QCD corrections to $t\bar{t}H$ production at Hadron Colliders

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RADCOR 2002

- Motivations (discovery, precision studies)
- $p\bar{p} \to t\bar{t}H$ (Run II)
- $pp \to t\bar{t}H$ (LHC)

Crucial to know first order QCD corrections

- reduce μ-dependence drastically
- increase precision of theoretical prediction

(with S. Dawson, L. Orr, and D. Wackeroth)

General Framework

- Strong case for a light scalar Higgs boson:

SM $\begin{cases} M_H > 114.1 \text{ GeV } (95\%\text{c.l.}) & \text{(direct)} \\ M_H < 196 \text{ GeV } (95\%\text{c.l.}) & \text{(indirect)} \end{cases}$

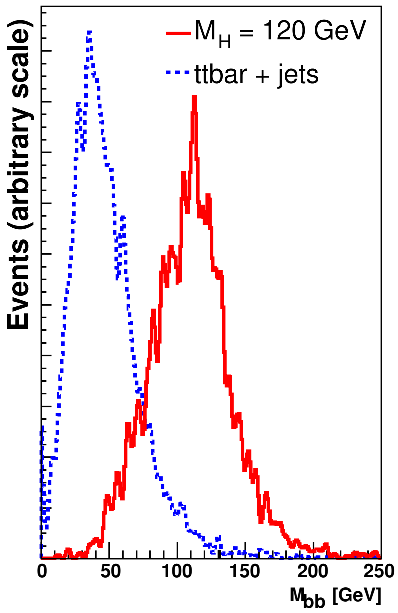
MSSM $\begin{cases} M_{h^0} > 91.0 \text{ GeV } (95\%\text{c.l.}) & \text{(direct)} \\ M_{A^0} > 91.9 \text{ GeV } (95\%\text{c.l.}) & \text{(direct)} \\ \\ M_{h^0} < 135 \text{ GeV} & \text{(theory)} \end{cases}$

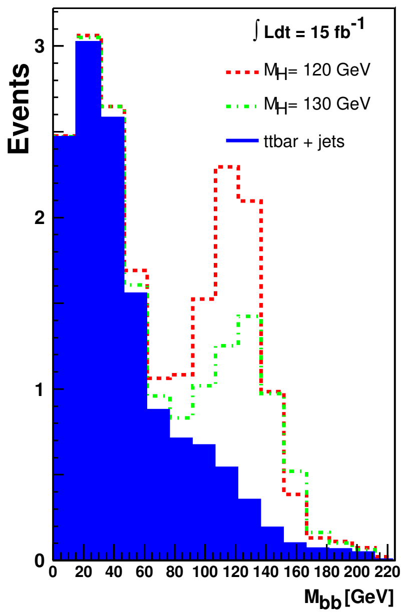
- Discovery: either the Tevatron or the LHC

- Precision studies (Higgs couplings):
  $\longrightarrow$ promising accuracies at the LHC
  $\longrightarrow$ ultimate precision at a high energy LC

- Model independent determination of the Higgs couplings:
  $\longrightarrow$ neat possibility at a high energy LC
  $\longrightarrow$ not obvious at the LHC

Tevatron, $p\bar{p} \to t\bar{t}H_{SM}$





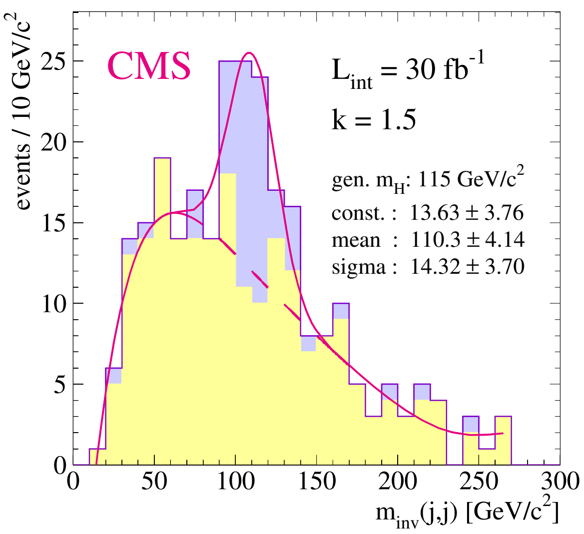
Goldstein etal.
PRL 86(2001)

$\Downarrow$

increased sensitivity for $M_{H_{SM}} < 140$ GeV

LHC, $pp \to t\bar{t}H_{SM}$

LHC top working group, ATLAS, CMS

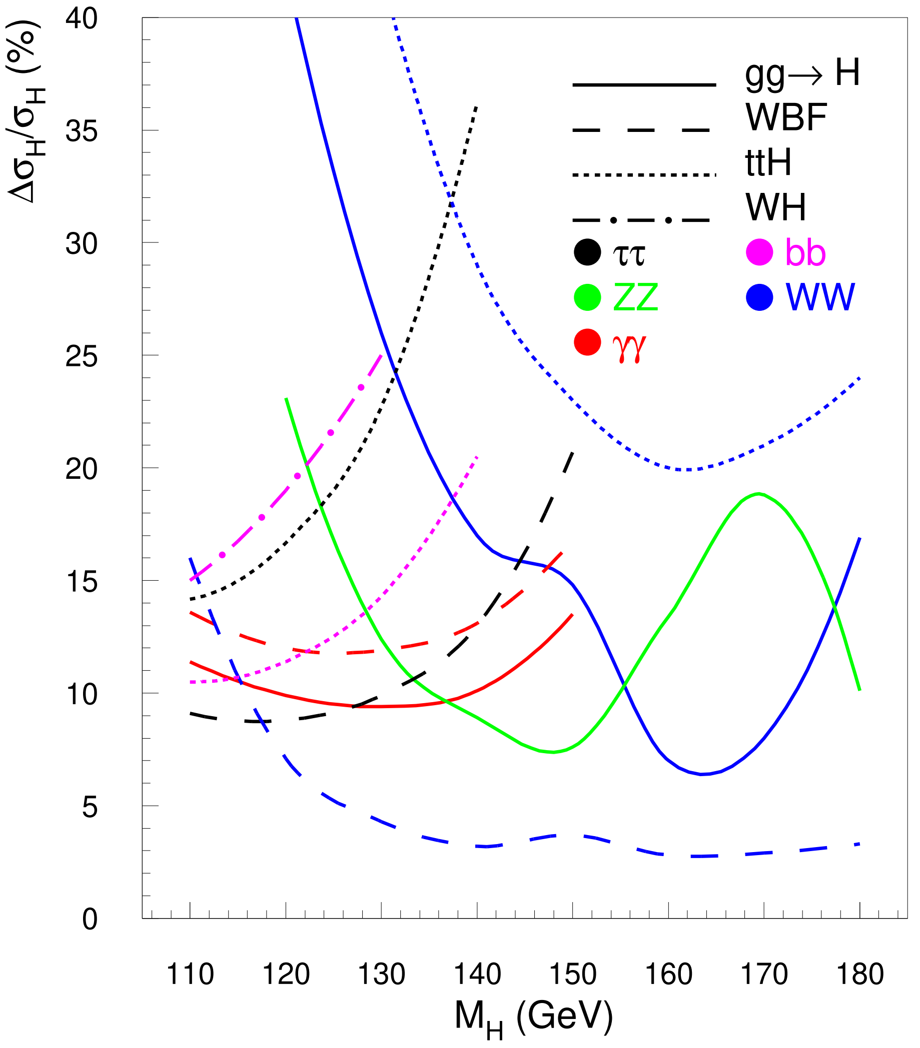


very important in the $100 < M_{H_{SM}} < 130$ GeV region

Measuring the Higgs couplings (LHC)

Consider all existing studies for a SM-like Higgs:

- ▷ $gg \to H,\ H \to \gamma\gamma,\ ZZ,\ WW$ (ATLAS,CMS)
- ▷ $qq \to qqH,\ H \to \gamma\gamma,\ \tau\tau,\ WW$ (Rainwater,Zeppenfeld,Plehn,Kauer)
- ▷ $q\bar{q}, gg \to t\bar{t}H,\ H \to b\bar{b}, WW, \tau\tau$ (ATLAS,CMS,Maltoni,Rainwater,Willenbrock,Belyaev,LR)
- ▷ $q\bar{q} \to WH,\ H \to b\bar{b}$ (CMS)



Crucial improvements with $t\bar{t}H$

(for $M_H < 140$ GeV)

- Determine individual couplings ($\Gamma_i \to y_i^2$), in particular y_t,y_b, and y_τ at the 7-25% level.

- Determine in a model independent way:

$$\frac{y_b}{y_\tau} \longrightarrow \frac{\Gamma_b}{\Gamma_\tau} = \frac{Z_b^{(t)}}{Z_\tau^{(t)}}$$

$$\frac{y_t}{y_g} \longrightarrow \frac{\Gamma_t}{\Gamma_g} = \frac{Z_\tau^{(t)} Z_\gamma^{(w)}}{Z_\tau^{(w)} Z_\gamma^{(g)}}$$

  at the $10-20\%$ level

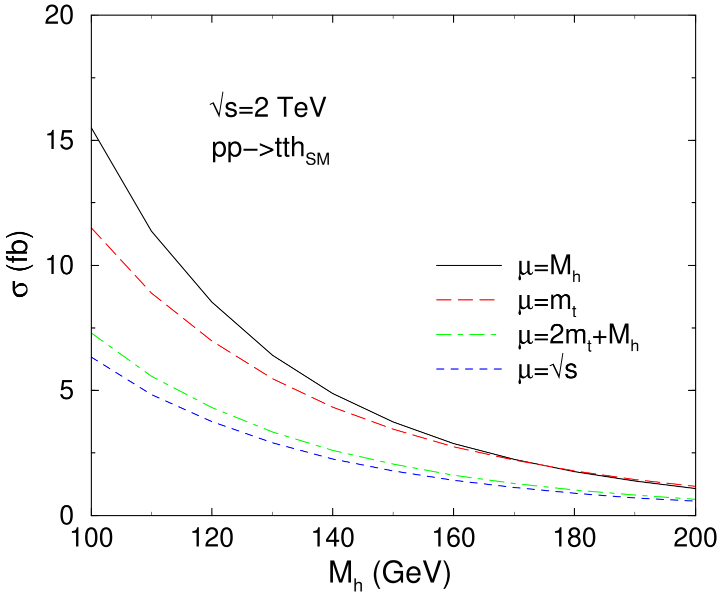
- With high luminosity ($\simeq$ 300 fb^{-1}), test Γ_z/Γ_w below $M_H = 140$ GeV by comparing

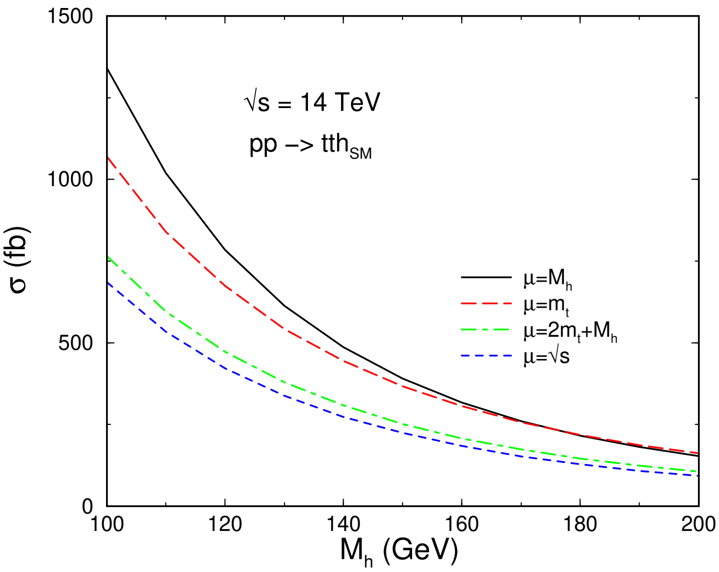
$$\frac{Z_b^{(wH)}}{Z_b^{(t)}} \quad \text{and} \quad \frac{Z_\tau^{(w)}}{Z_\tau^{(t)}}$$

- Explore Γ_g and Γ_γ beyond the SM

Need for NLO calculation

LO calculation have very strong μ-dependence





- $\sigma(p\bar{p} \to t\bar{t}H_{SM})$ calculated at $\mathcal{O}(\alpha_s^3)$
  - W.Beenakker, S.Dittmaier, M.Krämer, B.Plümper, M.Spira, P.Zerwas (PRL 87(2001))
  - L.Reina, S.Dawson (PRL 87(2001))
  - L.Reina, S.Dawson, D.Wackeroth (PRD 65(2002))

- $\sigma(pp \to t\bar{t}H_{SM})$ calculated at $\mathcal{O}(\alpha_s^3)$:
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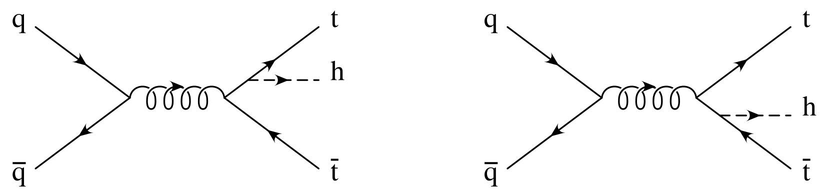
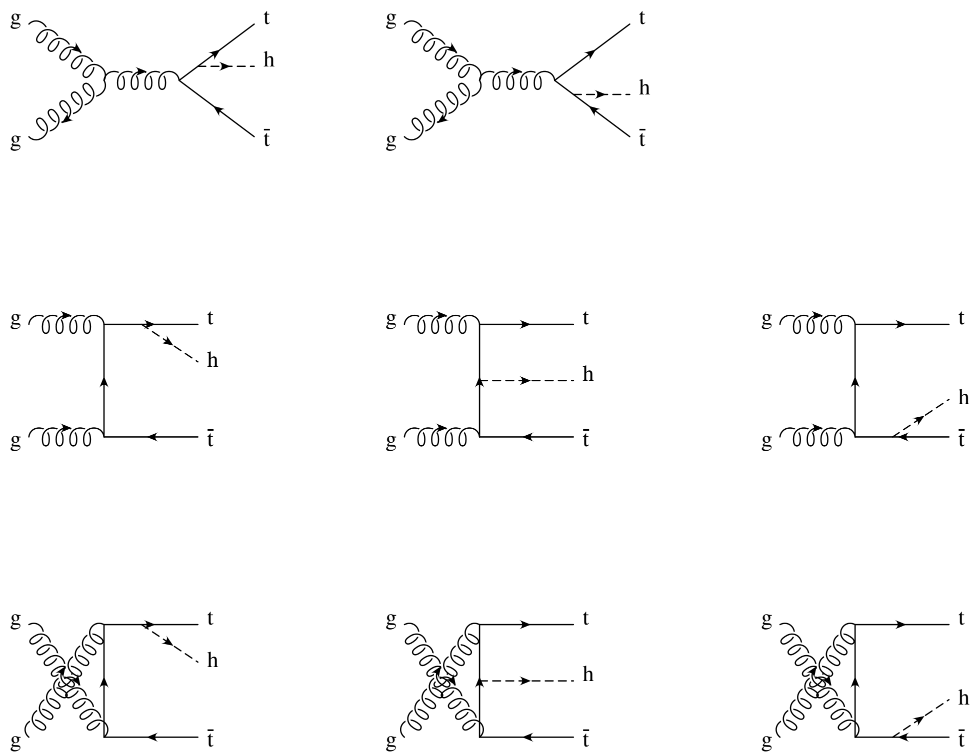
  - S.Dawson, L.Orr, L.Reina, D.Wackeroth (preliminary)

$q\bar{q} \to t\bar{t}H$



$gg \to t\bar{t}H$



General Approach

NLO total cross section:

$$\sigma^{NLO}_{p\bar{p},pp} = \sum_{i,j} \int dx_1 dx_2 \mathcal{F}_i^p(x_1)\mathcal{F}_j^{\bar{p},p}(x_2)\hat{\sigma}^{NLO}_{ij}(x_1,x_2)$$

where

$$\hat{\sigma}^{NLO}_{ij} = \hat{\sigma}^{LO}_{ij} + \frac{\alpha_s}{4\pi}\delta\hat{\sigma}^{NLO}_{ij}$$

NLO corrections made of:

$$\delta\hat{\sigma}^{NLO}_{ij} = \hat{\sigma}^{ij}_{virt} + \hat{\sigma}^{ij}_{real}$$

Specifically:

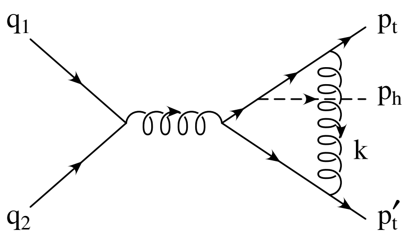
- one loop virtual corrections to $q\bar{q}, gg \to t\bar{t}H$
- real one gluon emission: $q\bar{q}, gg \to t\bar{t}H + g$

$\longrightarrow$ renormalize UV divergences

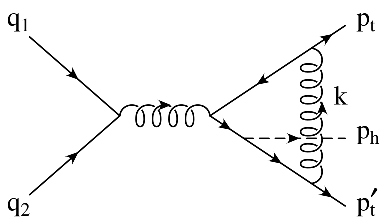
$\longrightarrow$ cancel IR divergences in $\hat{\sigma}_{virt} + \hat{\sigma}_{real}$

$\longrightarrow$ check μ-dependence of σ^{NLO}

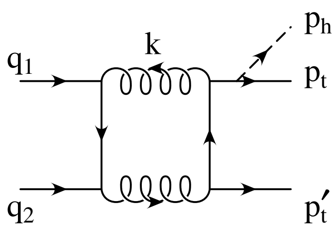
$$\hat{\sigma}_{virt} \propto \int d(PS) \sum_i 2\mathcal{R}e\left(\mathcal{A}_i \mathcal{A}_{LO}^*\right)$$



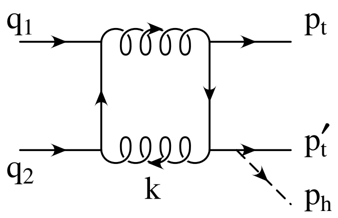
$B3^{(1)}$



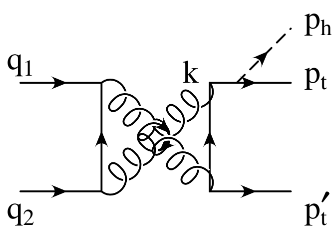
$B3^{(2)}$



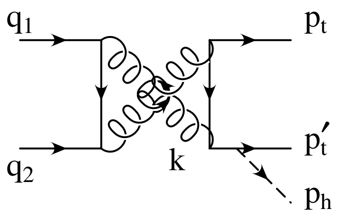
$B5^{(1)}$



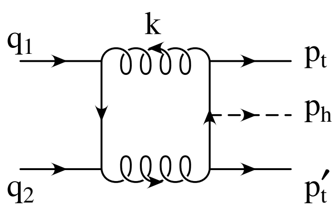
$B5^{(3)}$



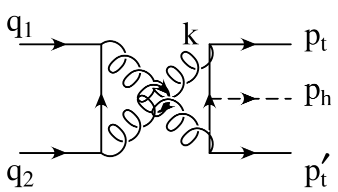
$B5^{(2)}$



$B5^{(4)}$



P1



P2

About $\sigma_{virtual}$

- Reduce each diagram in terms of scalar integrals of the form:

$$\int \frac{d^n k}{(2\pi)^n} \frac{1}{\left[(k^2 - m_1^2)\left((k+p_1)^2 - m_2^2\right)\cdots\right]}$$

$\rightarrow$ three external massive particle
$\rightarrow$ several massive internal propagators

- finite integrals: use existing libraries/packages

- UV-divergent integrals: analytical (easy)

- IR-divergent integrals : analytical (difficult)

  Example: pentagon scalar integral

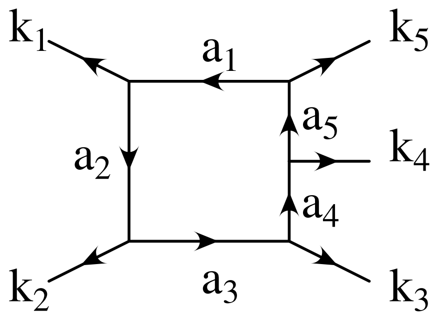
$$E0 = \int \frac{d^n k}{(2\pi)^n} \frac{1}{N_1 N_2 N_3 N_4 N_5}$$

use methods proposed by:

Bern, Dixon, and Kosower, PLB 302 (1993), NPB 412 (1994)

Denner, Fortschr. Phys. 41 (1993)

$\cdots$



$$E0 = \frac{-i}{16\pi^2}(4\pi\mu^2)^\epsilon \Gamma(3+\epsilon) \int d^5 a_i \frac{\delta(1-\sum_{i=1}^5 a_i)}{[\mathcal{D}(a_i)]^{3+\epsilon}}$$

where

$$\mathcal{D}(a_i) = \sum_{i,j=1}^{5} S_{ij} a_i a_j - i\eta$$

$$S_{ij} = \frac{1}{2}(M_i^2 + M_j^2 - p_{ij}) \quad , \quad p_{ij} = k_i + k_{i+1} + \cdots + k_{j-1}$$

scalar pentagon $\longrightarrow \sum$ scalar boxes

$$I_n = \frac{(-1)^n}{2}\left[\sum_{i=1}^{n} c_i I_{n-1}^{(i)} + (n-5+2\epsilon) c_0 I_n^{6-2\epsilon}\right]$$

$$\Downarrow$$

$$E0 = -\frac{1}{2}\left[\sum_{i=1}^{5} c_i D0^{(i)} + 2\epsilon c_0 E0^{6-2\epsilon}\right]$$

. . .

Pentagon tensor integrals present numerical instabilities, due to the dependence on inverse powers of the full Gram's determinant (GD)

$$GD(p_1+p_2 \to p_3+p_4+p_5) \simeq f(E_3, E_4, \sin\theta_3, \sin\theta_4, \sin\phi_4)$$

$GD \to 0$ when two momenta become degenerate. These are spurious divergences, which cause numerical instabilities.

We used two methods:

- Use kinematic cuts to avoid numerical instabilities and extrapolate to the unsafe region using reliable algorithms.

- Eliminate all pentagon tensor integrals *brute force*. Resulting expressions are very big but stable.

Consistent results within the errors of the numerical method.

About σ_{real}

Need to calculate IR singularities (soft/collinear) analytically, to verify that

$$\hat{\sigma}_{virt} + \hat{\sigma}_{real} = \text{finite}$$

$$\hat{\sigma}_{real} = \int d(PS_4)|\mathcal{A}_{real}(ij \to t\bar{t}H + g)|^2$$

Phase Space Slicing $\longrightarrow$ isolate the region of the $t\bar{t}H + g$ phase space where $s_{ig} \to 0$

$$s_{ig} = 2p_i \cdot p_g = 2E_iE_g(1 - \beta_i \cos\theta_{ig})$$

$\longrightarrow$ two cut-off method : $\begin{cases} \delta_s & (E_g < \delta_s\sqrt{s}/2) \\ \delta_c & (1 - \cos\theta_{ig}) < \delta_c) \end{cases}$

$\longrightarrow$ one cut-off method : s_{min} $(s_{ig} < s_{min})$

and compute $\hat{\sigma}_{real}$

- analytically, below the cut-off(s)
- numerically, above the cut-off(s)

Phase Space Slicing with two cut-offs

(Bergman, Baer, Ohnemus, Owens, Reno, ...)

$$\hat{\sigma}_{real}(ij \to t\bar{t}H + g) = \hat{\sigma}_{soft} + \hat{\sigma}_{hard}$$

$$\hat{\sigma}_{soft} \quad \longrightarrow \quad E_g < \frac{\sqrt{s}}{2}\delta_s$$

$$\hat{\sigma}_{hard} \quad \longrightarrow \quad E_g > \frac{\sqrt{s}}{2}\delta_s$$

In the soft limit $(E_g \to 0)$

$$d(PS_4) \stackrel{soft}{\longrightarrow} d(PS_3)d(PS_g) = d(PS_3)\frac{d^{d-1}k}{(2\pi)^{d-1}2E_g}$$

$$|\mathcal{A}_{real}(ij \to t\bar{t}H + g)|^2 \stackrel{soft}{\longrightarrow} (4\pi\alpha_s)|\mathcal{A}_{LO}|^2\Phi_{eik}$$

where the eikonal factor Φ_{eik} contains the soft poles

$$\Phi_{eik} \propto \sum_{ij}\left(\frac{s_{ij}}{s_{ig}s_{jg}} - \frac{m_i^2}{s_{ig}^2} - \frac{m_j^2}{s_{jg}^2}\right)$$

$$\Downarrow$$

$$\hat{\sigma}_{soft} \propto \int d(PS_3)\int_{soft} d(PS_g)\Phi_{eik}|\overline{A}_{LO}|^2$$

computed analytically ($\to$ soft singularities)

. . . moreover

$$\hat{\sigma}_{hard} = \hat{\sigma}_{hard/coll} + \hat{\sigma}_{hard/n-coll}$$

$$\begin{array}{rcl} \hat{\sigma}_{hard/coll} & \longrightarrow & (1-\cos\theta_{ig}) < \delta_c \\ \hat{\sigma}_{hard/n-coll} & \longrightarrow & (1-\cos\theta_{ig}) > \delta_c \end{array}$$

In the collinear limit
$(i \to i'g,\ p'_i = z p_i,\ p_g = (1-z)p_i)$

$$|\mathcal{A}_{real}(ij \to t\bar{t}H+g)|^2 \stackrel{collinear}{\longrightarrow} (4\pi\alpha_s)|\overline{A}_{LO}|^2 \frac{2P_{ii'}(z)}{z\, s_{ig}}$$

$$d(PS_4)(ij \to t\bar{t}H+g) \stackrel{collinear}{\longrightarrow} d(PS_3)(i'j \to t\bar{t}H) z d(PS_g)$$

($P_{ii'} \to$ AP-splitting functions)

$$\Downarrow$$

$$\hat{\sigma}_{hard/coll} \propto \int d(PS_3) \int_{coll} d(PS_g) \sum_i \frac{P_{ii'}}{s_{ig}} |\overline{A}_{LO}|^2$$

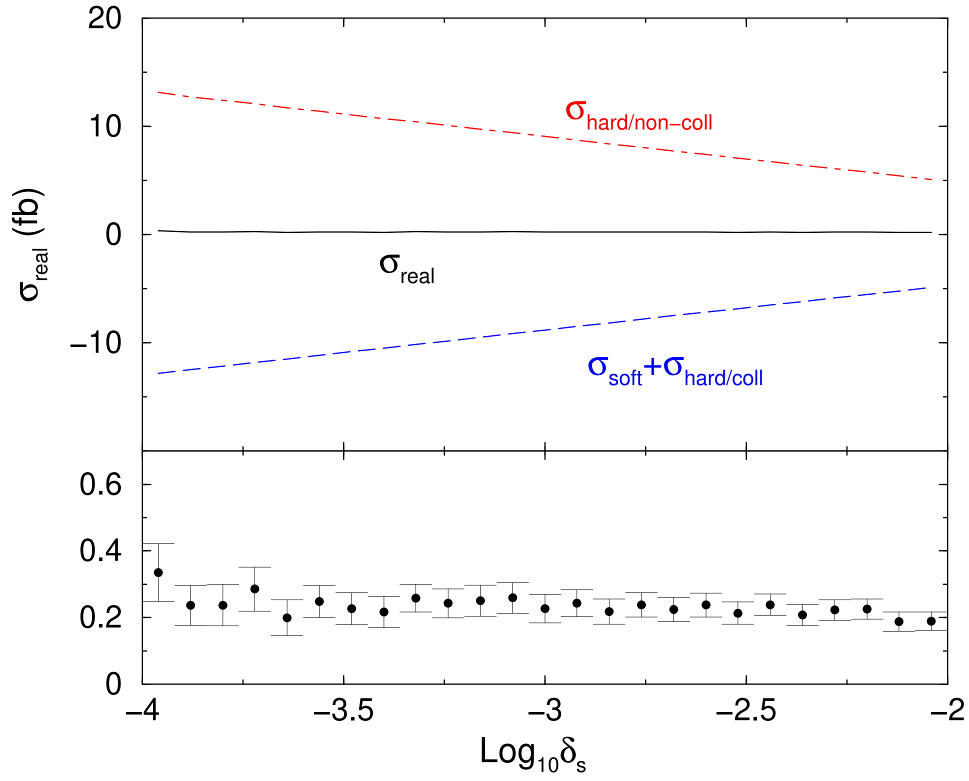
computed analytically (collinear singularities $\to$ mass factorization)

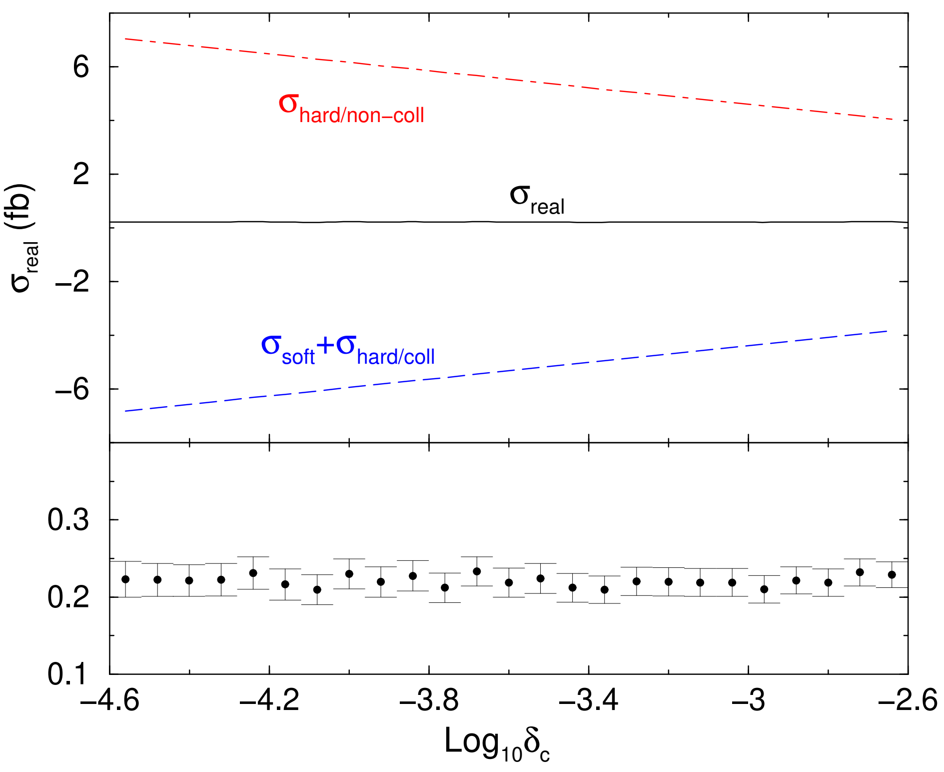
$$\hat{\sigma}_{hard/n-coll} = \int_{hard/n-coll} d(PS_4)|\mathcal{A}_{real}(q\bar{q} \to t\bar{t}H+g)|^2$$

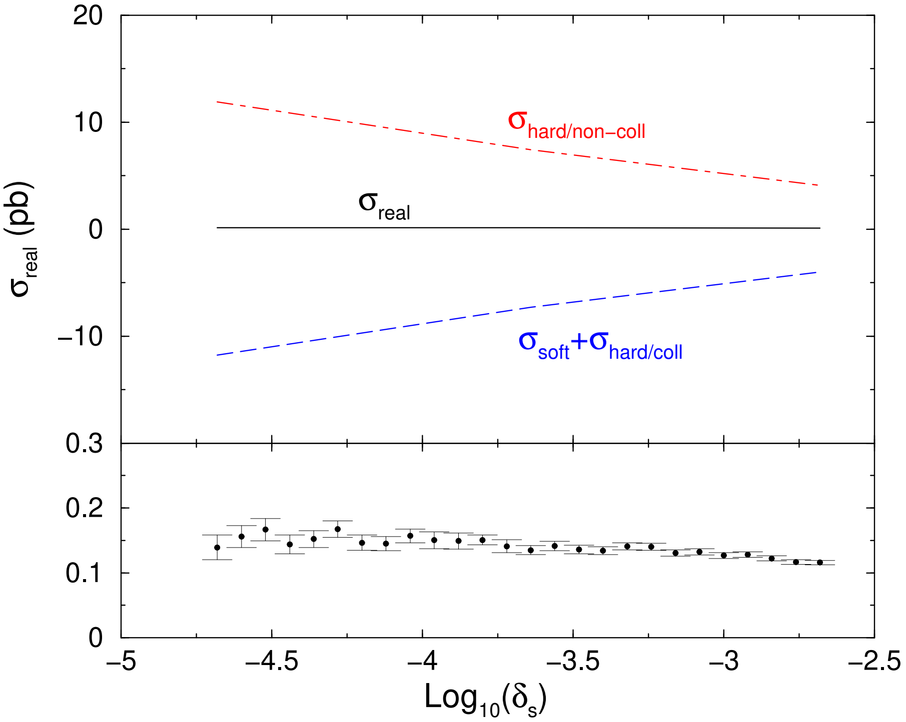
computed numerically (no singularities)

cut-off dependence cancel in:

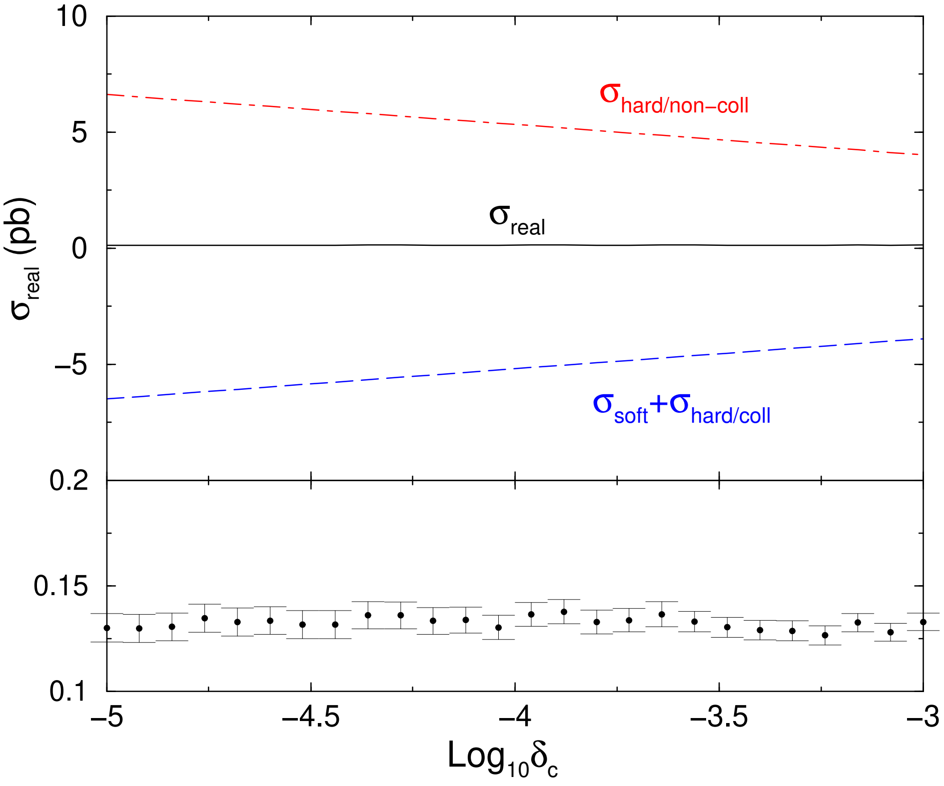
$$\sigma_{real} = \sigma_{soft} + \sigma_{hard/coll} + \sigma_{hard/n-coll}$$







$\sigma_{hard/non-coll}$
σ_{real}
$\sigma_{soft}+\sigma_{hard/coll}$
σ_{real} (pb)
$Log_{10}(\delta_s)$



$\sigma_{hard/non-coll}$
σ_{real}
$\sigma_{soft}+\sigma_{hard/coll}$
σ_{real} (pb)
$Log_{10}\delta_c$

Phase Space Slicing with one cut-off

(Giele, Glover, and Kosower; Keller and Laenen)

- Cross all partons to final state:

$$ij \to t\bar{t}H(+g) \ \text{ becomes } \ H \to ijt\bar{t}(+g)$$

- introduce a cut-off parameter s_{min}: the radiated gluon is considered soft/collinear if

$$s_{ig} < s_{min}$$

$\hat{\sigma}_{ir} = \hat{\sigma}_{soft} + \hat{\sigma}_{coll} \longrightarrow$ analytic

$\hat{\sigma}_{hard} \longrightarrow$ all $s_{ig} > s_{min}$, numerical

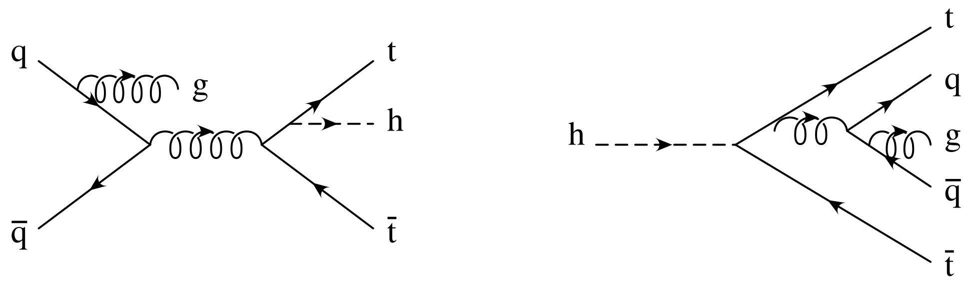
- Use color ordered amplitudes to systematically factor out soft/collinear divergences, and cancel them against $\hat{\sigma}_{virt}$

- Cross ij to initial state, h to final state:
  - flip signs of s_{ig} invariants accordingly
  - add $\hat{\sigma}_{crossing} \longrightarrow$ use crossing functions

$$\Downarrow$$

$$\hat{\sigma}_{real} = \hat{\sigma}_{hard} + \hat{\sigma}_{ir} + \hat{\sigma}_{crossing}$$

Crossing : $q\bar{q} \to t\bar{t}H$ case



Reduce to color ordered amplitudes using that

$$T^a_{c_1c_2}T^a_{c_3c_4} = \frac{1}{2}\left(\delta_{c_1c_4}\delta_{c_3c_2} - \frac{1}{N}\delta_{c_1c_2}\delta_{c_3c_4}\right)$$

$$\Downarrow$$

$$\mathcal{A}_{LO} \equiv \frac{1}{2}\left(\delta_{c_tc_{\bar{q}}}\delta_{c_qc_{\bar{t}}} - \frac{1}{N}\delta_{c_tc_{\bar{t}}}\delta_{c_qc_{\bar{q}}}\right)\delta_{f_qf_{\bar{q}}}\delta_{f_tf_{\bar{t}}}\mathcal{A}_0$$

and

$$\begin{aligned}\mathcal{A}_{real}(H \to q\bar{q}t\bar{t} + g) &\equiv \delta_{f_tf_{\bar{t}}}\delta_{f_qf_{\bar{q}}} \cdot \\ \Big\{\Big(\delta_{c_tc_{\bar{q}}}T^a_{c_qc_{\bar{t}}}B_1(t,\bar{t},q,\bar{q},g) &+ T^a_{c_tc_{\bar{q}}}\delta_{c_qc_{\bar{t}}}B_2(t,\bar{t},q,\bar{q},g)\Big) \\ -\frac{1}{N}\Big(\delta_{c_tc_{\bar{t}}}T^a_{c_qc_{\bar{q}}}B_3(t,\bar{t},q,\bar{q},g) &+ T^a_{c_tc_{\bar{t}}}\delta_{c_qc_{\bar{q}}}B_4(t,\bar{t},q,\bar{q},g)\Big)\Big\}\end{aligned}$$

$$\begin{aligned}|\,\mathcal{A}_{real}\,|^2 \;\propto\; &\left\{\frac{N}{2}\left[|B_1|^2 + |B_2|^2\right] + \right. \\ &\left.\frac{1}{2N}\left[-2|B_3+B_4|^2 + |B_3|^2 + |B_4|^2\right]\right\}\end{aligned}$$

In the soft limit

$$\int d(PS_4)|\mathcal{A}_{real}|^2 \xrightarrow{soft} (4\pi\alpha_s)\int d(PS_3)S_F|\mathcal{A}_{LO}|^2 \propto \hat{\sigma}_{soft}$$

$$\begin{aligned} S_F \;=\; & \frac{N}{2}\left(S_{q\bar{t}} + S_{\bar{q}t}\right) - \\ & \frac{1}{2N}\left[S_{q\bar{q}} + S_{t\bar{t}} - 2\left(S_{qt} - S_{\bar{q}t} - S_{q\bar{t}} + S_{\bar{q}\bar{t}}\right)\right] \end{aligned}$$

where

$$S_{ij} = \int_{soft} d(PS_g)\left(\frac{s_{ij}}{s_{ig}s_{jg}} - \frac{m_i^2}{s_{ig}^2} - \frac{m_j^2}{s_{jg}^2}\right)$$

In the collinear limit

$$\int d(PS_4)|\mathcal{A}_{real}|^2 \xrightarrow{coll} (4\pi\alpha_s)\int d(PS_3)C_F|\mathcal{A}_{LO}|^2 \propto \hat{\sigma}_{coll}$$

$$\begin{aligned} C_F \;=\; & \frac{N}{2}\left(C_{q\bar{t}} + C_{\bar{q}t}\right) - \\ & \frac{1}{2N}\left[C_{q\bar{q}} - 2\left(C_{qt} - C_{\bar{q}t} - C_{q\bar{t}} + C_{\bar{q}\bar{t}}\right)\right] \end{aligned}$$

where if $i \to i'g$ $(p_{i'} = zp_i,\ p_g = (1-z)p_i)$

$$C_{ij} = \int_{coll} d(PS_g)\frac{P_{ii'}}{s_{ig}}$$

with integration boundaries:

$$z_1 = 0 < z < 1 - z_2 = 1 - \frac{s_{min}}{s_{ij}}$$

Now:

$$\widehat{\sigma}_{ir} + \widehat{\sigma}_{virt} = \int d(PS_3)\left\{(4\pi\alpha_s)(S_F + C_F)|\mathcal{A}_{LO}|^2 + |\mathcal{A}_{virt}|^2\right\}$$

is finite and crossing can be performed, provided we correct for the mismatch between initial state and final state collinear singularities: crossing functions

$$C_i^{\mathcal{H}}(x) \simeq \sum_j \left[\int_x^1 \frac{dz}{z} f_j^{\mathcal{H}}\left(\frac{x}{z}\right) P_{j\to ig}(z) - f_i^{\mathcal{H}}(x)\int_0^1 dz P_{i\to jg}\right]\frac{s_{min}^{-\epsilon}}{\epsilon}$$

or, after mass factorization

$$C_i^{\mathcal{H}}(x,\mu) = \frac{N}{2\pi}\left[A_i^{\mathcal{H}}(x,\mu)\ln\left(\frac{s_{min}}{\mu^2}\right) + B_i^{\mathcal{H}}(x,\mu)\right]$$

$$\Downarrow$$

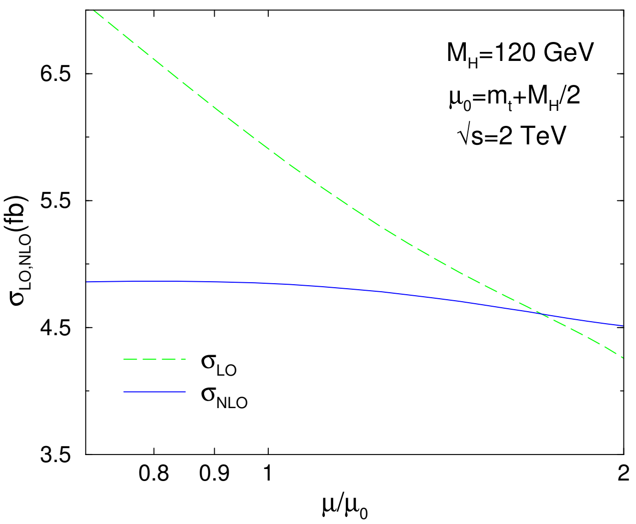
$$\mathcal{F}_i^{\mathcal{H}}(x) = f_i^{\mathcal{H}}(x,\mu_F) + \alpha_s C_i^{\mathcal{H}}(x,\mu_F) + O(\alpha_s^2)$$

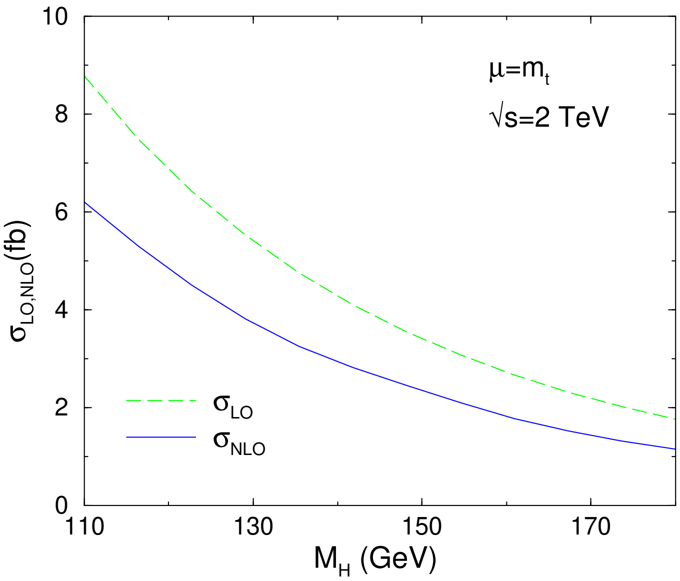
and finally add:

$$\sigma_{crossing} = \sum_{ij}\int dx_1 \int dx_2 \left\{C_i^p(x_1,\mu) f_j^{\bar{p}}(x_2,\mu) + f_i^p(x_1,\mu) C_j^{\bar{p}}(x_2,\mu)\right\}$$

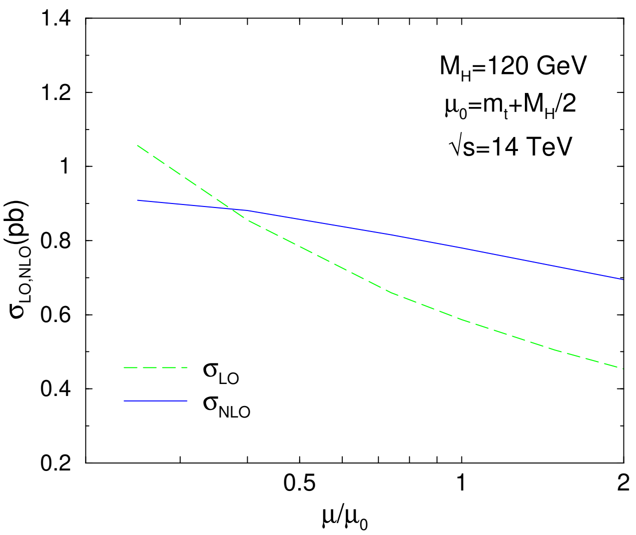
$p\bar{p}, pp \to t\bar{t}H_{\mathrm{SM}}$: NLO cross section

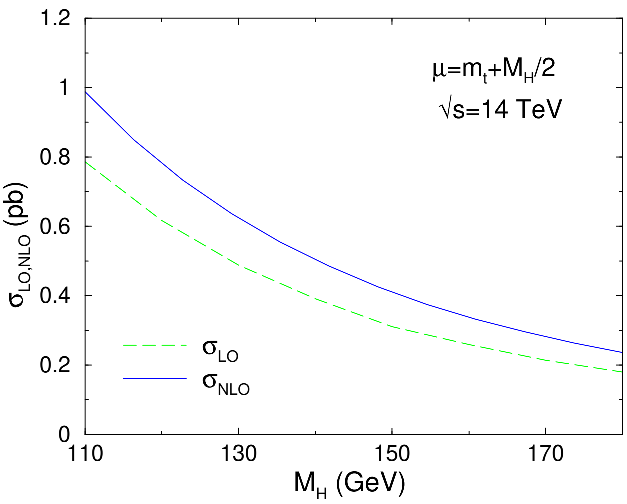
At the Tevatron





At the LHC





Theoretical uncertainty reduced to about 15-20%

Effective Higgs Approximation

$$M_h < M_t \ll \sqrt{s}$$

S.Dawson,L.R.

$$\hat{\sigma}_{ij}^{NLO}(x_1, x_2) = \int f_{t\to h}(x_h)\, \hat{\sigma}(ij \to t\bar{t})\, dx_h$$

Now we can compute

$$\text{QCD corrections} \begin{cases} f_{t\to h}(x_h) \simeq 1 - 4\frac{\alpha_s}{\pi} \\ \sigma(ij \to t\bar{t}) \to \delta_{QCD} \end{cases}$$

and estimate

$$K = \frac{\sigma_{NLO}}{\sigma_{LO}}$$

- $p\bar{p} \to t\bar{t}h_{\mathrm{SM}}$ ($\sqrt{s}=2$ TeV): $K \simeq 0.6 - 0.7$

$$\delta_{QCD}(p\bar{p} \to t\bar{t}) \simeq 2\frac{\alpha_s}{\pi}$$

- $pp \to t\bar{t}h_{\mathrm{SM}}$ ($\sqrt{s} = 14$ TeV) : $K \simeq 1.2 - 1.6$

$$\delta_{QCD}(pp \to t\bar{t}) \simeq 10\frac{\alpha_s}{\pi}$$

Conclusions

- $t\bar{t}H$ very interesting production mode at present and future colliders:

  ▷ discover/confirm the Higgs

  ▷ SM? New Physics?

- Crucial to know the impact of QCD corrections

- $\sigma(p\bar{p} \to t\bar{t}h_{SM})$ calculated at $\mathcal{O}(\alpha_s^3)$:
  K=0.7-1.0 (confirmed by two groups)

  Possible improvement $\to$ threshold studies

- $\sigma(pp \to t\bar{t}h_{SM})$ at $\mathcal{O}(\alpha_s^3)$:
  K=1.2-1.4 (K=1.3-1.5)
  (almost confirmed by two groups)