Six-fermion production at e^+e^- colliders*

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September 9, 2002

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*based of the results of hep-ph/0206070

Motivation

Many interesting subprocesses in $6f$ production:

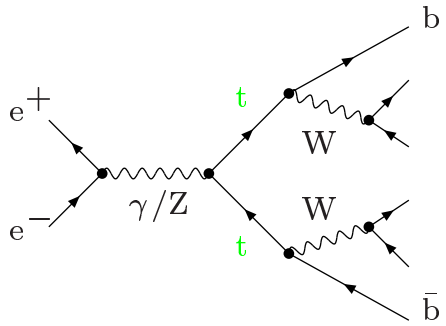
- Top-pair production	$\mathrm{e^+e^-} \to \mathrm{t\bar{t}}$
- Higgs production	$\mathrm{e^+e^-} \to \nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mathrm{H}$
	$\mathrm{e^+e^-} \to \mathrm{ZH}$
- Vector-boson scattering	$\mathrm{VV} \to \mathrm{VV}$
- Triple gauge-boson production	$\mathrm{e^+e^-} \to \mathrm{VVZ}$

General aspects:

- Treatment of unstable particles in resonance processes
- Challenge for numerical integration
  $\Rightarrow$ Detailed tuned comparison between Monte Carlos important!
- Final aim is inclusion of radiative corrections
  - Which radiative corrections have to be taken into account?
  - Pole expansion sufficient?
  - Equivalent vector-boson approximation in $\mathrm{VV} \to \mathrm{VV}$ sufficient?

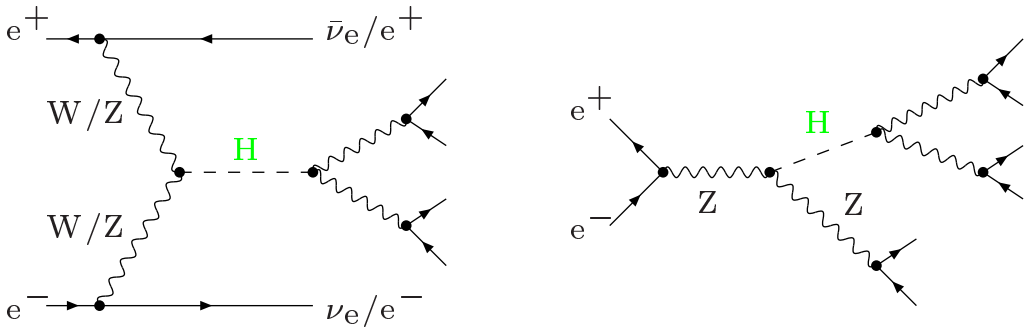
Interesting subprocesses

Top-pair production: similar to W-pair production



- Determination of m_{t} and Γ_{t}
- Investigation of top–gauge-boson couplings
- Access to top spin physics
- QCD effects in the $\mathrm{t\bar{t}}$ system (mainly near $\mathrm{t\bar{t}}$ threshold)

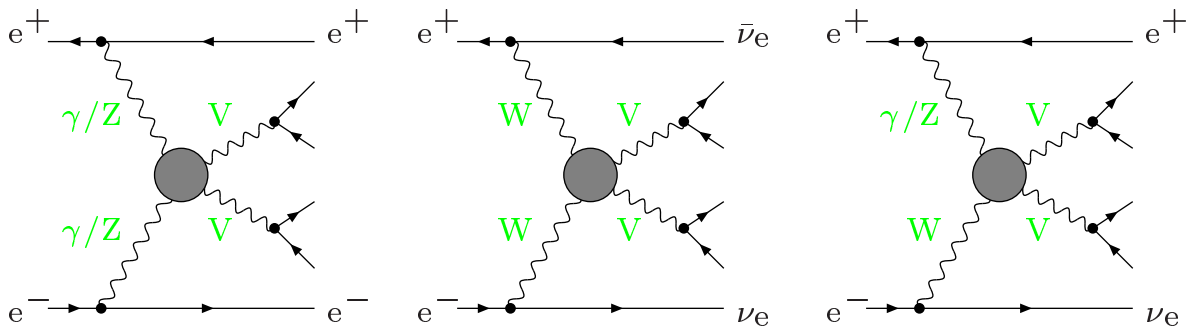
Higgs production:



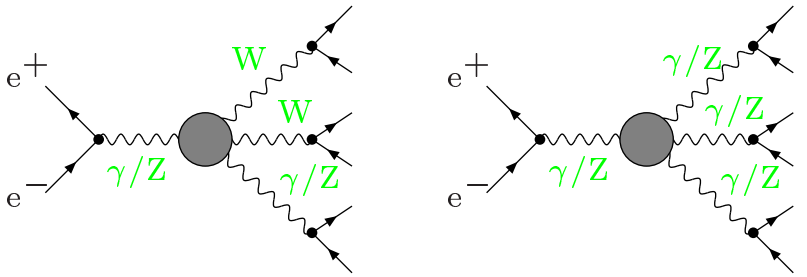
- Γ_{H} large for $M_{\mathrm{H}} \gtrsim 160\,\mathrm{GeV}$ ($\mathrm{H} \to \mathrm{WW}, \mathrm{ZZ}$ opens)
- Determination of M_{H}, Higgs–gauge-boson couplings

Included in vector-boson scattering and triple gauge-boson production.

Vector-boson scattering:



Triple gauge-boson production:



Investigation of quartic gauge-boson couplings:

- sensitive to non-standard couplings due to unitarity cancellations
- window in mechanism of spontaneous symmetry breaking (equivalence theorem)

Theoretical status of $6f$ production

Existing numerical studies:

- Accomando et al.'97,'98
  $\mathrm{e^+e^-} \to \mathrm{t\bar{t}}, \mathrm{ZH}, \mathrm{WWZ} \to 6f$
- Yuasa et al.'97, Kolodziej'01
  $\mathrm{e^+e^-} \to \mathrm{t\bar{t}} \to 6f$
- Montagna et al.'97–'99
  $\mathrm{e^+e^-} \to \mathrm{t\bar{t}}, \mathrm{ZH}, \nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mathrm{H} \to 6f$
- Gangemi'00, Chierici et al.'01
  $\mathrm{e^+e^-} \to \nu_\mathrm{e}\bar{\nu}_\mathrm{e}(\mathrm{WW} \to \mathrm{WW}) \to 6f$

$\Rightarrow$ Many final states not considered yet!

Multi-purpose event generators:

- GRACE (Ishikawa et al.'92)
- PHEGAS & HELAC (Papadopoulos,Kanaki'00)
- WHIZARD & MADGRAPH/O'MEGA
  (Kilian'01; Stelzer, Long'94; Moretti et al.'01)

Questions: numerical stability, limitations, efficiency ???

$\Rightarrow$ No detailed tuned comparison for $\mathrm{e^+e^-} \to 6f$ exists!

Monte Carlo generator LUSIFER

A LUcid approach to SIx-FERmion production

S. Dittmaier,M.R.'02

Features of Monte Carlo generator LUSIFER:

- Monte Carlo event generator for $e^+e^- \to 6f$ (and $4f, 4f+\gamma$)
- All $6f$ final states, full lowest-order amplitudes
- All fermions massless except for internal top quarks
- Multi-channel integration technique with adaptive optimization
- Initial-state radiation (ISR) via leading-log. structure functions

Main complications in $e^+e^- \to 6f$

- Fast evaluation of diagrams

Number of diagrams:	256	$\mu^-\bar{\nu}_\mu u\bar{d}b\bar{b}$
(unitary gauge)	1016	$\nu_e\bar{\nu}_e u\bar{u}u\bar{u}$
	⋮	
	13896	$e^-e^+e^-e^+e^-e^+$

- Extremely rich peaking structure
  ⇒ many different channels (=14-dim. phase space), one channel for each diagram
- Gauge invariance in presence of unstable particles

Generic calculation of amplitudes

- Analytic part: helicity amplitudes for generic diagrams

  Evaluation with Weyl–van der Waerden spinors
  and use of discrete symmetries

  $\Rightarrow$ one/two compact generic functions per generic diagram
  for all final states and polarizations !

- Combinatorics: field insertions for generic diagrams

  $\Rightarrow$ permutations of external fermions and
  insertions of internal fermion and boson fields

- Square amplitude: squared diagrams and interferences

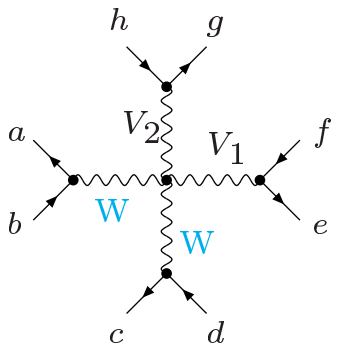
  $\Rightarrow$ Fermi signs and colour algebra
  (gluon diagrams implemented for up to four quarks)

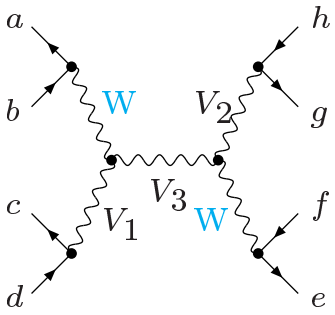
Introduction of decay widths

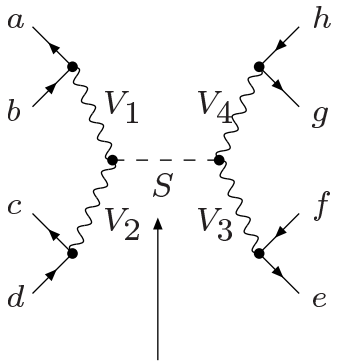
optional: constant, running width, complex-mass scheme

$\rightarrow$ important for controlling gauge invariance

Generic diagrams with 4 fermion currents:



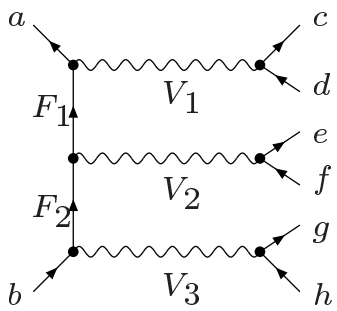


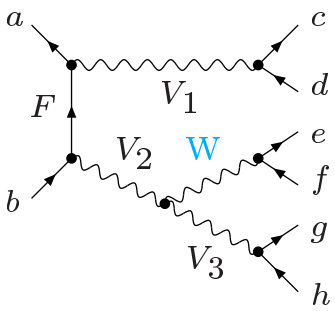


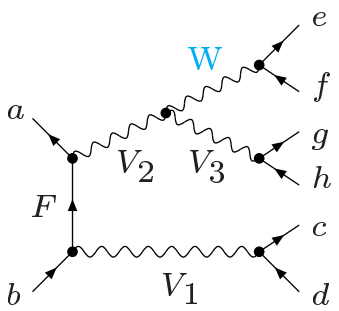
$V_{...} = \gamma, \mathrm{Z}, \mathrm{W}$

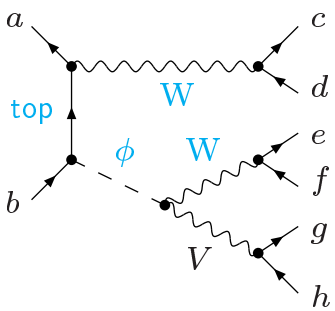
S = Higgs, Goldstone ϕ

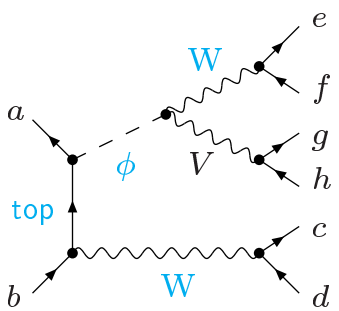
Generic diagrams with 3 fermion currents:





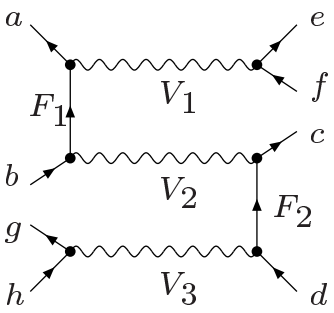


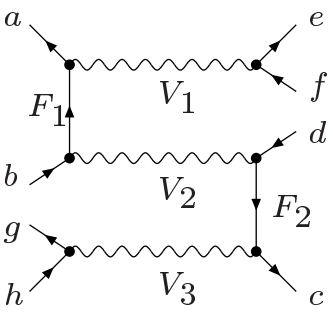


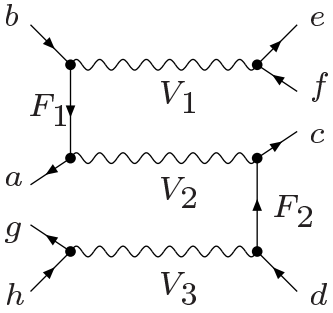


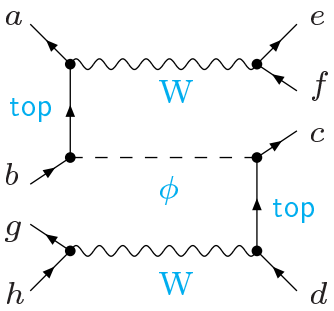
$F_{...}$ = massless fermion, top

Generic diagrams with 2 fermion currents:









Numerical integration

- Multi-channel Monte Carlo integration Berends,Daverveldt,Kleiss '85 Hilgart,Kleiss,Le Diberder '93

  A channel corresponds to each diagram, importance sampling for all propagators
  $\Rightarrow$ $\mathcal{O}(10^2$–$10^4)$ different channels
- Adaptive weight optimization Kleiss,Pittau '94
- Phase-space generators similar to RacoonWW Denner,Dittmaier,M.R.,Wackeroth'00,'01

  Now: applicable to arbitrary processes!

Features:

- A single compact generic phase-space generator for all diagrams
- Fast initialization by inserting internal fields one after an other and testing if diagram is excluded or not.
- Subparts of individual channels evaluated once for each event.
  $\Rightarrow$ Fast numerical integration!
- Standard cut already taken into account in event generation: invariant-mass cuts, energy cuts, angular cuts
  $\Rightarrow$ Important for the numerical stability and efficiency!

Importants sampling:

- Breit-Wigner propagator: Jacobian $\propto [(p^2 - M^2)^2 + M^2\Gamma^2]$
- Particle with vanishing width: Jacobian $\propto (p^2 - m^2)^\nu$,$\nu \approx 0.8$

Top-pair production - total cross section

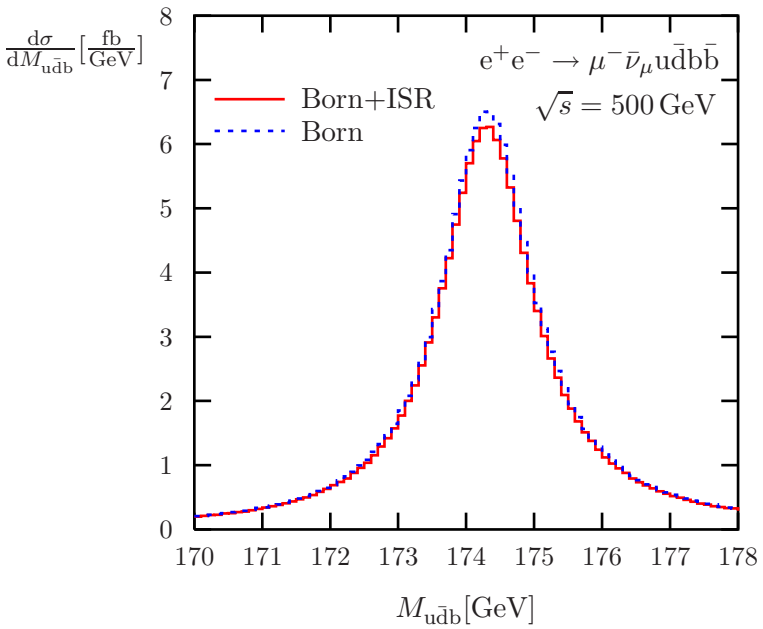
$\sqrt{s} = 500\,\mathrm{GeV}$, no gluon diagrams, phase-space cuts applied

$\sigma[\,\mathrm{fb}]$ $\mathrm{e^+e^-} \to$	LUSIFER		WHIZARD & MADGRAPH	
	Born	Born + ISR	Born	Born + ISR
$\mu^-\bar{\nu}_\mu\nu_\tau\tau^+\mathrm{b}\bar{\mathrm{b}}$	5.800(4)	5.584(4)	5.796(3)	5.589(3)
$\mathrm{e}^-\bar{\nu}_\mathrm{e}\nu_\mu\mu^+\mathrm{b}\bar{\mathrm{b}}$	5.819(5)	5.604(4)	5.827(3)	5.607(3)
$\mathrm{e}^-\mathrm{e}^+\nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mathrm{b}\bar{\mathrm{b}}$	5.853(7)	5.647(7)	5.875(3)	5.651(4)
$\mu^-\bar{\nu}_\mu\mathrm{u}\bar{\mathrm{d}}\mathrm{b}\bar{\mathrm{b}}$	17.095(11)	16.454(10)	17.103(8)	16.463(9)
$\mathrm{e}^-\bar{\nu}_\mathrm{e}\mathrm{u}\bar{\mathrm{d}}\mathrm{b}\bar{\mathrm{b}}$	17.187(21)	16.511(12)	17.148(8)	16.529(9)

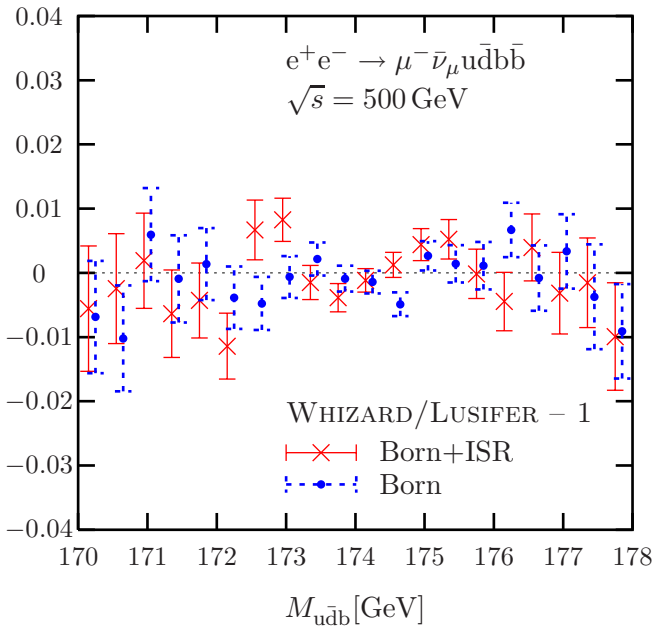
$\Rightarrow$ Good agreement!

- Deviation $\lesssim 3\sigma$ for complicated processes
- ISR reduces cross section by $\sim 4\%$
- Electron background diagrams $\lesssim 1\%$ for $\theta(\mathrm{e}^\pm, \mathrm{beam}) > 5^\circ$

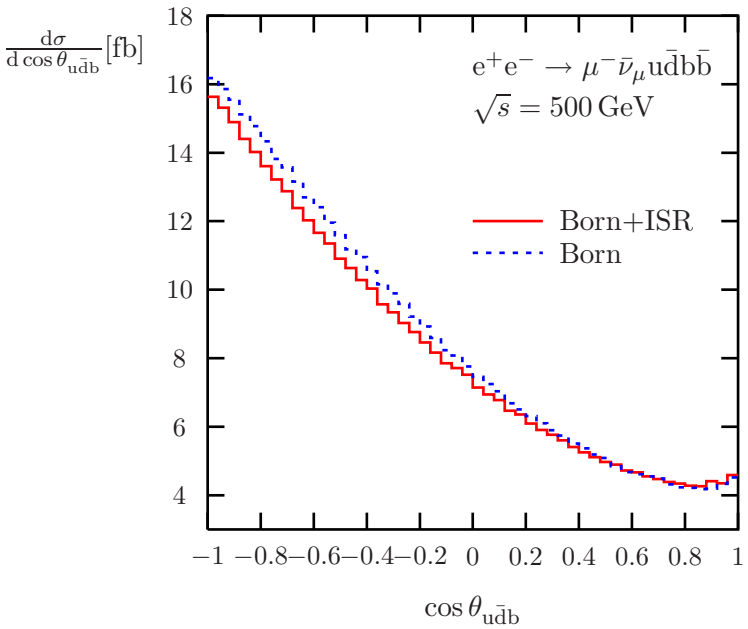
Invariant-mass distribution of top quark



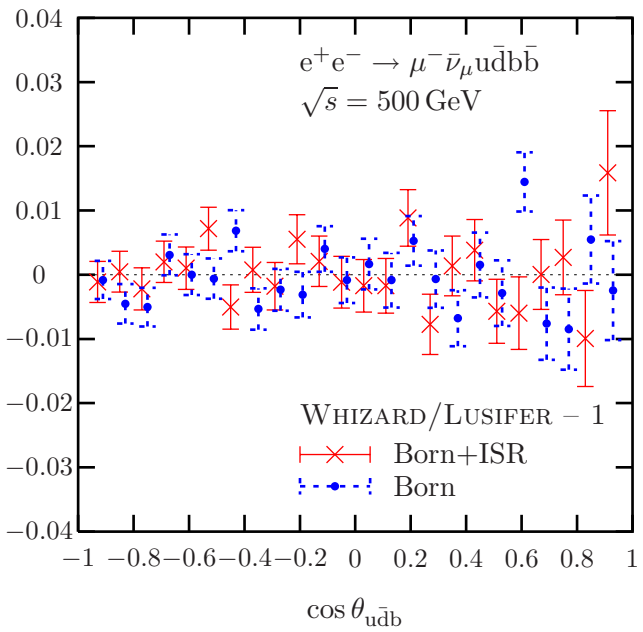
Comparison with WHIZARD & MADGRAPH



Production-angle distribution of top quark



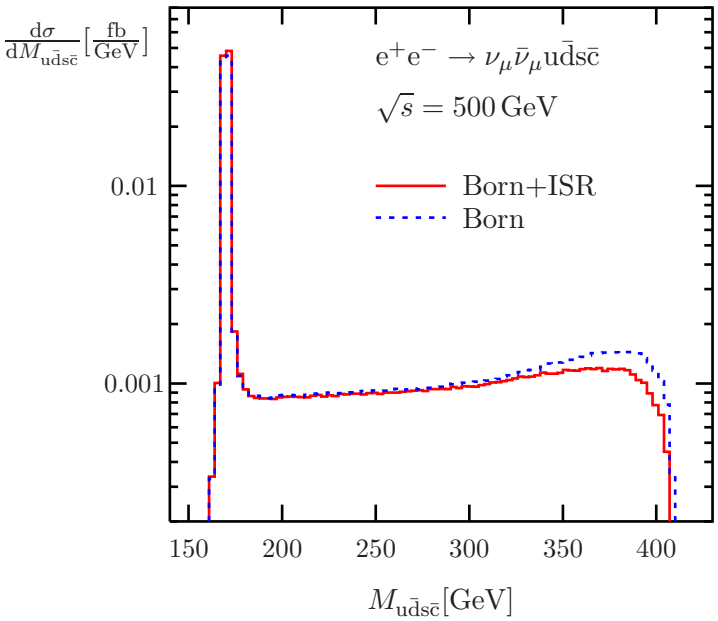
Comparison with WHIZARD & MADGRAPH



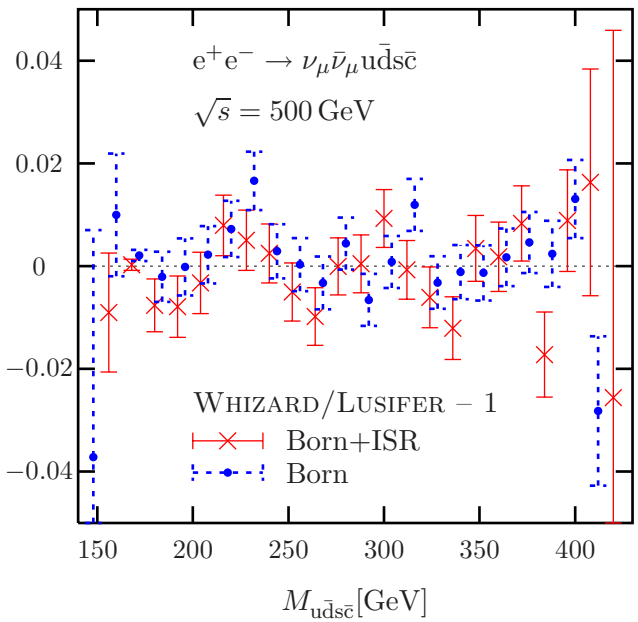
Invariant-mass distribution of Higgs boson

$e^+e^- \to ZH \to \nu_\mu \bar{\nu}_\mu u\bar{d}s\bar{c}$, $M_H = 170\,\mathrm{GeV}$,

phase-space cuts, no gluon diagrams



Comparison with WHIZARD & MADGRAPH



Production-angle distr. of Higgs boson

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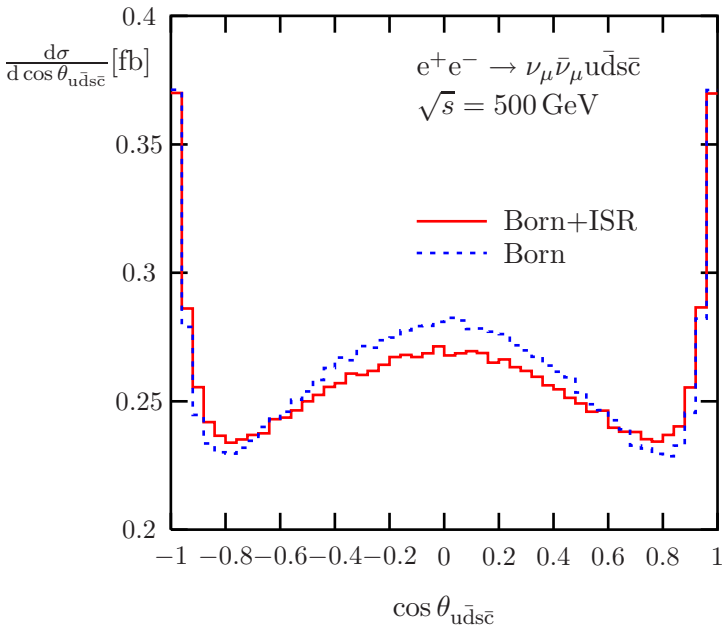
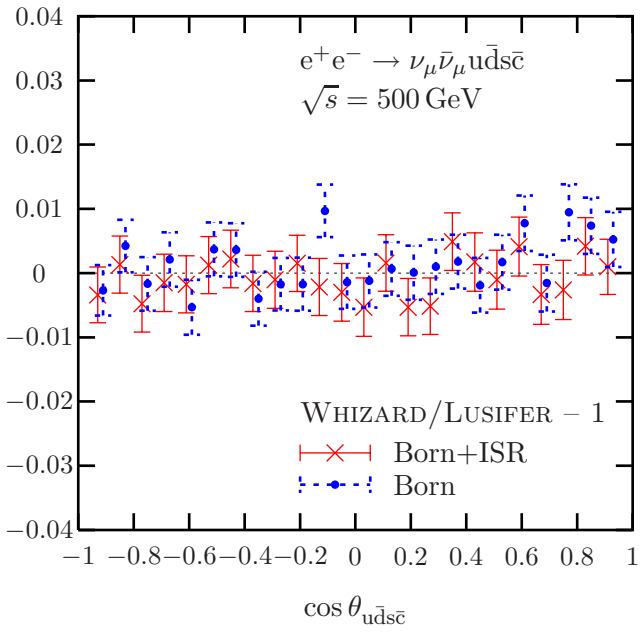
phase-space cuts, no gluon diagrams



Distortion of spectrum by ISR

Comparison with WHIZARD & MADGRAPH



Finite widths and gauge invariance

$\mathrm{WW} \to \mathrm{WW}$ scattering

Process: $\mathrm{e}^+\mathrm{e}^- \to \nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mu^-\bar{\nu}_\mu\mathrm{u}\bar{\mathrm{d}}$

$\sqrt{s}/\,\mathrm{GeV}$	500	800	1000	2000
LUSIFER ('t Hooft–Feynman gauge)				
fixed width	1.633(1)	4.105(4)	5.680(6)	11.74(2)
step width	1.633(1)	4.104(4)	5.672(6)	11.73(2)
running width	1.640(1)	4.132(4)	5.721(5)	12.88(1)
complex mass	1.633(1)	4.104(3)	5.671(5)	11.73(1)
WHIZARD&MADGRAPH (unitary gauge)				
step width	1.631(1)	4.105(4)	5.670(5)	11.74(1)

- Violations of unitary cancellations for running width $\Gamma(k^2) = \Gamma \times k^2/M^2 \times \theta(k^2)$
  $\Rightarrow$ Totally wrong high-energy behaviour
- Fixed width and step width in agreement with gauge-invariant result of complex-mass scheme

Tuned comparisons between LUSIFER and WHIZARD & MADGRAPH

Results of comparison:

- In general statistical errors comparable
- LUSIFER faster than WHIZARD
- WHIZARD has convergence problems for some $6f$ final states
  $\rightarrow$ error sometimes underestimated (typically $f_1\bar{f}_1 f_2\bar{f}_2 f_3\bar{f}_3$)
- No MADGRAPH amplitude in WHIZARD for some $6f$ final states
  (e.g. $\mathrm{e}^-\mathrm{e}^+\mathrm{e}^-\mathrm{e}^+ f\bar{f}$, $\mu^-\mu^+\mu^-\mu^+\mu^-\mu^+$)

$\Rightarrow$ In general good agreement between both programs!

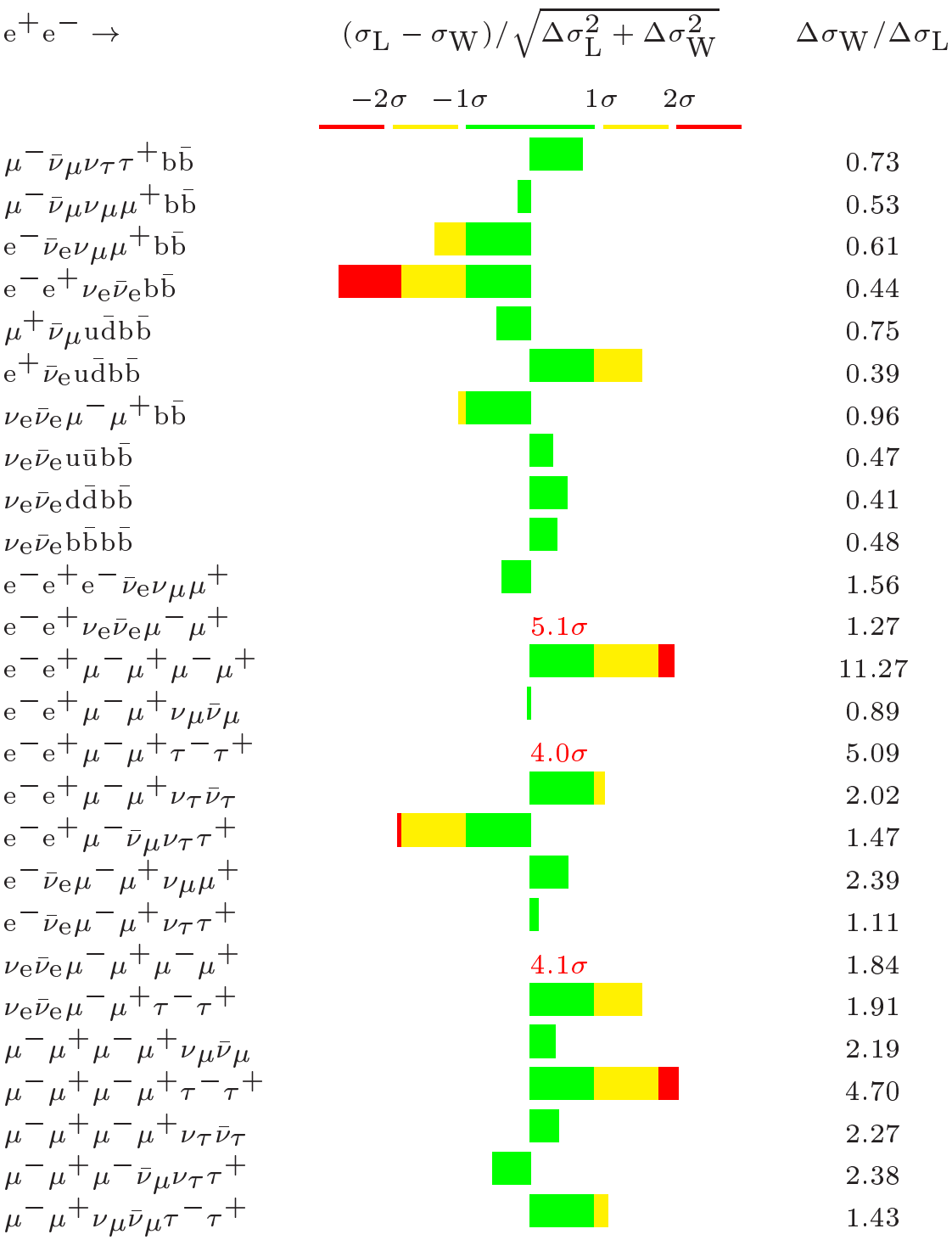
Some examples:

$\sqrt{s} = 500\,\mathrm{GeV}$, phase-space cuts applied

σ[fb] $\mathrm{e}^+\mathrm{e}^- \rightarrow$	LUSIFER		WHIZARD & MADGRAPH	
	Born	Born + ISR	Born	Born + ISR
$\mu^-\mu^+\nu_\mu\bar{\nu}_\mu\tau^-\tau^+$	0.04230(8)	0.04197(9)	0.04212(12)	0.04165(33)
$\mathrm{e}^-\mathrm{e}^+\mathrm{e}^-\bar{\nu}_\mathrm{e}\nu_\mu\mu^+$	0.1860(3)	0.1709(4)	0.1862(4)	0.1712(5)
$\nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mu^-\mu^+\tau^-\tau^+$	0.00737(2)	0.00679(2)	0.00730(3)	0.00650(7)
$\mu^-\bar{\nu}_\mu\nu_\tau\bar{\nu}_\tau\mathrm{u}\bar{\mathrm{d}}$	0.1841(1)	0.1820(1)	0.1843(1)	0.1820(1)
$\mathrm{e}^-\mathrm{e}^+\mathrm{e}^-\bar{\nu}_\mathrm{e}\mathrm{u}\bar{\mathrm{d}}$	0.5509(7)	0.5046(7)	0.5514(13)	0.5061(10)
$\mathrm{e}^-\mathrm{e}^+\nu_\mathrm{e}\bar{\nu}_\mathrm{e}(\mathrm{u}\bar{\mathrm{u}}+\mathrm{d}\bar{\mathrm{d}})$	0.3579(6)	0.3436(7)	0.3592(6)*	0.3437(4)*
$\mathrm{e}^-\mathrm{e}^+\mu^-\mu^+(\mathrm{u}\bar{\mathrm{u}}+\mathrm{d}\bar{\mathrm{d}})$	0.01547(6)	0.01489(5)	0.01523(11)*	0.01396(27)*

* factor 2 more WHIZARD events

Without ISR:


$e^+e^- \to$
$(\sigma_L - \sigma_W)/\sqrt{\Delta\sigma_L^2 + \Delta\sigma_W^2}$
$\Delta\sigma_W/\Delta\sigma_L$
-2σ
-1σ
1σ
2σ
$\mu^-\bar{\nu}_\mu\nu_\tau\tau^+b\bar{b}$ 0.73
$\mu^-\bar{\nu}_\mu\nu_\mu\mu^+b\bar{b}$ 0.53
$e^-\bar{\nu}_e\nu_\mu\mu^+b\bar{b}$ 0.61
$e^-e^+\nu_e\bar{\nu}_eb\bar{b}$ 0.44
$\mu^+\bar{\nu}_\mu u\bar{d}b\bar{b}$ 0.75
$e^+\bar{\nu}_e u\bar{d}b\bar{b}$ 0.39
$\nu_e\bar{\nu}_e\mu^-\mu^+b\bar{b}$ 0.96
$\nu_e\bar{\nu}_e u\bar{u}b\bar{b}$ 0.47
$\nu_e\bar{\nu}_e d\bar{d}b\bar{b}$ 0.41
$\nu_e\bar{\nu}_e b\bar{b}b\bar{b}$ 0.48
$e^-e^+e^-\bar{\nu}_e\nu_\mu\mu^+$ 1.56
$e^-e^+\nu_e\bar{\nu}_e\mu^-\mu^+$ 5.1σ 1.27
$e^-e^+\mu^-\mu^+\mu^-\mu^+$ 11.27
$e^-e^+\mu^-\mu^+\nu_\mu\bar{\nu}_\mu$ 0.89
$e^-e^+\mu^-\mu^+\tau^-\tau^+$ 4.0σ 5.09
$e^-e^+\mu^-\mu^+\nu_\tau\bar{\nu}_\tau$ 2.02
$e^-e^+\mu^-\bar{\nu}_\mu\nu_\tau\tau^+$ 1.47
$e^-\bar{\nu}_e\mu^-\mu^+\nu_\mu\mu^+$ 2.39
$e^-\bar{\nu}_e\mu^-\mu^+\nu_\tau\tau^+$ 1.11
$\nu_e\bar{\nu}_e\mu^-\mu^+\mu^-\mu^+$ 4.1σ 1.84
$\nu_e\bar{\nu}_e\mu^-\mu^+\tau^-\tau^+$ 1.91
$\mu^-\mu^+\mu^-\mu^+\nu_\mu\bar{\nu}_\mu$ 2.19
$\mu^-\mu^+\mu^-\mu^+\tau^-\tau^+$ 4.70
$\mu^-\mu^+\mu^-\mu^+\nu_\tau\bar{\nu}_\tau$ 2.27
$\mu^-\mu^+\mu^-\bar{\nu}_\mu\nu_\tau\tau^+$ 2.38
$\mu^-\mu^+\nu_\mu\bar{\nu}_\mu\tau^-\tau^+$ 1.43

$e^-e^+\nu_e\bar{\nu}_e(u\bar{u}+d\bar{d})$		$0.99\times\sqrt{2}$
$e^-e^+\mu^-\mu^+(u\bar{u}+d\bar{d})$		$1.86\times\sqrt{2}$
$e^-e^+\nu_\mu\bar{\nu}_\mu(u\bar{u}+d\bar{d})$		$0.76\times\sqrt{2}$
$e^-\bar{\nu}_e\nu_\mu\mu^+(u\bar{u}+d\bar{d})$		$3.07\times\sqrt{2}$
$\nu_e\bar{\nu}_e\mu^-\mu^+(u\bar{u}+d\bar{d})$		$0.75\times\sqrt{2}$
$\mu^-\mu^+\mu^-\mu^+(u\bar{u}+d\bar{d})$		$2.46\times\sqrt{2}$
$\mu^-\mu^+\nu_\mu\bar{\nu}_\mu(u\bar{u}+d\bar{d})$		$0.53\times\sqrt{2}$
$\mu^-\mu^+\tau^-\tau^+(u\bar{u}+d\bar{d})$		$1.84\times\sqrt{2}$
$\mu^-\mu^+\nu_\tau\bar{\nu}_\tau(u\bar{u}+d\bar{d})$		$0.72\times\sqrt{2}$
$\mu^-\bar{\nu}_\mu\nu_\tau\tau^+(u\bar{u}+d\bar{d})$		$0.94\times\sqrt{2}$
$e^-e^+e^-\bar{\nu}_e u\bar{d}$		1.76
$e^-\bar{\nu}_e\nu_e\bar{\nu}_e u\bar{d}$		0.88
$e^-e^+\mu^-\bar{\nu}_\mu u\bar{d}$		0.87
$e^-\bar{\nu}_e\mu^-\mu^+u\bar{d}$		1.21
$e^-\bar{\nu}_e\nu_\mu\bar{\nu}_\mu u\bar{d}$		1.02
$\nu_e\bar{\nu}_e\mu^-\bar{\nu}_\mu u\bar{d}$		1.05
$\mu^-\mu^+\mu^-\bar{\nu}_\mu u\bar{d}$		1.18
$\mu^-\bar{\nu}_\mu\nu_\mu\bar{\nu}_\mu u\bar{d}$		1.23
$\mu^-\mu^+\tau^-\bar{\nu}_\tau u\bar{d}$		0.51
$\mu^-\bar{\nu}_\mu\nu_\tau\bar{\nu}_\tau u\bar{d}$		1.26
$e^-e^+ + 4q$	-5.2σ	$0.52\times\sqrt{7}$
$\nu_e\bar{\nu}_e + 4q$		$0.35\times\sqrt{7}$
$\mu^+\mu^- + 4q$		$0.45\times\sqrt{7}$
$\nu_\mu\bar{\nu}_\mu + 4q$		$0.36\times\sqrt{7}$
$e^-\bar{\nu}_e + 4q$		$0.42\times\sqrt{7}$
$\mu^-\bar{\nu}_\mu + 4q$		$0.46\times\sqrt{7}$

Summary and outlook

Interesting subprocesses in $\mathrm{e^+e^-} \to 6f$:

- Top-pair production $\mathrm{e^+e^-} \to \mathrm{t\bar{t}}$
- Higgs production $\mathrm{e^+e^-} \to \mathrm{ZH}$
  $\mathrm{e^+e^-} \to \nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mathrm{H}$
- Vector-boson scattering $\mathrm{VV} \to \mathrm{VV}$
- Triple-gauge-boson production $\mathrm{e^+e^-} \to \mathrm{VVZ}$

Monte Carlo event generator LUSIFER:

- Available from
  `http://www-itp.physik.uni-karlsruhe.de/~roth/Lusifer/lusifer.html`
- All processes $\mathrm{e^+e^-} \to 6f$ (massless external fermions)
- Higher-order LL ISR taken into account via structure functions
- Tuned comparison with WHIZARD&MADGRAPH
  $\Rightarrow$ good agreement

Outlook

- Interface to PHYTHIA $\Rightarrow$ hadronization
- Implementation of 2-Higgs-doublet model (MSSM)
- Inclusion of non-universal radiative corrections