Decoupling behaviour of $\mathcal{O}(m_t^4)$ corrections to the h^0 self-couplings.

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From work in collaboration with:

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- Introduction

- Tree-level Higgs boson self-couplings

- Leading one-loop contributions from $t - \tilde{t}$ sector
  - Analytical studies

- Numerical analysis for heavy top-squarks sector

- Conclusions

MOTIVATIONS/GOALS

- To establish the Higgs mechanism experimentally, the Higgs self-interaction potential must be reconstructed.

  This task requires the measurement of the trilinear and quartic self-couplings, as predicted in the Standard Model or in supersymmetric theories

- Use radiative corrections to obtain information for establishing the Higgs potential and thus the Higgs mechanism

  - Three-point one-loop radiative corrections for the neutral Higgs system in the MSSM have been calculated within the effective potential approximation and in some limiting situations

  V. Barger et al., Phys.Rev. D45 (1992)
  P.Osland and P.N.Pandita, Phys.Rev. D59 (1999), hep-ph/9806351

  - Phenomenological studies have addressed the issue of the measurements of some of the Higgs self-couplings in the MSSM

  M. Mühlleitner, hep-ph/0008127 and references therein

- Our intention is to investigate how far the MSSM Higgs potential reproduces the SM potential when the non-standard particles are heavy

- We want to explore decoupling behaviour, both numerically and analytically, of the radiative corrections to h^0 self-couplings at the one-loop level

- We have started by

  – Leading contributions from $t-\tilde{t}$ sector
  ($m_t > m_{h^0}$ in the MSSM)
  hep-ph/0108245

  – Contributions from the Higgs sector itself
  hep-ph/0208014

Tree-level Higgs boson self-couplings

Trilinear and quartic SM and MSSM Higgs boson self-couplings

ϕ		$\lambda_{\phi\phi\phi}$	$\lambda_{\phi\phi\phi\phi}$
SM	H	$\frac{3gM_H^2}{2\,M_W}$	$\frac{3g^2M_H^2}{4\,M_W^2}$
MSSM	h^o	$\frac{3gM_Z}{2c_W}\cos 2\alpha \sin(\beta+\alpha)$	$\frac{3g^2}{4c_W^2}\cos^2 2\alpha$

Decoupling limit in the Higgs sector,
Haber & Nir 1990

$M_A \gg M_Z$

$$M_{H^o} \simeq M_{H^\pm} \simeq M_A \gg M_Z$$
$$M_{h^o} \simeq M_Z |\cos 2\beta|$$

$$\alpha \to \beta - \frac{\pi}{2} \quad \Rightarrow \quad \cos 2\alpha \to -\cos 2\beta \quad \sin(\beta+\alpha) \to -\cos 2\beta$$

$$\Downarrow \quad \phi \equiv h^0$$

$$\lambda^0_{\phi\phi\phi} \simeq \frac{3g}{2\,M_W} M^{2\,\mathrm{tree}}_{h^0}\,, \quad \lambda^0_{\phi\phi\phi\phi} \simeq \frac{3g^2}{4\,M_W^2} M^{2\,\mathrm{tree}}_{h^0}$$

Tree-level couplings lead to equal results in the MSSM and in the SM in the *decoupling limit*
$\Rightarrow$ Decoupling at tree-level

$\mathcal{O}(m_t^4)$ one-loop contributions

Work in collaboration with W.Hollik

- We investigate the differences between the corrections to the h^0 self-couplings with respect to the $H_{\rm SM}$ self-couplings
- Analytical results for the $n-$point renormalized vertex functions in the MSSM and in the SM
  Is there decoupling of heavy particles beyond tree-level?
- On-shell renormalization procedure
- Large masses limit

  $m_t > M_{h^0}$ in the MSSM

  $\tilde{t}$ masses are very large as compared to the external momenta and to the EW scale,

$$m^2_{\tilde{t}_1}\,, m^2_{\tilde{t}_2} \gg M_Z^2\,, M^2_{h^0}$$

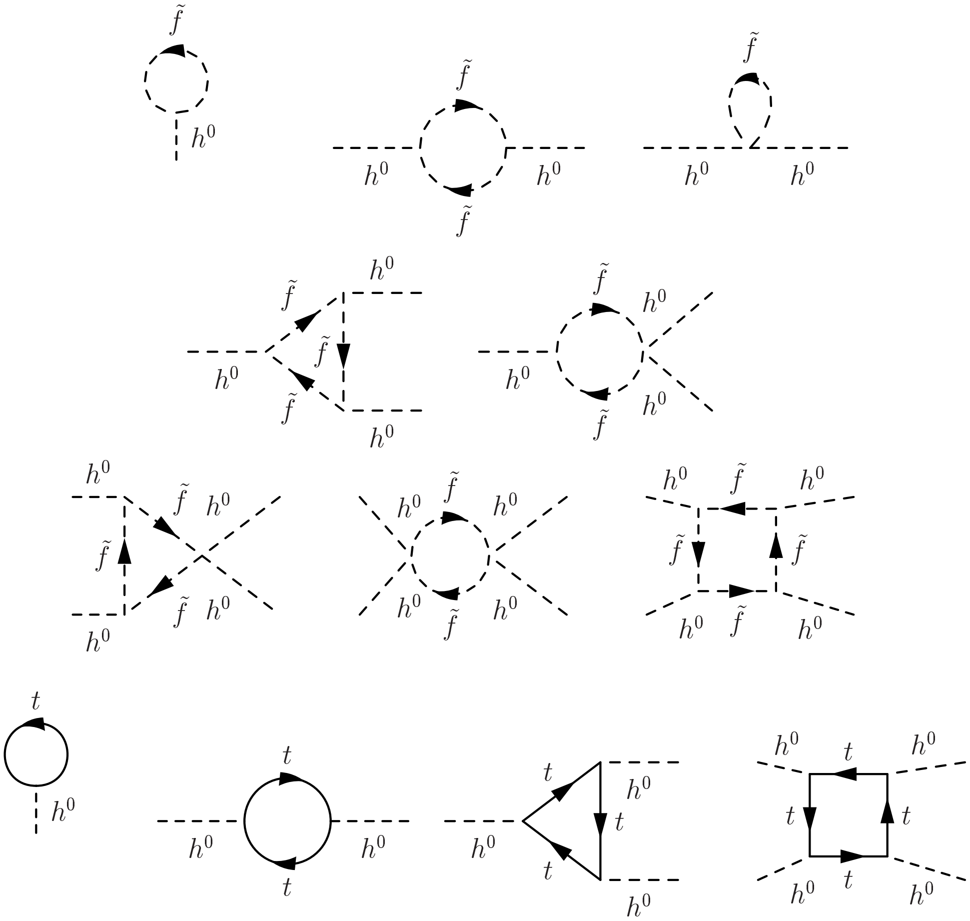
  - close to each other (Analytical studies)

$$|m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}| \ll |m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2}|$$

  - stop mass splitting of the order of SUSY mass scale $M_{\tilde{Q}}$ (Numerical analysis)

$$|m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}| \simeq |m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2}|$$

One-loop $t - \tilde{t}$ diagrams



Green Functions Counterterms: $\delta\Gamma^{(1)}_{h^0}\,,\delta\Gamma^{(2)}_{h^0}\,,\delta\Gamma^{(3)}_{h^0}\,,\delta\Gamma^{(4)}_{h^0}$

$$\Rightarrow\ \delta Z_{H_{1,2}}\,,\delta v\,,\delta g^2\,,\delta g'^2\,,\delta m_1^2\,,\delta m_2^2\,,\delta m_{12}^2$$

$\delta Z_{H_{1,2}}\,,\delta v\,,\delta g^2\,,\delta g'^2 \ \longrightarrow\ \Sigma_\gamma\,,\Sigma_{\gamma Z}\,,\Sigma_Z\,,\Sigma_{W^\pm}\,,\Sigma_{A^0}\,,\Sigma_{A^0 Z}$

$\delta m_1^2\,,\delta m_2^2\,,\delta m_{12}^2 \ \longrightarrow$ - A^0 boson with the pole mass M_{A^0}

- vanishing renormalized tadpoles for $h^0\,,H^0$

RENORMALIZED VERTEX FUNCTIONS

- We consider the large masses limit

$$m_{\tilde{t}_1}^2 \,, m_{\tilde{t}_2}^2 \gg M_Z^2 \,, M_{h^0}^2 \,,$$
$$|m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2| \ll |m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2| \,.$$

- By using the standard on-shell renormalization procedure, we obtain the vertex functions counterterms.

A. Dabelstein, Z. Phys. **C67** (1995) 495; Nucl. Phys. **B456** (1995) 25;
M. Böhm, H. Spiesberger, W. Hollik, Fortsch. Phys. **34** (1986) 687;
W. Hollik, Fortsch. Phys. **38** (1990) 165.

$$\boxed{\text{Results for } \Delta\hat{\Gamma}_{h^0}^{t,\tilde{t}\,(n)} = \Delta\Gamma_{h^0}^{t,\tilde{t}\,(n)} + \delta\Gamma_{h^0}^{t,\tilde{t}\,(n)} \quad {\scriptstyle (n=2,3,4)}}$$

$$\Delta\hat{\Gamma}_{h^0}^{t,\tilde{t}\,(2)} = -\frac{3}{8\pi^2}\frac{g^2}{M_W^2}\, m_t^4 \,\log\frac{m_t^2}{m_{\tilde{t}_1} m_{\tilde{t}_2}} \,,$$
$$\Delta\hat{\Gamma}_{h^0}^{t,\tilde{t}\,(3)} = -\frac{3}{v}\,\frac{3}{8\pi^2}\frac{g^2}{M_W^2}\, m_t^4 \,\log\frac{m_t^2}{m_{\tilde{t}_1} m_{\tilde{t}_2}} - \frac{3}{8\pi^2}\frac{g^3}{M_W^3}\, m_t^4 \,,$$
$$\Delta\hat{\Gamma}_{h^0}^{t,\tilde{t}\,(4)} = -\frac{3}{v^2}\,\frac{3}{8\pi^2}\frac{g^2}{M_W^2}\, m_t^4 \,\log\frac{m_t^2}{m_{\tilde{t}_1} m_{\tilde{t}_2}} - \frac{3}{4\pi^2}\frac{g^4}{M_W^4}\, m_t^4 \,.$$

- Comments

- The UV-divergence cancels out, but the logarithmic and lineal heavy mass terms, which look like a non-decoupling effects of the heavy particles, remains.

- $t - \tilde{t}$ large correction to the h^0 self-couplings, even for large M_{A^0}.

- Leading one-loop corrections from $t, \tilde{t}-$sector to the h^0 Higgs boson mass

$$\Delta M_{h^0}^2 = -\frac{3}{8\pi^2}\frac{g^2}{M_W^2}\, m_t^4 \,\log\frac{m_t^2}{m_{\tilde{t}_1} m_{\tilde{t}_2}}$$

- MSSM Renormalized vertex functions

$$\boxed{\begin{aligned} \Delta\hat{\Gamma}_{h^0}^{t,\tilde{t}\,(2)} &= \Delta M_{h^0}^2\,, \\ \Delta\hat{\Gamma}_{h^0}^{t,\tilde{t}\,(3)} &= \frac{3g}{2M_Z c_W}\,\Delta M_{h^0}^2 - \frac{3}{8\pi^2}\frac{g^3}{M_W^3}\,m_t^4\,, \\ \Delta\hat{\Gamma}_{h^0}^{t,\tilde{t}\,(4)} &= \frac{3g^2}{4M_Z^2 c_W^2}\,\Delta M_{h^0}^2 - \frac{3}{4\pi^2}\frac{g^4}{M_W^4}\,m_t^4\,. \end{aligned}}$$

where: $\Delta M_{h^0}^2 = -\frac{3}{8\pi^2}\frac{g^2}{M_W^2}\,m_t^4\,\log\frac{m_t^2}{m_{\tilde{t}_1}m_{\tilde{t}_2}}$

- The UV-divergence cancel out in the renormalization procedure, such that the mass correction $\Delta M_{h^0}^2$ is finite

- The logarithmic terms in the heavy-squark masses disappear when the vertices are expressed in terms of M_{h^0} $\Rightarrow$ they decouple

- but linear heavy mass terms $\mathcal{O}(m_t^4)$ remain

- Without the non-logarithmic top-mass term, the trilinear and quartic h^0 self-couplings at the one-loop level have the same form as the tree level couplings, with the tree-level Higgs mass replaced by the corresponding one-loop mass $M_{h^0}^2 = M_{h^0}^{2\,\mathrm{tree}} + \Delta M_{h^0}^2$

- SM Renormalized trilinear and quartic self-couplings

$$\boxed{\Delta\hat{\Gamma}_H^{(3)} = -\frac{3g^3}{8\pi^2 M_W^3}\,m_t^4\,, \qquad \Delta\hat{\Gamma}_H^{(4)} = -\frac{3g^4}{4\pi^2 M_W^4}\,m_t^4\,.}$$

The non-logarithmic top-mass terms are common to both h^0 and H_{SM}

Renormalized MSSM and SM Higgs self-interactions

- Tree-level couplings are equal in the $M_{A^0} \gg M_Z$ limit
- Finite terms are the same in both models
- Difference between Renormalized MSSM and SM Higgs self-interactions

$$\boxed{\begin{aligned}
\Gamma^{(2)}_{R\ h^0}{}^{\rm MSSM} - \Gamma^{(2)}_{R\ H_{\rm SM}}{}^{\rm SM} &= \Delta M^2_{h^0}\,, \\
\Gamma^{(3)}_{R\ h^0}{}^{\rm MSSM} - \Gamma^{(3)}_{R\ H_{\rm SM}}{}^{\rm SM} &= \frac{3}{v}\,\Delta M^2_{h^0}\,, \\
\Gamma^{(4)}_{R\ h^0}{}^{\rm MSSM} - \Gamma^{(4)}_{R\ H_{\rm SM}}{}^{\rm SM} &= \frac{3}{v^2}\,\Delta M^2_{h^0}\,.
\end{aligned}}$$

- One-loop MSSM contributions to the h^0 vertex functions
  either represent a shift in the h^0 mass and in the h^0 triple and quartic self-couplings, which can be absorbed in M_{h^0},
  or reproduce the SM one-loop corrections.

$\Downarrow \qquad \Downarrow \qquad \Downarrow$

The triple and quartic h^0 couplings thereby acquire the structure of the SM Higgs-boson self-couplings.

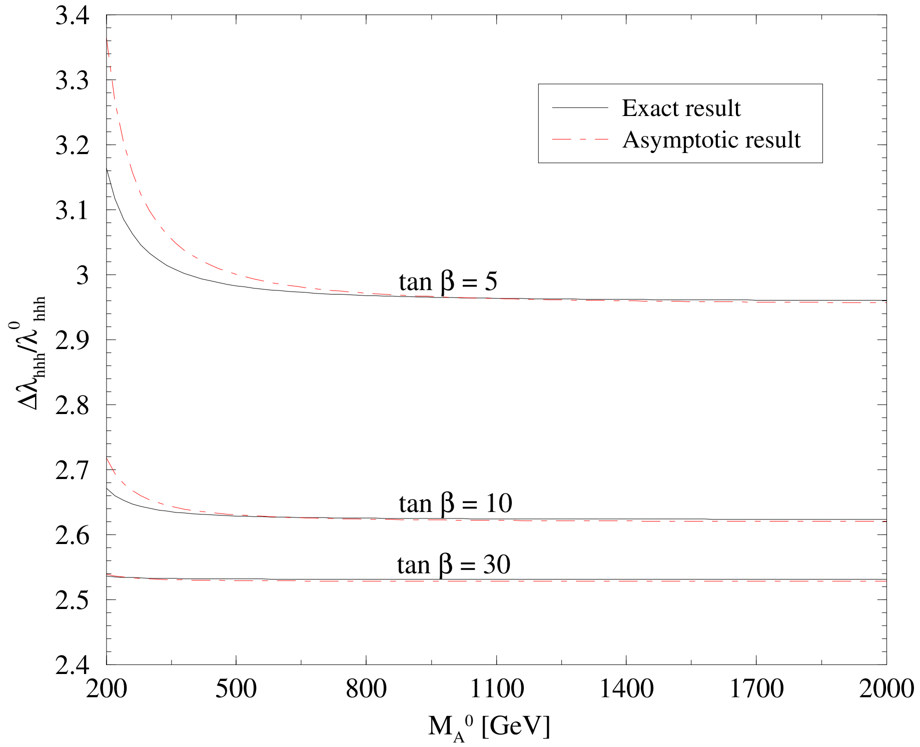
- Decoupling if

$$M_{A^0} \gg M_Z \text{ and } |m^2_{\tilde t_1} - m^2_{\tilde t_2}| \ll |m^2_{\tilde t_1} + m^2_{\tilde t_2}|$$

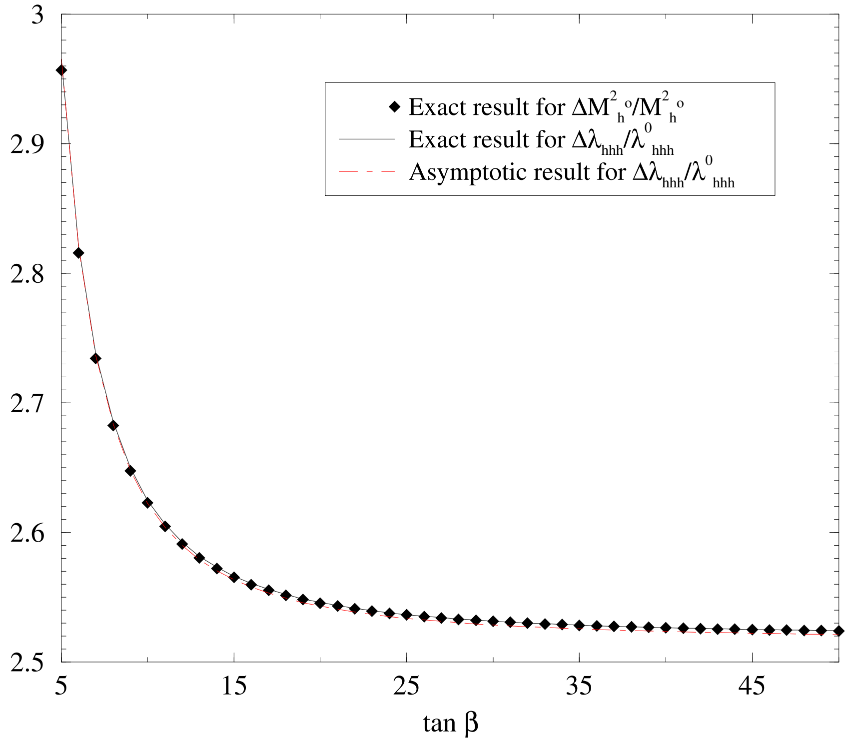
$\mathcal{O}(m_t^4)$ contributions to $\Delta\lambda_{hhh}/\lambda^0_{hhh}$

$$M_{\tilde{Q}} \sim M_{\tilde{U}} \sim 15 \text{ TeV}\,, \;\; \mu \sim |A_t| \sim 1.5 \text{ TeV} \;\; \Rightarrow \;\; |m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}| \ll |m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2}|$$

$\longrightarrow$ Large top-quark/squark corrections, even for large M_{A^0}



$\longrightarrow$ The radiative corrections dissappear when λ^0_{hhh} is expressed in terms of M_{h^0}



Trilinear h^0 self-couplings

- Exact analytical results for $t - \tilde{t}$ contributions to the trilinear h^0 self-couplings.

$$\Delta\lambda_{hhh} = \frac{3g^3}{32\pi^2}\frac{1}{M_W^3}\, m_t^4\, \frac{\cos^3\alpha}{\sin^3\beta}\left\{3\log\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4} + \dots\right\}$$

We agree with the results given in
V. Barger, M. S. Berger, A. L. Stange, R. J. Phillips, Phys. Rev. **D45** (1992) 4128;
P. Osland, P. N. Pandita, Phys. Rev. **D59** (1999) 055013; hep-ph/9911295; hep-ph/9902270

- Numerical analysis:

  - The SUSY parameters have been taken to be

$$M_{\tilde{Q}} \sim 1 \text{ TeV}\,, \quad M_{\tilde{U}} \sim \mu \sim |A_t| \sim 500 \text{ GeV}$$

$$\Updownarrow$$

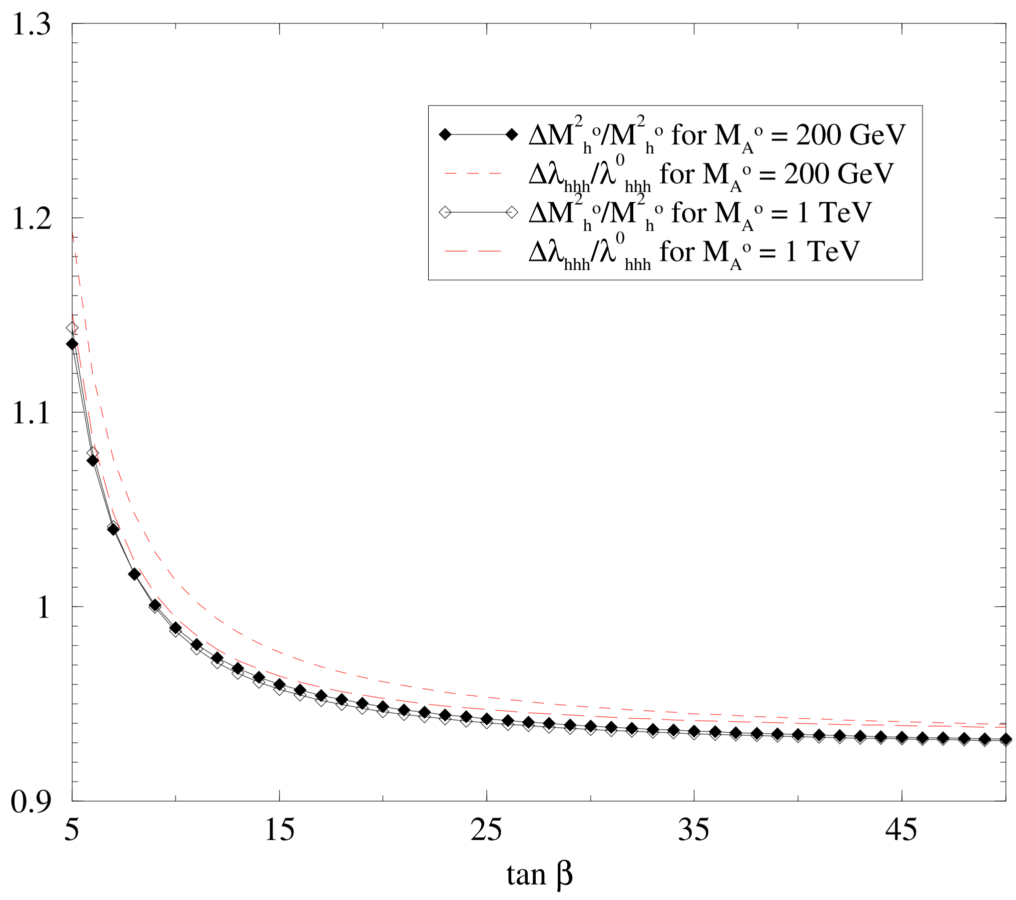
$m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$, are heavy as compared to the to the electroweak scale, but their difference is of $\mathcal{O}(M_{\tilde{U}})$

$$m_{\tilde{t}_1}^2\,, m_{\tilde{t}_2}^2 \gg M_Z^2\,, M_{h^0}^2\,,$$
$$|m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2| \simeq |m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2|$$

  - The radiative correction to the α angle is included.

$\mathcal{O}(m_t^4)$ contributions to $\Delta\lambda_{hhh}/\lambda^0_{hhh}$ and $\Delta M^2_{h^0}/M^{2\,\mathrm{tree}}_{h^0}$

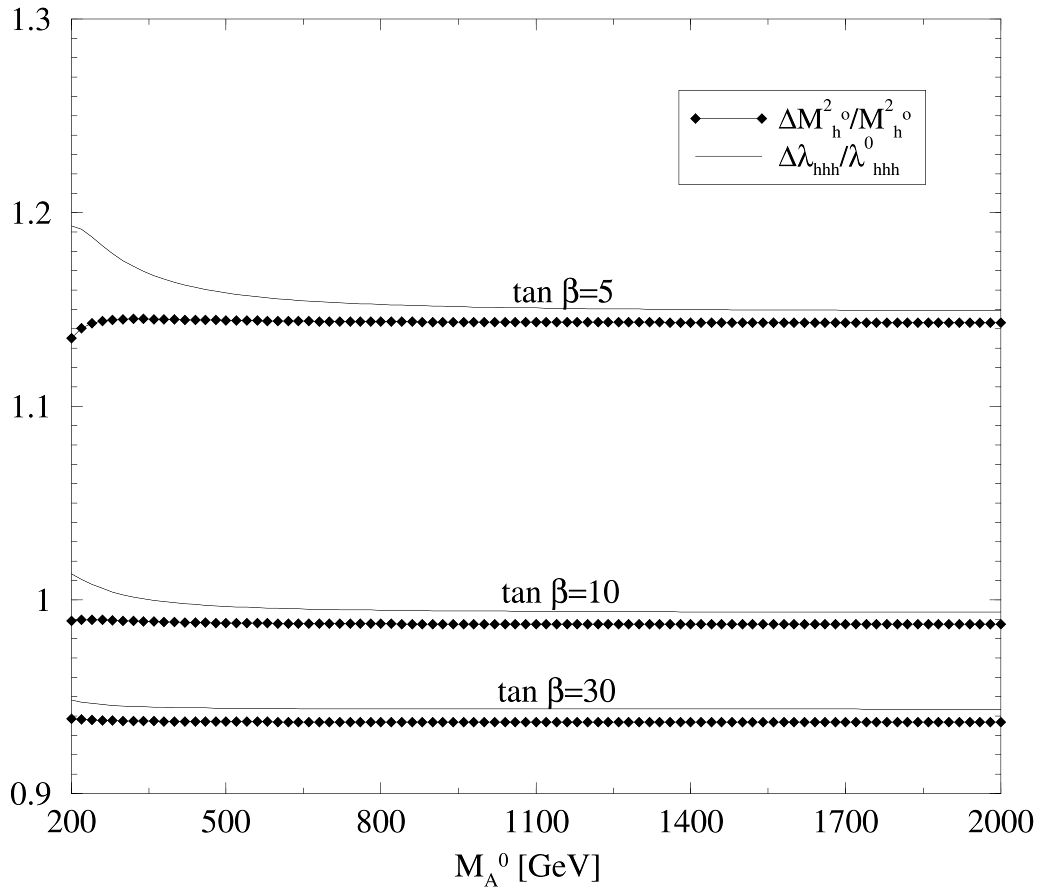
$$|m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}| \simeq |m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2}|$$



Large correction which decrease with $\tan\beta$

For heavy stop system with large mass splitting, $\mathcal{O}(M_{SUSY})$, the $\mathcal{O}(m_t^4)$ corrections to the trilinear h^0 self-couplings are large, but their main part can again be absorbed in M_{h^0}.

(SM terms $\sim m_t^4$ are substracted)



$\Delta\lambda_{hhh}/\lambda^0_{hhh} \approx \Delta M^2_{h^0}/M^{2\,\mathrm{tree}}_{h^0}$ is fulfilled up to a small difference which remains also for large M_A

- The genuine loop corrections to the triple couplings, after re-expressing them in terms of M_{h^0}, is of the order of a few per cent

  $\rightarrow$ They are largest for low $\tan\beta$ and M_{A^0}, typically 5%.

  $\rightarrow$ For large M_{A^0}, they decrease to the level of 1%.

- Not possible to measure at TESLA

  For a SM-like Higgs boson with $m_h = 120$ GeV at $1000 fb^{-1}$, a precision of $\delta\lambda_{hhh}/\lambda_{hhh} = 23\%$ is possible.

  D.J.Miller *et al.*, hep-ph/0001194; C.Castanier *et al.*, hep-ex/0101028

- Similar results have been obtained for the quartic h^0 self-coupling

CONCLUSIONS

- Sizeable corrections to the h^0 self-couplings
- We showed analytically that $\mathcal{O}(m_t^4)$ one-loop contributions to the h^0 self-couplings:

  - Decouple when the self-couplings are expressed in terms of the Higgs-boson mass, in the limit of

$$M_{A^0} \gg M_Z \text{ and } |m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2| \ll |m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2|$$

  $\Rightarrow$ The triple and quartic h^0 couplings acquire the structure of the SM Higgs-boson self-couplings.

- For large mass splitting in the stop sector, the corrections to the triple couplings, after re-expressing them in terms of M_{h^0}, is of the order of a few per cent

  Examples: For low $\tan\beta$ and M_{A^0}, typically 5%.

  For large M_{A^0}, they decrease to the level of 1%.

- Similar results have been obtained for the quartic h^0 self-coupling.

The h^0 self-interactions are very close to those of the SM Higgs boson for the heavy stop sector and would need high-precision experiments for their experimental verification.
Not possible at TESLA