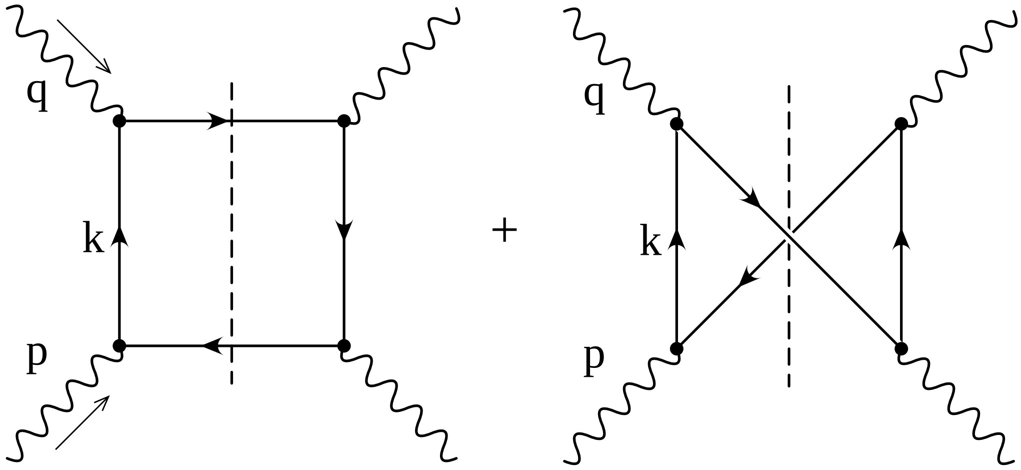
Λ : the QCD scale parameter

[Questions]

1. Does the BC sum rule holds for g_2^γ ?
2. Are there the twist-3 contributions to g_2^γ?
   If so
3. Are they small or sizable?
4. Does the WW relation also holds for g_2^γ ?
   and can the twist-3 part be extracted?
5. Any complication in the QCD analysis for g_2^γ ?

2. g_2^γ in parton model

⊙ **The box-diagram contributions (massless quark-loop)**



Neglecting the power corrections of P^2/Q^2, we obtain

$$g_1^{\gamma(\mathrm{Box})}(x,Q^2,P^2) = \frac{3\alpha}{\pi}N_f\langle e^4\rangle\left[(2x-1)\ln\frac{Q^2}{P^2} -2(2x-1)(\ln x+1)\right]$$

$$g_2^{\gamma(\mathrm{Box})}(x,Q^2,P^2) = \frac{3\alpha}{\pi}N_f\langle e^4\rangle\left[-(2x-1)\ln\frac{Q^2}{P^2} +2(2x-1)\ln x+6x-4\right]$$

where

$x = Q^2/(2p\cdot q)$, $\quad \alpha = e^2/4\pi$, $\quad \langle e^4\rangle = \Sigma_{i=1}^{N_f} e_i^4/N_f$,

N_f : the number of active quark flavors

⊙ $g_2^{\gamma(\mathrm{Box})}$ **satisfies the BC sum rule**

$$\int_0^1 dx g_2^{\gamma(\mathrm{Box})}(x, Q^2, P^2) = 0$$

From the OPE analysis, we will see that the BC sum rule for g_2^γ generally holds in the deep-inelastic region $Q^2 \gg P^2$

⊙ **Apply the WW relation to** $g_1^{\gamma(\mathrm{Box})}$ **and** $g_2^{\gamma(\mathrm{Box})}$

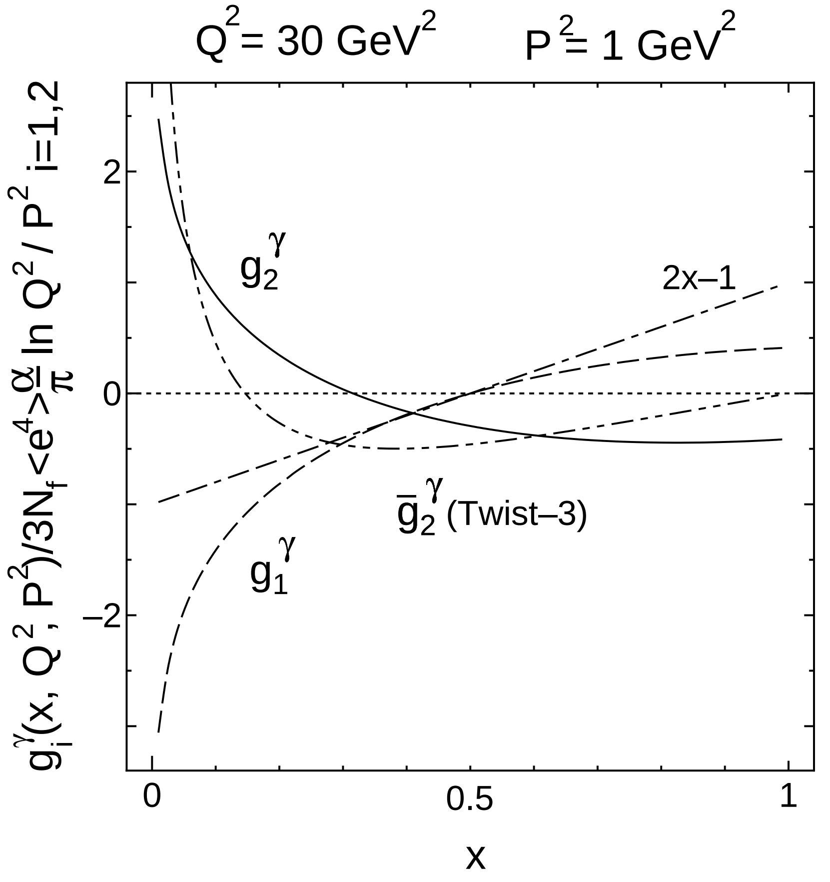
$$g_2^{\gamma\mathrm{WW},(\mathrm{Box})}(x, Q^2, P^2) \equiv -g_1^{\gamma(\mathrm{Box})}(x, Q^2, P^2) + \int_x^1 \frac{dy}{y} g_1^{\gamma(\mathrm{Box})}(y, Q^2, P^2)$$

$$\overline{g}_2^{\gamma(\mathrm{Box})} \equiv g_2^{\gamma(\mathrm{Box})} - g_2^{\gamma\mathrm{WW},(\mathrm{Box})}$$

$$= \frac{3\alpha}{\pi} N_f \langle e^4 \rangle \left[(2x - 2 - \ln x) \ln \frac{Q^2}{P^2} -2(2x-1)\ln x + 2(x-1) + \ln^2 x\right]$$

We will see later that

$g_2^{\gamma\mathrm{WW},(\mathrm{Box})}$ **contains the twist-2 contribution**
$\overline{g}_2^{\gamma(\mathrm{Box})}$ **is indeed the twist-3 contribution.**



$\bar{g}_2^{\gamma(\mathrm{Box})}$**(Twist-3) is comparable in magnitude with** $g_2^{\gamma(\mathrm{Box})}$ **for large regions of** x

3. OPE and pure QED effects on g_2^γ

♣ The OPE reads

$$
\begin{aligned}
i \int d^4x e^{iq\cdot x} T(J_\mu(x)J_\nu(0))^A \\
&= -i\epsilon_{\mu\nu\lambda\sigma} q^\lambda \sum_{n=1,3,\cdots} \left(\frac{2}{Q^2}\right)^n q_{\mu_1}\cdots q_{\mu_{n-1}} \\
&\times \left\{ \sum_i E^n_{(2)i} R^{\sigma\mu_1\cdots\mu_{n-1}}_{(2)i} + \sum_i E^n_{(3)i} R^{\sigma\mu_1\cdots\mu_{n-1}}_{(3)i} \right\}
\end{aligned}
$$

$R^n_{(2)i}$: twist-2 operators
$\sigma\mu_1\cdots\mu_{n-1}$ totally symmetric

$R^n_{(3)i}$: twist-3 operators
$\mu_1\cdots\mu_{n-1}$ totally symmetric
but antisymmetric under $\sigma \leftrightarrow \mu_i$

$E^n_{(2)i}$, $E^n_{(3)i}$: coefficient functions

- The "matrix elements" of $R^n_{(2)i}$ and $R^n_{(3)i}$

$$
\langle 0|T(A_\rho(-p)R^{\sigma\mu_1\cdots\mu_{n-1}}_{(2)i}A_\tau(p))|0\rangle_{\text{Amp}} = -ia^n_{(2)i}\epsilon_{\rho\tau\alpha}{}^{\{\sigma}p^{\mu_1}\cdots p^{\mu_{n-1}\}}p^\alpha
$$

$$
\langle 0|T(A_\rho(-p)R^{\sigma\mu_1\cdots\mu_{n-1}}_{(3)i}A_\tau(p))|0\rangle_{\text{Amp}} = -ia^n_{(3)i}\epsilon_{\rho\tau\alpha}{}^{[\sigma,}p^{\{\mu_1]}\cdots p^{\mu_{n-1}\}}p^\alpha
$$

with

$$
\begin{aligned}
\epsilon_{\rho\tau\alpha}{}^{[\sigma,}p^{\{\mu_1]}\cdots p^{\mu_{n-1}\}} &= \frac{n-1}{n}\epsilon_{\rho\tau\alpha}{}^{\sigma}p^{\mu_1}\cdots p^{\mu_{n-1}} \\
&- \frac{1}{n}\sum_{j=1}^{n-1}\epsilon_{\rho\tau\alpha}{}^{\mu_j}p^{\mu_1}\cdots p^{\sigma}\cdots p^{\mu_{n-1}}
\end{aligned}
$$

♣ The moment sum rules for g_1^γ and g_2^γ

$$\int_0^1 dx x^{n-1} g_1^\gamma(x, Q^2, P^2) = \sum_i a^n_{(2)i} E^n_{(2)i}(Q^2)$$

$$\int_0^1 dx x^{n-1} g_2^\gamma(x, Q^2, P^2) = \frac{n-1}{n}\Big\{-\sum_i a^n_{(2)i} E^n_{(2)i}(Q^2) + \sum_i a^n_{(3)i} E^n_{(3)i}(Q^2)\Big\}$$

♣ The OPE analysis leads to

1. The BC sum rule holds for g_2^γ

2. The twist-2 contribution to g_2^γ is expressed by the WW relation

$$-\frac{n-1}{n}\sum_i a^n_{(2)i} E^n_{(2)i}(Q^2) = \int_0^1 dx x^{n-1} g_2^{\gamma \mathrm{WW}}(x, Q^2, P^2)$$

with

$$g_2^{\gamma \mathrm{WW}}(x, Q^2, P^2) \equiv -g_1^\gamma(x, Q^2, P^2) + \int_x^1 \frac{dy}{y} g_1^\gamma(y, Q^2, P^2)$$

3. The difference, $\overline{g}_2^\gamma = g_2^\gamma - g_2^{\gamma \mathrm{WW}}$, contains only the twist-3 contributions

$$\int_0^1 dx x^{n-1} \overline{g}_2^\gamma(x, Q^2, P^2) = \frac{n-1}{n}\Big[\sum_i a^n_{(3)i} E^n_{(3)i}(Q^2)\Big]$$

♣ The analysis in pure QED (in the lowest order of α)

- **The relevant twist-3 operators**

$$R_{(3)q}^{\sigma\mu_1\cdots\mu_{n-1}} = i^{n-1}e_q^2\,\overline{\psi}\gamma_5\gamma^{[\sigma,}D^{\{\mu_1]}\cdots D^{\mu_{n-1}\}}\psi - \text{traces}$$

$$R_{(3)\gamma}^{\sigma\mu_1\cdots\mu_{n-1}} = \frac{1}{4}i^{n-1}\epsilon^{[\sigma,}{}_{\alpha\beta\gamma}F^{\alpha\{\mu_1]}\partial^{\mu_2}\cdots\partial^{\mu_{n-1}\}}F^{\beta\gamma} - \text{traces}$$

with $D_\mu = \partial + ieA_\mu$

- **The solution of the RG eq.**

$$\begin{pmatrix} E_{(3)q}^n(Q^2,\alpha) \\ E_{(3)\gamma}^n(Q^2,\alpha) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{\alpha}{8\pi}K_{(3)q}^n \ln\frac{Q^2}{\mu^2} & 1 \end{pmatrix} \begin{pmatrix} E_{(3)q}^n(1,\alpha) \\ E_{(3)\gamma}^n(1,\alpha) \end{pmatrix}$$

The mixing anomalous dimension: $K_{(3)q}^n = -24e_q^4\frac{1}{n(n+1)}$

$$E_{(3)q}^n(1,\alpha) = 1 + \mathcal{O}(\alpha), \quad E_{(3)\gamma}^n(1,\alpha) = \frac{\alpha}{4\pi}3e_q^4 B_{(3)\gamma}^n$$

$$a_{(3)q}^n = \frac{\alpha}{4\pi}\left(-\frac{1}{2}K_{(3)q}^n \ln\frac{P^2}{\mu^2} + 3e_q^4 A_{(3)q}^n\right), \qquad a_{(3)\gamma}^n = 1$$

$$A_{(3)q}^n + B_{(3)\gamma}^n = 8\left\{\frac{1}{(n+1)^2} - \frac{1}{n^2}\right\}$$

- **The pure QED result for the twist-3 part of g_2^γ**

$$\overline{g}_2^{\gamma,\ n}|_{\rm QED} = \frac{n-1}{n}\frac{\alpha}{4\pi}3e_q^4\left\{-\frac{4}{n(n+1)}\ln\frac{Q^2}{P^2} + \frac{8}{(n+1)^2} - \frac{8}{n^2}\right\}$$

$$\Longrightarrow \overline{g}_2^{\gamma(\rm Box)}$$

4. QCD effects on g_2^γ

(As in the case of nucleon, $\overline{g}_2^{\rm nucl}$,)

The increase of participating twist-3 operators with spin
The mixing among these operators

$\Longrightarrow$

The Q^2 evolution equation for the moments of $\overline{g}_2^\gamma$ cannot be written in a simple form, but in a sum of terms, the number of which increases with spin.

- **The n-th moment of $\overline{g}_2^\gamma$ in LO**

$$\int_0^1 dx x^{n-1} \overline{g}_2^\gamma(x, Q^2, P^2) = \frac{n-1}{n} \frac{2\pi\alpha}{\beta_0} [K_n^{(0)}]_i \Big[\int_{\bar{g}^2}^{g^2} \frac{dg'^2}{(g'^2)^2} \times \exp\left(\frac{\hat{\gamma}_n^{(0)}}{2\beta_0} \ln \frac{\bar{g}^2}{g'^2}\right)\Big]_{ij} \left[E_{(3)}^n(1,0)\right]_j$$

i, j : run over hadronic (quark and gluon) sector only.
$[K_n^{(0)}]_i$: 1-loop mixing anomalous dimension between twist-3 hadronic and photon operators
$[\hat{\gamma}_n^{(0)}]_{ij}$: 1-loop mixing anomalous dimension among twist-3 hadronic operators

- **We consider two cases:**

(1) the flavor nonsinglet part of $\overline{g}_2^\gamma$ for large N_C

(2) the non-trivial lowest moment ($n = 3$) of $\overline{g}_2^\gamma$

4.1 Flavor nonsinglet part of $\overline{g}_2^\gamma$ for large N_C

Q: $N_f \times N_f$ quark charge matrix

Quark operators $\propto Q^2$

$Q^2 = \langle e^2 \rangle 1 + Q^{\rm ch} \qquad \langle e^2 \rangle = \Sigma_q e_q^2 / N_f$

$\langle e^2 \rangle 1 \longrightarrow$ flavor singlet part

$Q^{\rm ch} \longrightarrow$ flavor nonsinglet part

♣ The relevant operators for $\overline{g}_2^{\gamma(NS)}$

$$
\begin{aligned}
R_{(3)F}^{\sigma\mu_1\cdots\mu_{n-1}} &= i^{n-1} S' \overline{\psi} \gamma_5 \gamma^\sigma D^{\mu_1} \cdots D^{\mu_{n-1}} Q^{ch} \psi, \\
R_{(3)l}^{\sigma\mu_1\cdots\mu_{n-1}} &= \frac{1}{2n} \{ (V_l - V_{n-1-l} + U_l + U_{n-1-l}) \\
&\qquad + (\widetilde{V}_l - \widetilde{V}_{n-1-l} + \widetilde{U}_l + \widetilde{U}_{n-1-l}) \} \\
&\qquad\qquad (l = 1, \cdots, n-2) \\
R_{(3)m}^{\sigma\mu_1\cdots\mu_{n-1}} &= i^{n-2} m S' \overline{\psi} \gamma_5 \gamma^\sigma D^{\mu_1} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} Q^{ch} \psi \\
R_{(3)E}^{\sigma\mu_1\cdots\mu_{n-1}} &= i^{n-2} \frac{n-1}{2n} S' [\, \overline{\psi} \gamma_5 \gamma^\sigma D^{\mu_1} \cdots D^{\mu_{n-2}} \\
&\qquad \times \gamma^{\mu_{n-1}} (i \not{D} - m) Q^{ch} \psi \\
&\qquad + \overline{\psi} (i \not{D} - m) \gamma_5 \gamma^\sigma D^{\mu_1} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} Q^{ch} \psi]
\end{aligned}
$$

S' : symmetrization on the indices $\mu_1 \mu_2 \cdots \mu_{n-1}$ and antisymmetrization on $\sigma \mu_i$

- **They are not all independent**

$$
\begin{aligned}
R_{(3)F}^{\sigma\mu_1\cdots\mu_{n-1}} &= \frac{n-1}{n} R_{(3)m}^{\sigma\mu_1\cdots\mu_{n-1}} + \sum_{l=1}^{n-2} (n-1-l) R_{(3)l}^{\sigma\mu_1\cdots\mu_{n-1}} \\
&\quad + R_{(3)E}^{\sigma\mu_1\cdots\mu_{n-1}}
\end{aligned}
$$

- **The operators V_l, V_l, $\widetilde{V}_l$, $\widetilde{U}_l$**

$$
\begin{aligned}
V_l &= +i^n g S' \overline{\psi} \gamma_5 D^{\mu_1} \cdots G^{\sigma\mu_l} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} Q^{ch} \psi \\
U_l &= -i^{n-1} g S' \overline{\psi} D^{\mu_1} \cdots \widetilde{G}^{\sigma\mu_l} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} Q^{ch} \psi
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{V}_l &= -i^n e S' \overline{\psi} \gamma_5 D^{\mu_1} \cdots F^{\sigma\mu_l} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} Q^{ch} \psi \\
\widetilde{U}_l &= +i^{n-1} e S' \overline{\psi} D^{\mu_1} \cdots \widetilde{F}^{\sigma\mu_l} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} Q^{ch} \psi
\end{aligned}
$$

$G_{\mu\nu}$, $\quad \widetilde{G}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} G^{\alpha\beta}$: gluon field strength and its dual

$F_{\mu\nu}$, $\quad \widetilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}$: photon field strength and its dual

$$
\begin{aligned}
D_\mu &= \partial_\mu - igA_\mu^a T^a + ieA_\mu \\
[D_\mu, D_\nu] &= -igG_{\mu\nu}^a T^a + ieF_{\mu\nu}
\end{aligned}
$$

Inclusion of $\widetilde{V}_l$ and $\widetilde{U}_l$ is important

$$\Downarrow$$

$$K_{n,F}^{(0)} = \Sigma_{l=1}^{n-2}(n-1-l)K_{n,l}^{(0)}, \qquad \langle\gamma|R_{(3)E}^n|\gamma\rangle = 0$$

$$\Downarrow$$

The choice of the hadronic operators as independent bases is irrelevant to the final result for the n-th moment of $\overline{g}_2^{\gamma(NS)}$

♣ The large N_C limit

At large N_C, the operators having the gluon field strength $G_{\mu\nu}$ decouple from the evolution eq. for $\overline{g}_2^{\gamma(NS)}$

The whole contribution in LO is represented by one type of operator $R_{(3)F}^n$

Ali, Braun, Hiller, PLB 266(1991), Sasaki, PRD 58(1998)

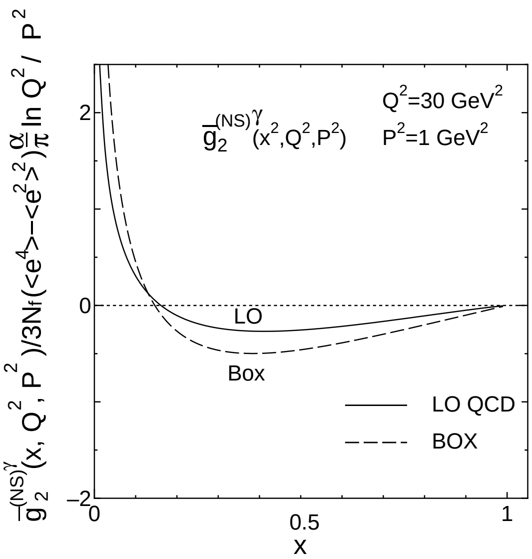
$$\hat{\gamma}^{(0)}_{n,FF} = 8C_F(S_n - \frac{1}{4} - \frac{1}{2n}) + \mathcal{O}(1/N_C^2) \qquad \text{with} \quad S_n = \sum_{j=1}^{n} \frac{1}{j}$$

$$\hat{\gamma}^{(0)}_{n,lF} = \mathcal{O}(1/N_C^2)\ , \qquad \text{for} \quad l = 1, \cdots, n-2$$

$$K^{(0)}_{n,F} = -24N_f(\langle e^4\rangle - \langle e^2\rangle^2)\frac{1}{n(n+1)}$$

$$\int_0^1 dx x^{n-1}\overline{g}_2^{\gamma(NS)}(x, Q^2, P^2)$$

$$= \frac{n-1}{n}\frac{\alpha}{4\pi}\frac{2\pi}{\beta_0\alpha_s(Q^2)}K^{(0)}_{n,F}\frac{1}{1+\hat{\gamma}^{(0)}_{n,FF}/2\beta_0}$$
$$\times \left\{1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{\hat{\gamma}^{(0)}_{n,FF}/2\beta_0+1}\right\}$$



4.2 The third ($n = 3$) moment of $\overline{g}_2^{\gamma}$

◇ Flavor nonsinglet part

Take, for example, $R^{n=3}_{(3)1}$, $R^{n=3}_{(3)m}$, $R^{n=3}_{(3)E}$, besides $R^{n=3}_{(3)\gamma}$ as independent operators

$$
\begin{aligned}
\overline{g}^{\gamma(NS)}_{2,n=3} &= \int_0^1 dx x^2 \overline{g}_2^{\gamma(NS)}(x, Q^2, P^2) \\
&= \frac{2}{3}\frac{\alpha}{4\pi}\frac{2\pi}{\beta_0 \alpha_s(Q^2)} K_1^{(0)NS} \frac{1}{1 + \hat{\gamma}_{11}^{(0)NS}/2\beta_0} \\
&\quad \times \left\{ 1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\hat{\gamma}_{11}^{(0)NS}/2\beta_0 + 1} \right\}
\end{aligned}
$$

with

$$
\begin{aligned}
K_1^{(0)NS} &= -24 N_f (\langle e^4 \rangle - \langle e^2 \rangle^2) \frac{1}{3 \cdot 4} \\
\hat{\gamma}_{11}^{(0)NS} &= 6 C_G - \frac{2}{3} C_F
\end{aligned}
$$

Other choices for the independent operators give the same result.

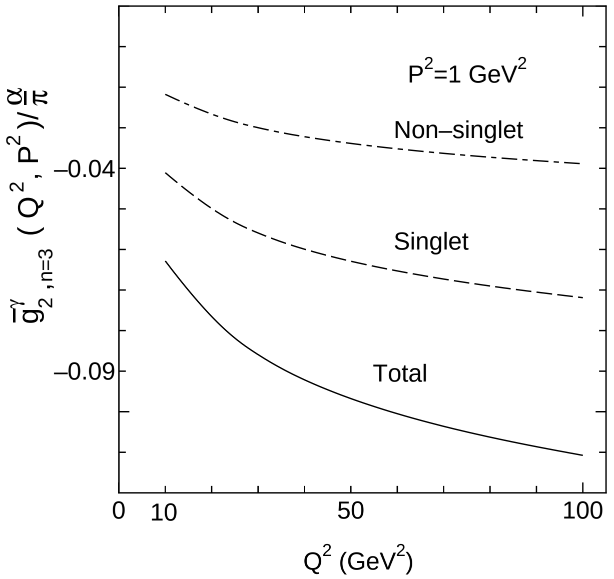
◇ Flavor singlet part

Take, for example, $R^{S,n=3}_{(3)1}$, $R^{S,n=3}_{(3)m}$, $R^{S,n=3}_{(3)E}$, $T^{n=3}_{(3)B}$, $T^{n=3}_{(3)E}$, as independent operators,
from the work of Kodaira et al. PTP99 (1998)

$$
\begin{aligned}
\overline{g}^{\gamma(S)}_{2,n=3} &= \int_0^1 dx x^2 \overline{g}^{\gamma(S)}_2(x,Q^2,P^2) \\
&= \frac{2}{3}\,\frac{\alpha}{4\pi}\,\frac{2\pi}{\beta_0\alpha_s(Q^2)} K_1^{(0)S} \frac{\langle e^2\rangle}{1+\hat{\gamma}_{11}^{(0)S}/2\beta_0} \\
&\quad \times \left\{ 1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{\hat{\gamma}_{11}^{(0)S}/2\beta_0+1} \right\}
\end{aligned}
$$

with

$$
\begin{aligned}
K_1^{(0)S} &= -24 N_f \langle e^2\rangle \frac{1}{3\cdot 4} \\
\hat{\gamma}_{11}^{(0)S} &= 6C_G - \frac{2}{3}C_F + \frac{4}{3}N_f
\end{aligned}
$$



5. Conclusion

1. Does the BC sum rule holds for g_2^γ ? Yes
2. Are there the twist-3 contributions to g_2^γ? Yes
   If so
3. Are they small or sizable? Sizable
4. Does the WW relation also holds for g_2^γ ? Yes
   and can the twist-3 part be extracted? Yes
5. Any complication in the QCD analysis for g_2^γ ?

   Yes. We have studied for two cases:

   (1) the flavor nonsinglet part of $\overline{g}_2^\gamma$ for large N_C

   (2) the non-trivial lowest moment ($n = 3$) of $\overline{g}_2^\gamma$