Electroweak corrections at high energies

Stefano Pozzorini

Paul Scherrer Institut, Villigen, Switzerland

in collaboration with
A. Denner, M. Melles

RADCOR, 9. September 2002

- Large logarithmic corrections
- Existing 1-loop and 2-loop results
- New (preliminary) 2-loop results

  $\rightarrow$ massive 2-loop integrals in logarithmic approximation

Introduction

- at energies $\sqrt{s} \gg M_W$ the **EW corrections are enhanced** by large $\log(s/M_W^2)$

- at N-loop level

$$\begin{aligned} \mathrm{LL}: &\quad \alpha^N \log^{2N}\left(\frac{s}{M_W^2}\right) \\ \mathrm{NLL}: &\quad \alpha^N \log^{2N-1}\left(\frac{s}{M_W^2}\right) \\ \mathrm{NNLL}: &\quad \alpha^N \log^{2N-2}\left(\frac{s}{M_W^2}\right) \end{aligned}$$

  grow with energy

- at future TeV colliders they exceed the few percent level for $N \leq 2$

- high-precision theoretical predictions require 1- and 2-loop corrections

General remarks

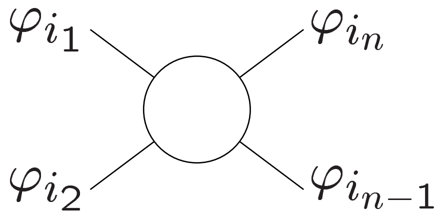
- multi-scale problem: large $\log(r_{kl}/M_i^2)$ with several ratios of energy and mass scales

$$\underbrace{(p_k+p_l)^2}_{r_{kl}} \gg M_{\mathrm{t,H}}^2 \sim M_{\mathrm{W,Z}}^2 \gg m_f^2 \gg \underbrace{\lambda^2}_{\mathrm{IR-reg}}$$

- we restrict ourselves to exclusive processes and virtual corrections

- we consider EW logarithmic corrections to on-shell matrix elements

$$\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} =$$



with arbitrary polarized external particles

$$f^{\mathrm{L}}, f^{\mathrm{R}}, \quad \gamma_{\mathrm{T}}, \mathrm{Z}_{\mathrm{T}}, \mathrm{W}_{\mathrm{T}}^{\pm}, \quad \mathrm{Z}_{\mathrm{L}}, \mathrm{W}_{\mathrm{L}}^{\pm}, \quad H$$

High-energy limit for EW matrix elements

In the limit $M^2/s \to 0$ we consider logarithmic mass singularities

$$\frac{\delta\mathcal{M}_N}{\mathcal{M}_0} \sim \alpha^N \left[\log^{2N-k}\frac{|r_{kl}|}{M^2} + \mathcal{O}\left(\frac{M^2}{|r_{kl}|}\log^{2N-k}\frac{|r_{kl}|}{M^2}\right)\right]$$

and neglect **mass-suppressed** corrections

- M terms are neglected in the loop numerators

- $1/M$ terms from longitudinal gauge bosons

$$\varepsilon_{\mathrm{L}}^{\mu}(k,M) = \frac{k^\mu}{M} + \mathcal{O}\left(\frac{M}{k_0}\right)$$

  are avoided by using the Goldstone-boson equivalence theorem

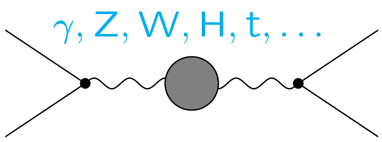
$$\mathcal{M}^{\varphi_{i_1}\cdots \mathrm{Z_L}\cdots\varphi_{i_n}} \simeq \mathrm{i}(1+\delta A^\chi)\mathcal{M}^{\varphi_{i_1}\cdots\chi\cdots\varphi_{i_n}}$$
$$\mathcal{M}^{\varphi_{i_1}\cdots \mathrm{W}^\pm_{\mathrm{L}}\cdots\varphi_{i_n}} \simeq \pm(1+\delta A^\phi)\mathcal{M}^{\varphi_{i_1}\cdots\phi^\pm\cdots\varphi_{i_n}}$$

- NLL corrections $\delta A^{\chi,\phi}$ to the LO GBET have to be taken into account

Origin of $\log(r_{kl}/M_i^2)$ corrections

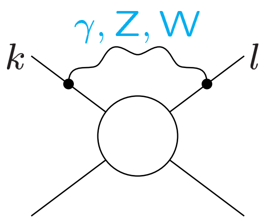
UV singularities $\Leftrightarrow$ renormalization of couplings at the scale M_i

- $\alpha \log(r_{kl}/M_i^2)$ from running of couplings

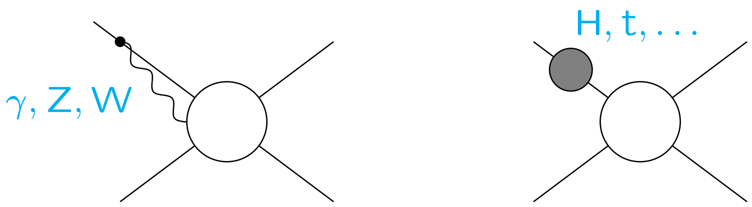


Mass singularities $\Leftrightarrow$ virtual particles coupling to **on-shell external legs**

- $\alpha \log^2(r_{kl}/M_i^2)$ from soft collinear γ, Z, W



- $\alpha \log(r_{kl}/M_i^2)$ from collinear particles



Existing results at 1 loop

Universality of EW logarithms [Denner and P. (2001)]

$$\mathcal{M}_1 = \left(1 + \underbrace{\delta^1_{\mathrm{EW}}}_{\mathrm{soft,coll.}}\right) \underbrace{\mathcal{M}_0}_{g\to g(s)}, \qquad \delta^1_{\mathrm{EW}} = \frac{\alpha}{4\pi} \sum_{\mathbf{legs}\,k} \delta^1_{\mathrm{EW}}(k)$$

- symmetric electroweak part

$$\delta^1_{\mathrm{sew}}(k) = \delta^1_{\mathrm{EW}}(k)\Big|_{\lambda=M_{\mathrm{W}}}$$
$$= -\frac{1}{2} C^{\mathrm{ew}}(k) \log^2 \frac{s}{M_{\mathrm{W}}^2} + \gamma^{\mathrm{ew}}(k) \log \frac{s}{M_{\mathrm{W}}^2}$$
$$+ \sum_{l\neq k} \sum_{a=\gamma,\mathrm{Z},\mathrm{W}^\pm} I^a(k) I^{\bar{a}}(l) \log \frac{r_{kl}}{s} \log \frac{s}{M_{\mathrm{W}}^2}$$

- subtracted electromagnetic part

$$\delta^1_{\mathrm{sem}}(k) = \delta^1_{\mathrm{EM}}(k) - \delta^1_{\mathrm{EM}}(k)\Big|_{\lambda=M_{\mathrm{W}}}$$
$$= -\frac{1}{2} Q^2(k) \Big[2 \log \frac{s}{m_k^2} \log \frac{M_{\mathrm{W}}^2}{\lambda^2} - \log^2 \frac{M_{\mathrm{W}}^2}{m_k^2}$$
$$-2 \log \frac{M_{\mathrm{W}}^2}{\lambda^2} - \log \frac{M_{\mathrm{W}}^2}{m_k^2} \Big] + \sum_{l\neq k} Q(k) Q(l) \log \frac{r_{kl}}{s} \log \frac{M_{\mathrm{W}}^2}{\lambda^2}$$

In agreement with explicit calculations

$e^+e^- \to W^+W^-$ [Beenakker et.al (1993)],
$e^+e^- \to f\bar{f}, b\bar{b}, t\bar{t}$ [Beccaria et.al. (2000,2001)],
$\gamma\gamma \to f\bar{f}$ [Layssac and Renard (2001)],
$e^+e^- \to \gamma\gamma, \gamma Z, ZZ$ [Gounaris et.al. (2002)].

Electroweak Casimir operator

$$C^{\mathrm{ew}} = \frac{1}{c_{\mathrm{W}}^2}\left(\frac{Y}{2}\right)^2 + \frac{1}{s_{\mathrm{W}}^2} C^{\mathrm{SU(2)}}$$

Anomalous dimensions for fermions

$$\gamma^{\mathrm{ew}}(k) = \begin{cases} \frac{3}{2} C^{\mathrm{ew}}(k) & \text{light fermions} \\ \frac{3}{2} C^{\mathrm{ew}}(k) - \frac{m_{\mathrm{t}}^2}{8 s_{\mathrm{W}}^2 M_{\mathrm{W}}^2} & \mathrm{t_L}, \mathrm{b_L} \\ \frac{3}{2} C^{\mathrm{ew}}(k) - \frac{m_{\mathrm{t}}^2}{4 s_{\mathrm{W}}^2 M_{\mathrm{W}}^2} & \mathrm{t_R} \end{cases}$$

Anomalous dimensions for bosons

$$\gamma^{\mathrm{ew}}(k) = \begin{cases} \frac{1}{2} U b^{\mathrm{ew}} U^{-1} + \delta \hat{U} U^{-1} & \mathrm{W_T^{\pm}}, \mathrm{Z_T}, \gamma_{\mathrm{T}} \\ 2 C^{\mathrm{ew}}(k) - \frac{3 m_{\mathrm{t}}^2}{4 s_{\mathrm{W}}^2 M_{\mathrm{W}}^2} & \mathrm{H}, \mathrm{W_L^{\pm}}, \mathrm{Z_L} \end{cases}$$

Typical size at $\sqrt{s} \simeq 1$ TeV

1-Loop: angular **independent**

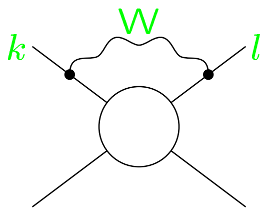
$$\left(\frac{\delta\sigma_{1,\text{sew}}}{\sigma_0}\right)_{\text{LL}} \simeq -\frac{\alpha}{\pi s_{\text{W}}^2}\log^2\frac{s}{M_{\text{W}}^2} \simeq -26.4\%$$

$$\left(\frac{\delta\sigma_{1,\text{sew}}}{\sigma_0}\right)_{\text{NLL}} \simeq +\frac{3\alpha}{\pi s_{\text{W}}^2}\log\frac{s}{M_{\text{W}}^2} \simeq +15.6\%$$

$$\left(\frac{\delta\sigma_{1,\text{sew}}}{\sigma_0}\right)_{\text{NLL}} \simeq -\frac{\alpha}{4\pi s_{\text{W}}^2}\frac{m_{\text{t}}^2}{M_{\text{W}}^2}\log\frac{s}{m_{\text{t}}^2} \simeq -4.3\%$$

- contributions of $\mathcal{O}(10\%)$
- large cancellations between LL and NLL

1-Loop: angular **dependent**



$$\Rightarrow \quad \frac{\alpha}{2\pi}\log\frac{r_{kl}}{s}\log\frac{s}{M_{\text{W}}^2} I^+(k)I^-(l)\mathcal{M}_0$$

- involve $I^{\pm}(k)$ $\Rightarrow$ more difficult to estimate in a process-independent way
- in general similar size and same sign of NLL corrections $\Rightarrow$ non negligible!

$$\boxed{\text{Typical size at } \sqrt{s} \simeq 1\ \text{TeV}}$$

2-Loop: angular **independent**

$$\left(\frac{\delta\sigma_{1,\mathrm{sew}}}{\sigma_0}\right)_{\mathrm{LL}} \simeq +\frac{\alpha^2}{2\pi^2 s_{\mathrm{W}}^4}\log^4\frac{s}{M_{\mathrm{W}}^2} \simeq 3.5\%$$

$$\left(\frac{\delta\sigma_{1,\mathrm{sew}}}{\sigma_0}\right)_{\mathrm{NLL}} \simeq -\frac{3\alpha^2}{\pi^2 s_{\mathrm{W}}^4}\log^3\frac{s}{M_{\mathrm{W}}^2} \simeq -4.1\%$$

$$\left(\frac{\delta\sigma_{1,\mathrm{sew}}}{\sigma_0}\right)_{\mathrm{NLL}} \simeq +\frac{\alpha^2}{4\pi^2 s_{\mathrm{W}}^4}\frac{m_{\mathrm{t}}^2}{M_{\mathrm{W}}^2}\log\frac{s}{m_{\mathrm{t}}^2}\log^2\frac{s}{M_{\mathrm{W}}^2} \simeq 1.2\%$$

- contributions of $\mathcal{O}(1\%)$
- NLL $\gtrsim$ LL
- large cancellations

EW logarithms at 2 loops

Analogy between **spontaneously broken** (EW) and **symmetric** (QED,QCD) **gauge thories**

- "in the high-energy limit the EW theory is in the symmetric phase"

$\Rightarrow$ Extension of QED and QCD results

- **evolution equations**: $SU(3)_c \rightarrow SU(2) \times U(1)_Y$
- all-order resummation of logarithms

Nontrivial effects from **spontaneous symmetry breaking**, in particular in the gauge sector

- mass gap: $M_\mathrm{W}, M_\mathrm{Z} \gg \lambda$
- mixing: $(\mathrm{W}^3, \mathrm{B}) \rightarrow (\gamma, \mathrm{Z})$
- longitudinal gauge bosons

$\Rightarrow$ Calculations within the EW theory

- consider all mass-singular **2-loop diagrams**
- evaluate **massive loop integrals** in logarithmic approximation

Existing Results at 2 loops

LL exponentiation for arbitrary processes

- $\mu_\perp$–IREE: $SU(2) \times U(1)_Y \to U(1)_{\text{em}}$

  [Fadin et.al. (2000)]

- Coulomb gauge for massive GB's: 2-loop calculation

  [Beenakker and Werthenbach (2000,2002)]

NLL exponentiation for arbitrary processes

- matching of $SU(2) \times U(1)_Y$ and $U(1)_{\text{em}}$ at the EW scale [Melles (2001,2002)]

$\Rightarrow$ ang-indep NLL
$\Rightarrow$ *ang-dep* NLL

NNLL exponentiation for massless $f\bar{f} \to f'\bar{f}'$

- QCD evolution equation [Kühn et.al. (2000,2001)]

$\Rightarrow$ large *ang-dep* logarithms!

do the

angular-dependent NLL corrections

for **arbitrary processes**

exponentiate?

New 2-loop electroweak calculation

- LL from **soft-collinear** $\gamma, \mathrm{Z}, \mathrm{W}^{\pm}$

$$\alpha^2 \log^4 \frac{r_{ij}}{M^2}, \qquad r_{ij} = (p_i + p_j)^2$$

- including **NLL** **angular dependence** from external-leg correlations

$$\log^4 \frac{r_{ij}}{M^2} \to \log^4 \frac{s}{M^2} + 4 \log \frac{r_{ij}}{s} \log^3 \frac{s}{M^2}$$

Outline of the calculation

(1) Feynman diagrams of order $\alpha^2 \log^4 \frac{r_{ij}}{M^2}$

(2) eikonal approximation

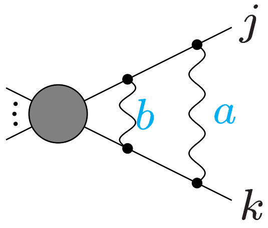
(3) **loop integrals to logarithmic accuracy**

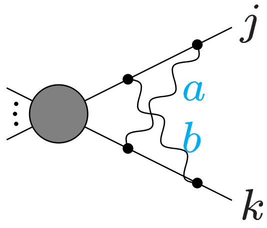
(4) sum of diagrams using global gauge invariance

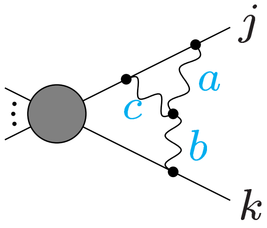
(1) Feynman diagrams of order $\alpha^2 \log^4 \frac{r_{ij}}{M^2}$

Feynman–t'Hooft gauge $\Rightarrow$ diagrams with soft-collinear $\gamma, \mathrm{Z}, \mathrm{W}^{\pm}$ coupling to:

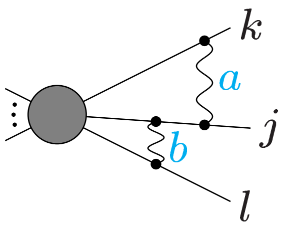
- 2 different external legs

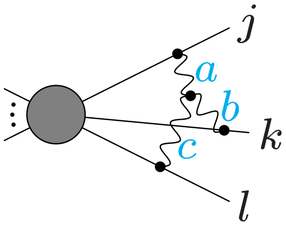
(a) 

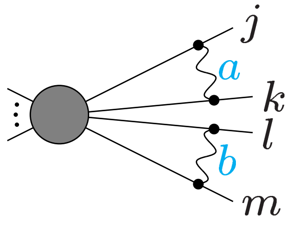
(b) 

(c) 

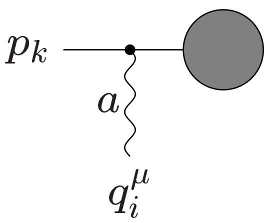
- 3 and 4 different external legs

(d) 

(e) 

(f) 

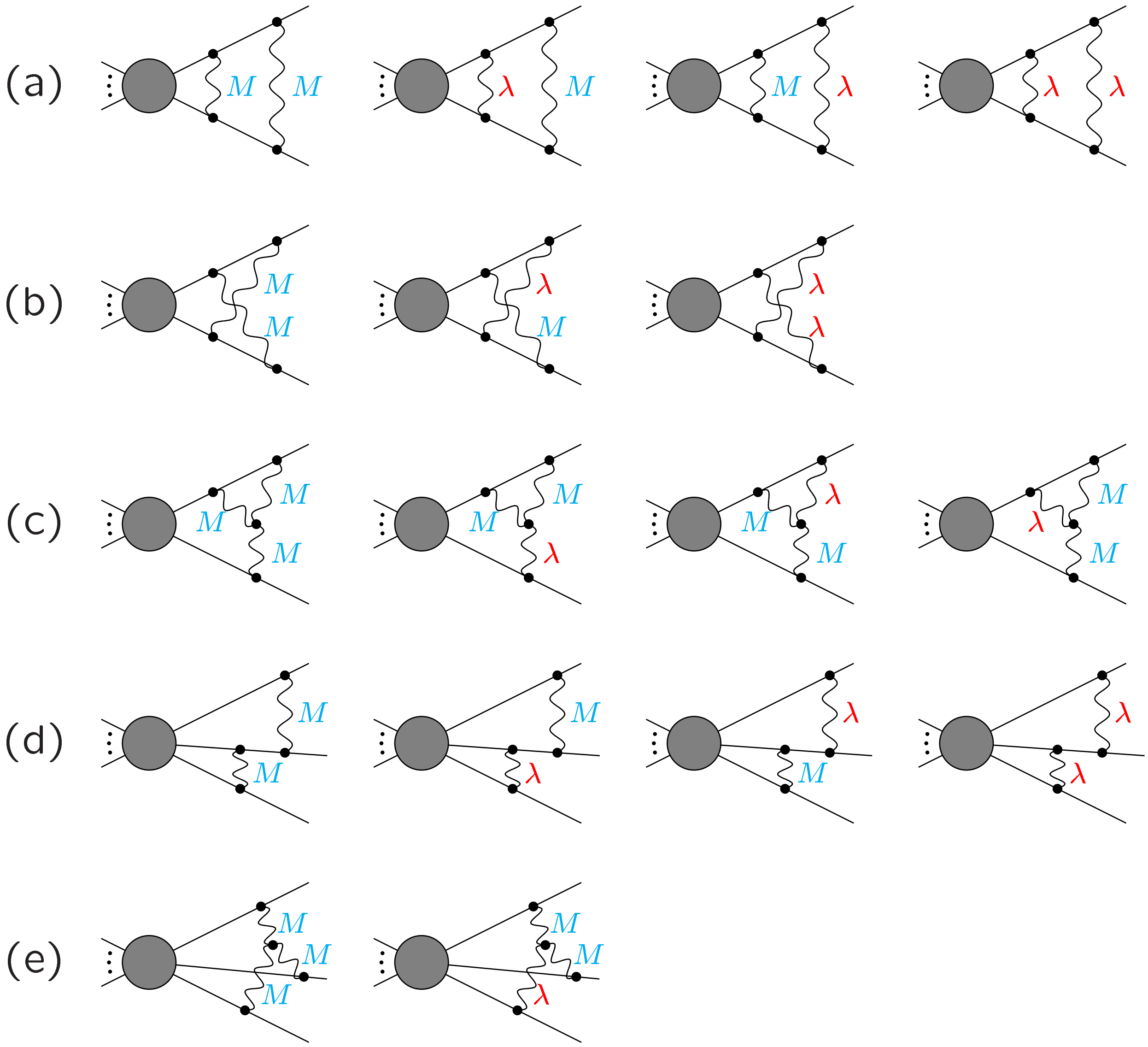
(2) Eikonal approximation



$$\Rightarrow \frac{-2ep_k^\mu I^a(k)}{\left((p_k - q_i)^2 - m_k^2 + \mathrm{i}\epsilon\right)} \frac{-\mathrm{i}}{\left(q_i^2 - M_a^2 + \mathrm{i}\epsilon\right)} \mathcal{M}_0 \bigg|_{q_i=0}$$

- universal factor $-2ep_k^\mu\, I^a(k)$ for eikonal vertices
- hard part of $\mathcal{M}$ in the soft limit $q_i = 0$

Mass assignements for EW diagrams



- gauge-boson masses

$$M = M_{\mathrm{W}} \simeq M_{\mathrm{Z}} \gg \lambda = M_{\gamma}$$

- external-leg masses

$$M \geq \mathbf{m}_{\mathrm{ext}} \gg \lambda$$

(3) 2-loop massive integrals in logarithmic app.

A) Sudakov method

- **Sudakov parametrization**

$$q_1^\mu = x_1 p_i^\mu + y_1 p_j^\mu + q_{1,\perp}^\mu$$
$$q_2^\mu = x_2 p_k^\mu + y_2 p_l^\mu + q_{2,\perp}^\mu$$

- $q_\perp^\mu \to 0$ in ladder diagrams

- **soft-collinear sigularities** $\sim 1/(x_1 y_1 x_2 y_2)$

B) Sector decomposition method

- **Feynman parametrization**

- factorization of **generic mass singularities**

- 1-loop massive integrals [Denner and Roth (1996)]
  3-loop massless integrals [Binoth and Heinrich (2000)]

(3.B) Sector decomposition method

1) Arbitrary massive 2-loop integral

$$I = \int \frac{\mathrm{d}^d q_1}{(2\pi)^d} \int \frac{\mathrm{d}^d q_2}{(2\pi)^d} \frac{N(\{q_j\},\{p_l\})}{\prod_{i=1}^{n+1}(k_i^2 - m_i^2 + \mathrm{i}\epsilon)}$$

2) Feynman parametrization

$$I = \int_{[0,1]^n} \mathrm{d}\vec{x} \frac{f(\vec{x})}{\left[sP_s(\vec{x}) + M^2 P_M(\vec{x}) + \ldots + \lambda^2 P_\lambda(\vec{x})\right]^e}$$

- hierarchy $s \gg M^2 \gg \cdots \gg \lambda^2$

- **mass singularities** from **zeros** of type

$$P(\vec{x}) = \sum_{i=1}^{m} x_i P_i(\vec{x}) = 0 \quad \text{at } x_1 = \cdots = x_m = 0$$

3) Decomposition of $[0,1]^m$ into m sectors Ω_j

$$\Omega_j : x_j > x_{i \neq j} \qquad\qquad T_j : x_i \to x_j x_i'$$

$\Rightarrow$ remapping $\Omega_j \to [0,1]^m$ and **factorization** of x_j

$$P(\vec{x}) = \left[P_j(\vec{x}) + \sum_{i \neq j} x_i' P_i(\vec{x})\right] x_j$$

4) Recursive application : $I = \sum_{\vec{j}} I_{\vec{j}}$

$$P(\vec{x}) = \hat{P}(\vec{x}) \prod_k x_k, \quad \text{with} \quad \hat{P}(\vec{0}) \neq 0$$

all zeros are factorized at each scale $s, M^2, \ldots \lambda^2$

$$I_{\vec{j}} = \int_{[0,1]^n} \mathrm{d}\vec{x}\, \hat{f}(\vec{x}) \prod_{i=1}^{n} x_i^{a_i} \times$$
$$\left\{ \left[s\hat{P}_s(\vec{x}) \prod_k x_k + M^2 \hat{P}_M(\vec{x}) \right] \prod_l x_l + \ldots + \lambda^2 \hat{P}_\lambda(\vec{x}) \right\}^{-e}$$

$\Rightarrow$ simple x-power-counting for logarithms

$\Rightarrow$ $\hat{P}(\vec{x}) \simeq \hat{P}(\vec{0})$ to LL accuracy

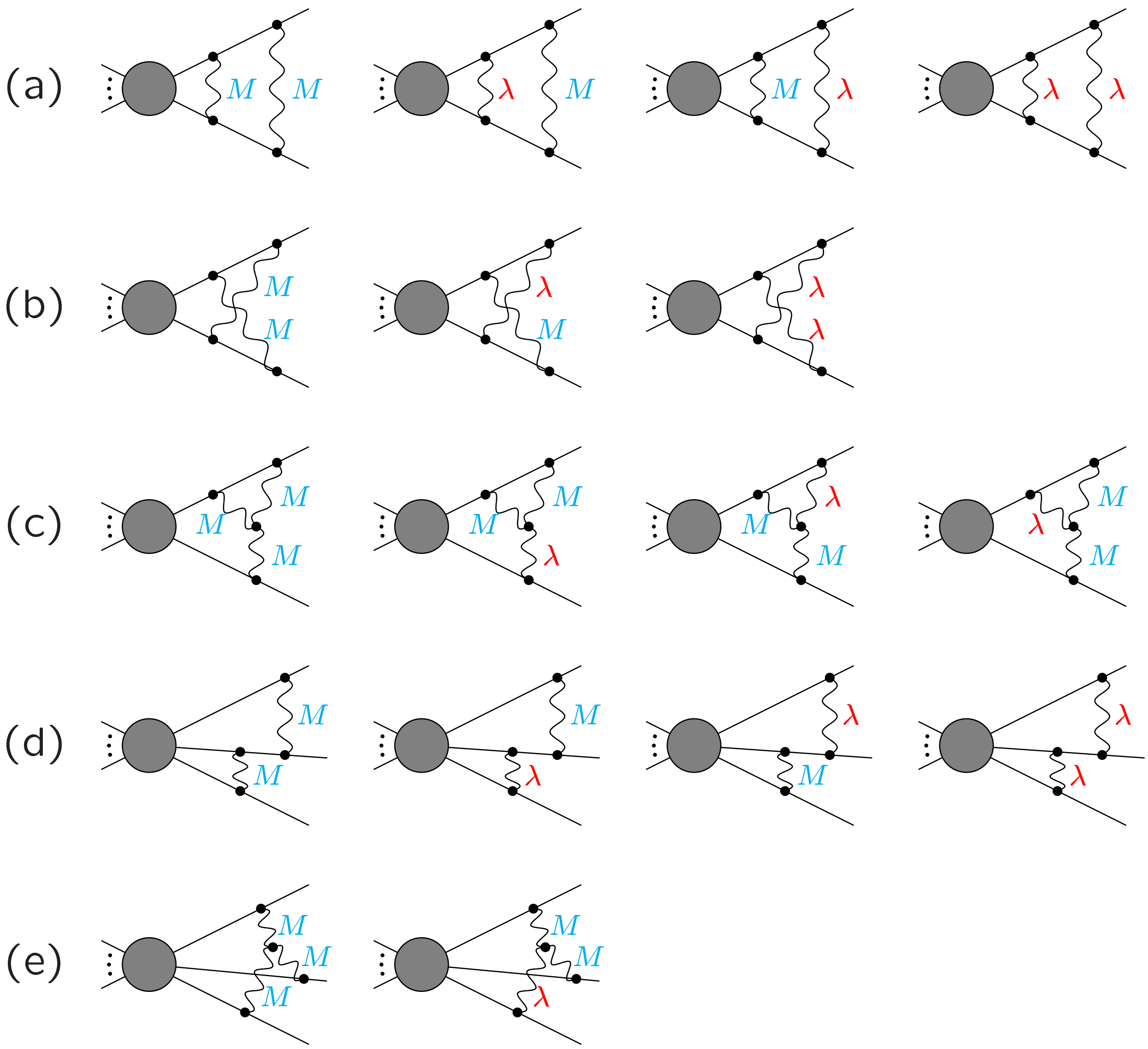
5) Set of standard integrals: LL analytical evaluation

$$\int_0^1 \mathrm{d}^l\vec{x}\, \mathrm{d}^m\vec{y}\, \mathrm{d}^n\vec{z} \frac{s^e \left[\prod x_i \prod y_j \prod z_k\right]^{e-1}}{\left\{\left[\left(s \prod x_i + M^2\right) \prod y_j + m^2\right] \prod z_k + \lambda^2\right\}^e}$$
$$= \sum_{p=0}^{m} \sum_{q=0}^{n+m-p} \frac{1}{p!} \frac{1}{q!} \frac{1}{(N-p-q)!}$$
$$\times \log^p\left(\frac{m^2}{\lambda^2}\right) \log^q\left(\frac{M^2}{m^2}\right) \log^{N-p-q}\left(\frac{s}{M^2}\right)$$

Algorithm for sector decomposition

- arbitrary 2-loop topologies
- LL accuracy, can be extended to NLL accuracy
- N-scale hierarhcies, with $N \leq 4$

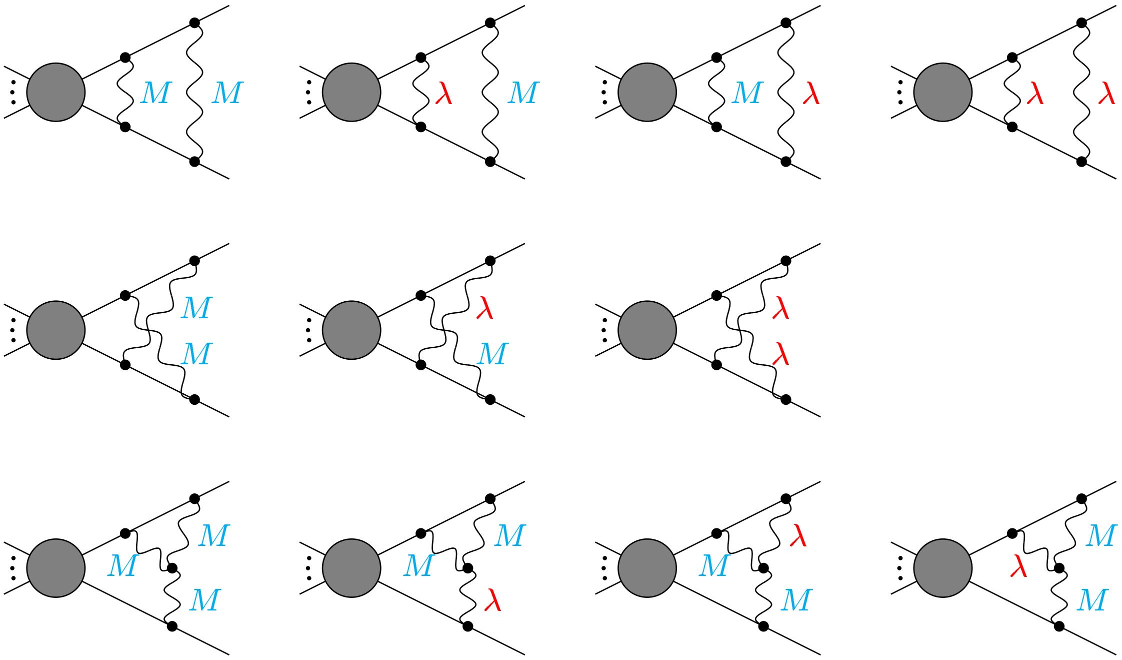
(3) Results for loop integrals



- 11/17 cases with sector decomposition
- 15/17 cases with Sudakov method
- 17/17 cases calculated
- 9/17 cases cross-checked successfully

Results for loop integrals

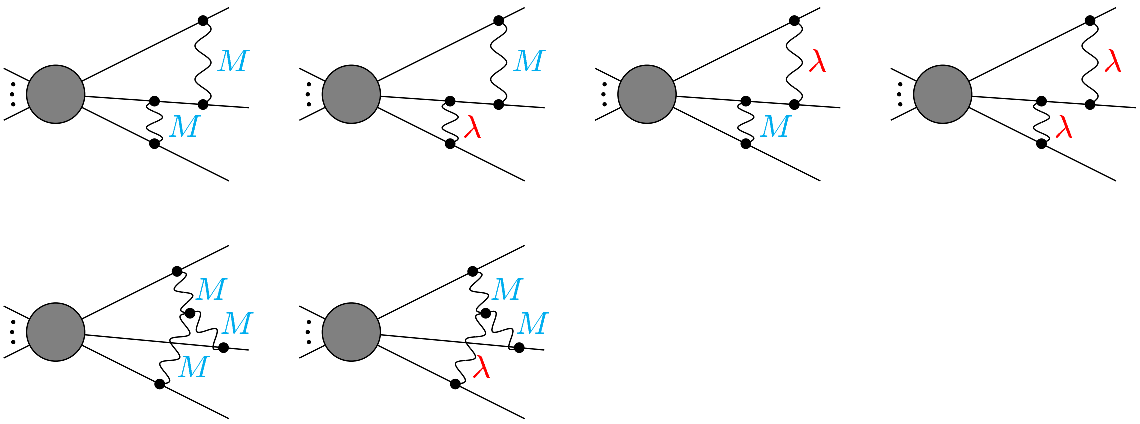
(a) (b) (c)



Diagrams (a–c) in agreement with form-factor calculation for **massless** external particles

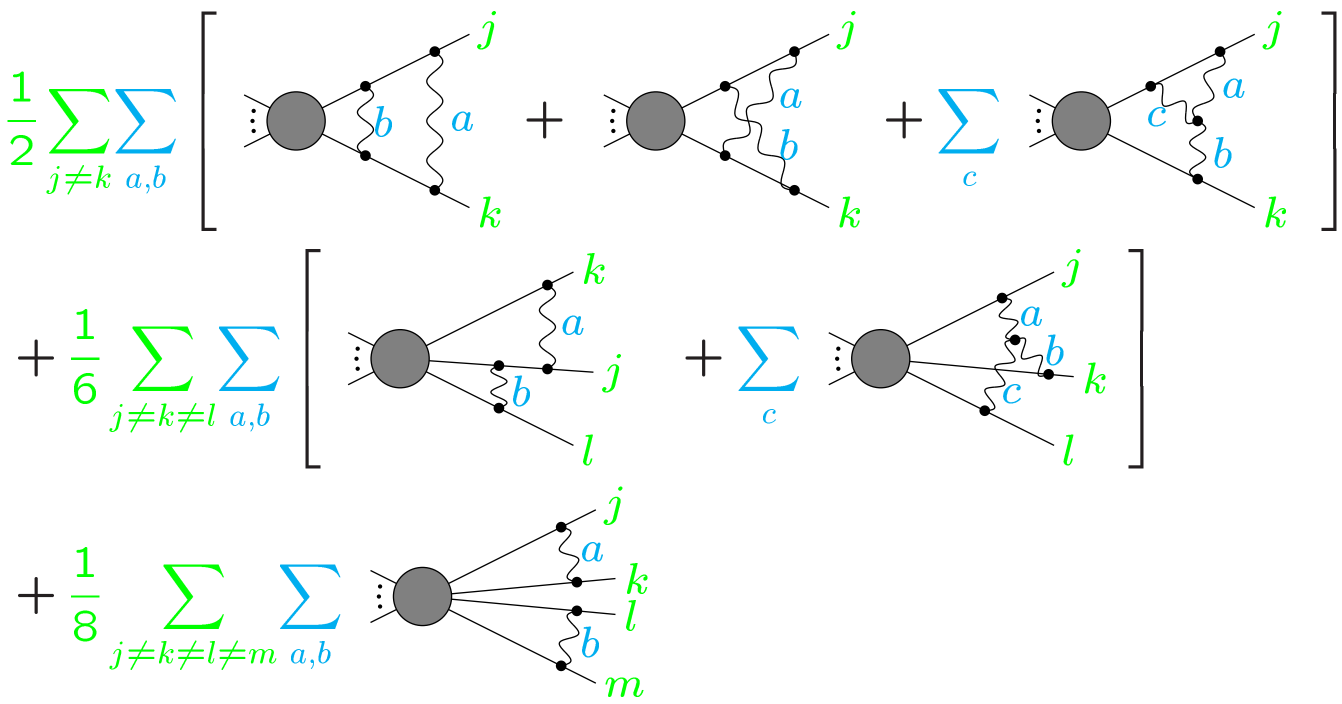
[Hori, Kawamura and Kodaira (2000)]

(d) (e)



(4) Sum of eikonal contributions

summing over external legs $j, k, l, m, = 1, \ldots, n$ and gauge bosons $\gamma, \mathrm{Z}, \mathrm{W}^{\pm}$



simplifying the sum with **global gauge symmetry**

$$\sum_{k=1}^{n} I^a(k)\mathcal{M} = \mathcal{O}\left(\frac{M^2}{s}\right) \times \mathcal{M} \simeq 0$$

and relating the result to the 1-loop eikonal corrections we find that . . .

Result

- the **2-loop eikonal corrections** are given by the **exponentiated** 1-loop eikonal corrections

$$\mathcal{M}_{2,\mathrm{eik}} = \exp\left(\delta^{1,\mathrm{eik}}_{\mathrm{sem}}\right)\, \exp\left(\delta^{1,\mathrm{eik}}_{\mathrm{sew}}\right)\, \mathcal{M}_0$$

- the symmetric electroweak corrections

$$\delta^1_{\mathrm{sew}} = \delta^1_{\mathrm{EW}}\Big|_{\lambda=M_{\mathrm{W}}}$$

and the subtracted electromagnetic corrections

$$\delta^1_{\mathrm{sem}} = \delta^1_{\mathrm{EM}} - \delta^1_{\mathrm{EM}}\Big|_{\lambda=M_{\mathrm{W}}}$$

exponentiate separately.

Comparison

- At **LL** level: confirms results from IREE [Fadin et.al. (2000)] and Coulomb gauge [Beenakker and Werthenbach (2000,2002)]

- At **angular-dependent NLL** level: in agreement with the *matching prescriptions* for $f\bar{f} \to f'\bar{f}'$ [Kühn et.al. (2000,2001)] and arbitrary processes [Melles (2002)]

Matching

$$\lambda = M \Rightarrow \delta^1_{\mathrm{sem}} = 0 \Rightarrow \mathcal{M} = \exp\left(\delta^1_{\mathrm{sew}}\right) \mathcal{M}_0$$
$$s = M \Rightarrow \delta^1_{\mathrm{sew}} = 0 \Rightarrow \mathcal{M} = \exp\left(\delta^1_{\mathrm{sem}}\right) \mathcal{M}_0$$

- provides a consistency check of our result
- but is not sufficient to determine the interference terms $\delta^1_{\mathrm{em}}\delta^1_{\mathrm{sym}}$
- in particular

$$\left[\exp\left(\delta^1_{\mathrm{sem}}\right), \exp\left(\delta^1_{\mathrm{sew}}\right)\right] \simeq \left[Q^2(k), I^+(k)I^-(l)\right] \times \mathcal{O}\left(\alpha^2 \log\frac{r_{ij}}{s}\log^2\frac{s}{M^2}\log\frac{M^2}{\lambda}\right)$$

2-loop calculation

- confirms the ansatz

$$\mathcal{M} = \exp\left(\delta^1_{\mathrm{sem}}\right) \exp\left(\delta^1_{\mathrm{sew}}\right) \mathcal{M}_0$$
$$\neq \exp\left(\delta^1_{\mathrm{sem}} + \delta^1_{\mathrm{sew}}\right) \mathcal{M}_0$$

- and the physical picture that *"the electromagnetic corrections below the electroweak scale are only sensitive to the external-leg charges"*

Conclusions

We have calculated 2-loop virtual electroweak corrections

- to **arbitrary processes**
- in the **high-energy** limit $r_{kl} \gg M_{\mathrm{W}}^2$
- using the **eikonal approximation**

We have evaluated 2-loop massive integrals

- to LL accuracy, including NLL **angular-dependent** contributions
- using two independent methods: **Sudakov method** and **sector decomposition**

The 2-loop LL and angular-dependent NLL

- result from **exponentiation** of the 1-loop corrections
- the *symmetric electroweak* and *subtracted electromagnetic* parts exponentiate separately