Charm Quark Loops in $\bar{B} \to X_s\,\gamma$

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1. **SM predictions for $\mathbf{BR}[\bar{B} \to X_s\,\gamma]$ vs. experiment**
2. **Perturbative calculations of $b \to X_s^{\rm parton}\,\gamma$**
   - (i) **completed**
   - (ii) **future $\leftrightarrow$ charm quark loops**
3. **Non-perturbative effects**
4. **Summary**

Starting point:

$$\Gamma[\bar{B} \to X_s\gamma] \simeq \Gamma[b \to X_s^{\rm parton}\gamma]$$
$$\equiv \Gamma[b \to s\gamma] \;+\; \Gamma[b \to s\gamma\ \text{gluon}] \;+\; \ldots$$

SM prediction:

$$BR[\bar{B} \to X_s\gamma]^{\text{subtr. } \psi}_{E_\gamma > E_0} = \begin{cases} (3.57 \pm 0.30) \times 10^{-4}, & E_0 = 1.6 \text{ GeV} \\ 3.70 \times 10^{-4}, & E_0 = \frac{1}{20} m_b \end{cases}$$

Experimental results (for $E_0 = \frac{1}{20} m_b$):

BABAR [hep-ex/0207076]

$$\text{BR}[\bar{B}_d \to X_s\gamma] = \left[3.88 \pm 0.36_{\text{stat}} \pm 0.37_{\text{sys}} \left(^{+0.43}_{-0.23}\right)_{\text{theory}}\right] \times 10^{-4}$$

CLEO [Phys. Rev. Lett. 87 (2001) 251807, hep-ex/0108032]

$$\text{BR}[\bar{B}_d \to X_s\gamma] = \left[3.21 \pm 0.43_{\text{stat}} \pm 0.27_{\text{sys}} \left(^{+0.18}_{-0.10}\right)_{\text{theory}}\right] \times 10^{-4}$$

BELLE [Phys. Lett. B 511 (2001) 151, hep-ex/0103042]

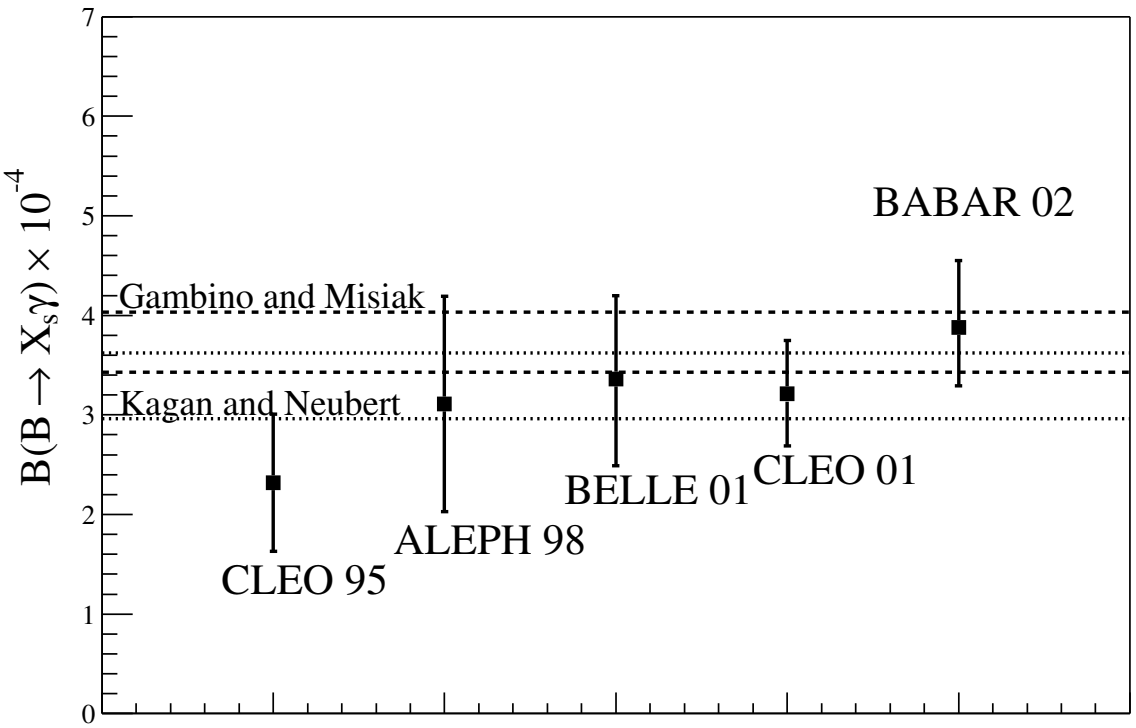
$$\text{BR}[\bar{B}_d \to X_s\gamma] = \left[3.36 \pm 0.53_{\text{stat}} \pm 0.42_{\text{sys}} \left(^{+0.50}_{-0.54}\right)_{\text{theory}}\right] \times 10^{-4}$$

ALEPH [Phys. Lett. B429 (1998) 169]

$$\text{BR}[b \to s\gamma] = (3.11 \pm 0.80_{\text{stat}} \pm 0.72_{\text{sys}}) \times 10^{-4}$$

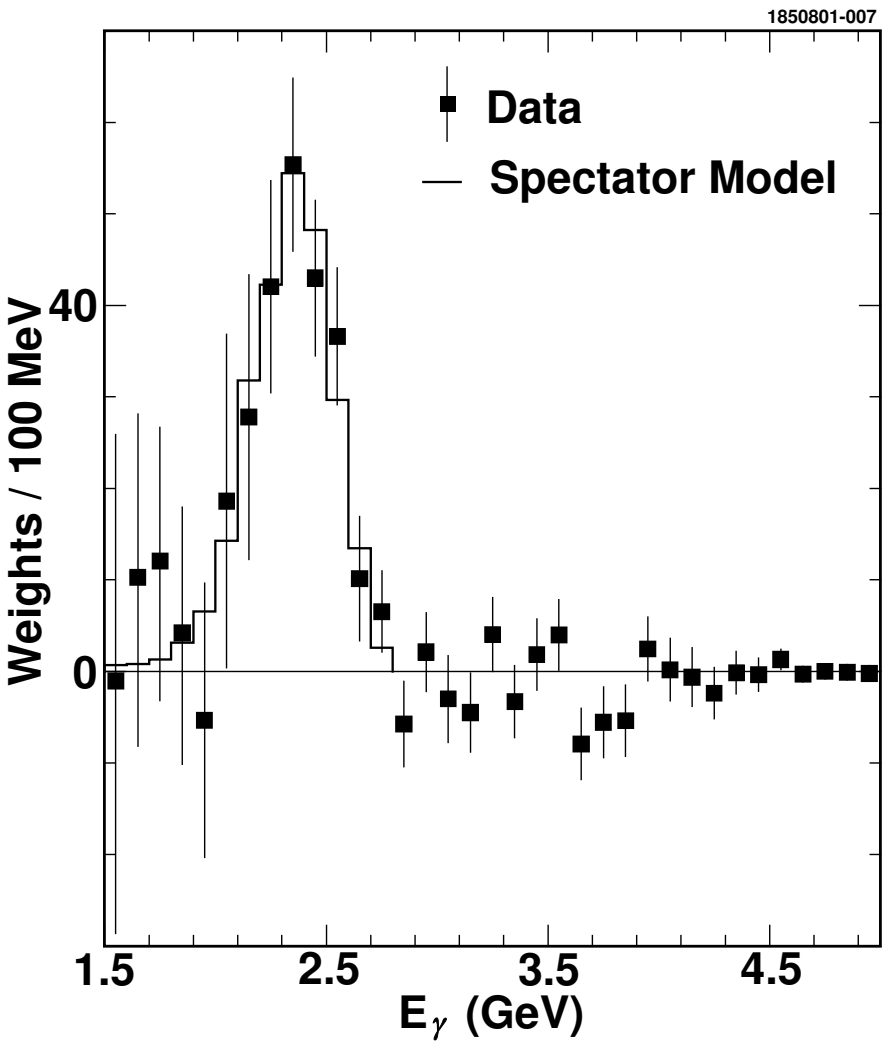
Weighted average:

$$\text{BR}[\bar{B} \to X_s\gamma]_{\text{exp}} = \left[3.40 \underbrace{\left(^{+0.42}_{-0.37}\right)}_{\sim \pm 12\%}\right] \times 10^{-4}$$

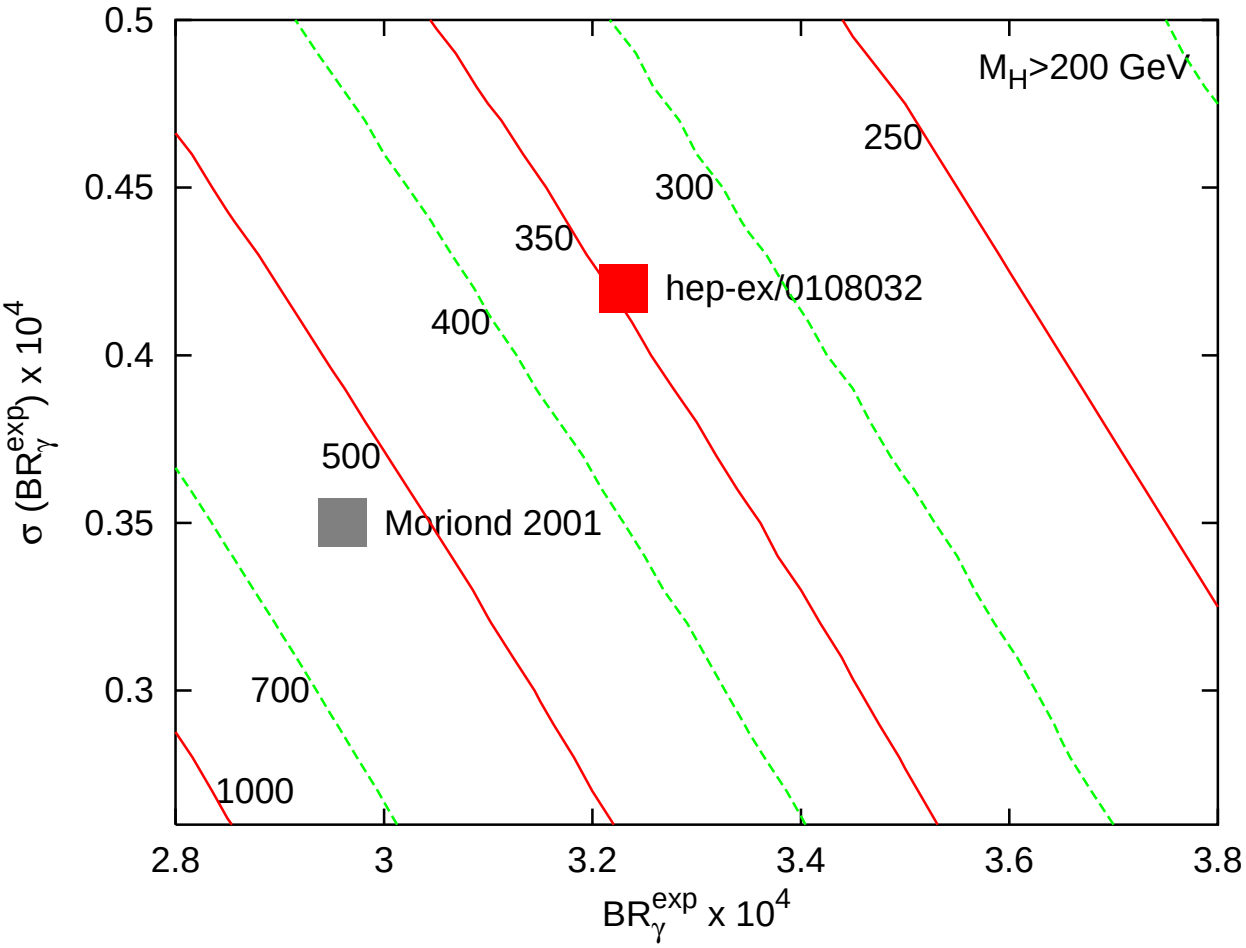


Summary of experimental results and SM predictions (BABAR [hep-ex/0207076])

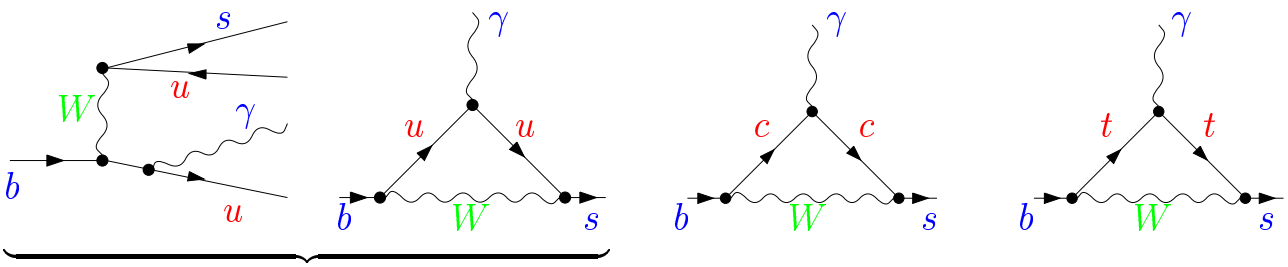
CLEO measurement of the $\bar{B} \to X_s \gamma$ photon spectrum



$\bar{B} \to X_s\,\gamma$ constraints on the charged scalar mass in the Two-Higgs-Doublet Model (II)



Electroweak transitions mediating $\bar{B} \to X_s\gamma$:

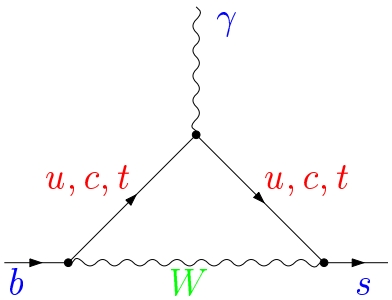


$$\left|\frac{V_{ub}V_{us}}{V_{cb}}\right| \simeq \left|\frac{V_{ub}V_{us}}{V_{ts}}\right| \simeq 2\% \qquad \sim +200\% \qquad \sim -100\%$$

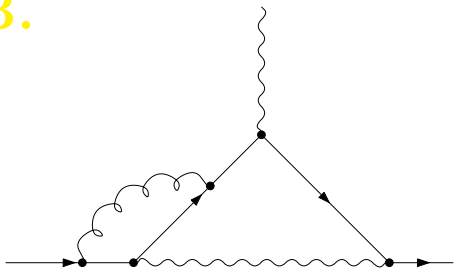
In the amplitude, after including LO QCD effects.

Examples of Feynman diagrams contributing to $b \to s\gamma$ at various orders in the renormalization-group-improved perturbation theory:

A.

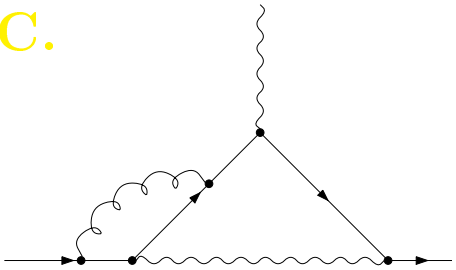


B.

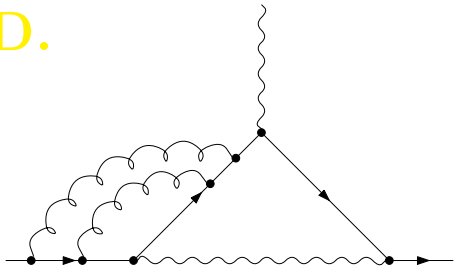


$\alpha_s \ln \frac{M_W^2}{m_b^2}$**: $\sim +65\%$ in amplitude**
$\sim +175\%$ in BR

C.



D.



non − logarithmic logarithmic

$\sim +20\%$ in BR

The effective Lagrangian:

$$\mathcal{L} \;=\; \mathcal{L}_{QCD\times QED}(u,d,s,c,b) \;+\frac{4G_F}{\sqrt{2}}V_{ts}^*V_{tb}\sum_{i=1}^{8}C_i(\mu)O_i$$

$$O_i=\begin{cases}(\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b), & i=1,2, & |C_i(m_b)|\sim 1\\ (\bar{s}\Gamma_i b)\boldsymbol{\Sigma}_q(\bar{q}\Gamma_i' q), & i=3,4,5,6, & |C_i(m_b)|<0.07\\ \frac{em_b}{16\pi^2}\bar{s}_L\sigma^{\mu\nu}b_R F_{\mu\nu}, & i=7, & C_7(m_b)\sim -0.3\\ \frac{gm_b}{16\pi^2}\bar{s}_L\sigma^{\mu\nu}T^a b_R G^a_{\mu\nu}, & i=8, & C_8(m_b)\sim -0.15\end{cases}$$

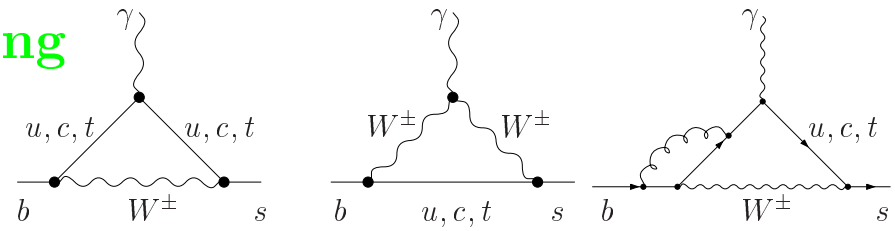
Perturbative expansion of the Wilson coefficients:

$$C_i(\mu) \;=\; C_i^{(0)}(\mu) \;+\frac{\alpha_s(\mu)}{4\pi}C_i^{(1)}(\mu) +\frac{\alpha_{em}}{\alpha_s(\mu)}C_i^{em(0)}(\mu) \;+\frac{\alpha_{em}}{4\pi\ \sin^2\theta_W}C_i^{ew(1)}(\mu)+\ldots$$

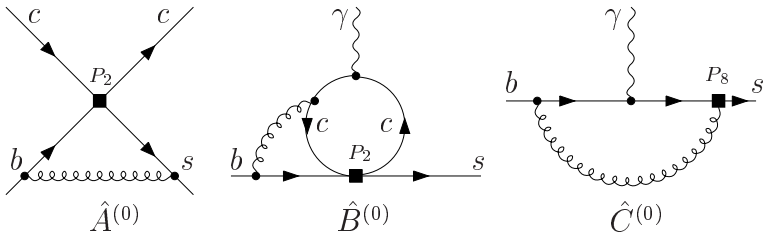
$$C_7^{(1)eff}(m_b)\simeq +0.5 \;\Rightarrow\; -\mathbf{4.9}\%\ \text{in BR}\quad [\mathbf{C},\mathbf{D}]$$
$$C_7^{em(0)eff}(m_b)\simeq +0.03 \;\Rightarrow\; -\mathbf{0.8}\%\ \text{in BR}\quad [\mathbf{E}]$$
$$C_7^{ew(0)eff}(m_b)\simeq +1.9 \;\Rightarrow\; -\mathbf{1.6}\%\ \text{in BR}\quad [\mathbf{F}]$$

Three steps of the calculation:

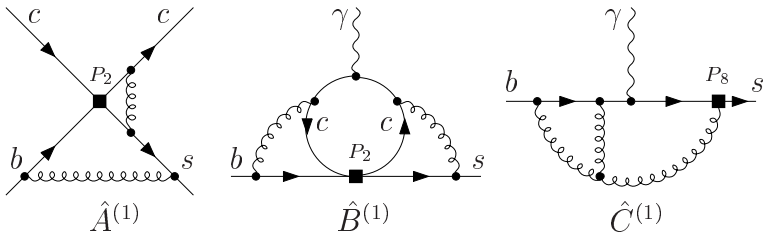
Matching



Mixing

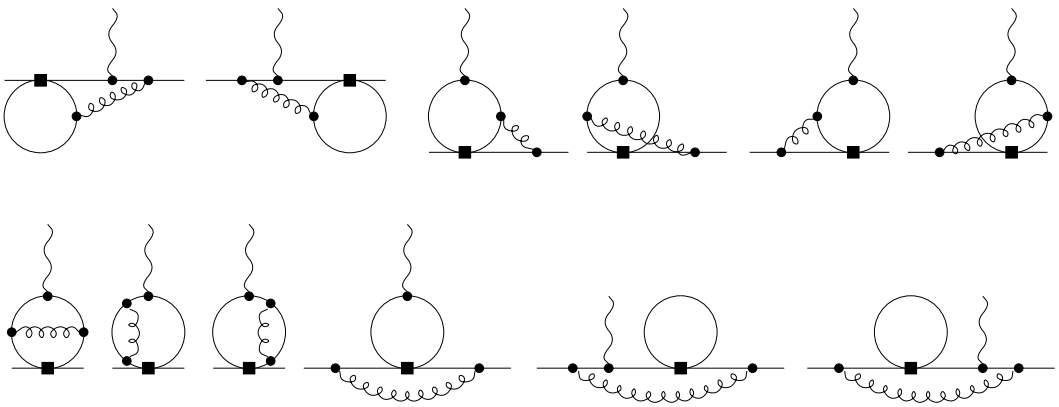


(LO)



(NLO)

Matrix elements:



Three methods have been used for calculating 2-loop matrix elements with charm-quark loops:

1. **Mellin-Barnes transform of Feynman-parameter integrals $\Rightarrow$ Expansion in m_c/m_b**
   [Greub, Hurth, Wyler, 1996]

2. **Asymptotic expansions $\Rightarrow$ Expansion in m_c/m_b**
   [Buras, Czarnecki, Misiak, Urban, 2001]

3. **"Brute force" $\Rightarrow$ No expansion in m_c/m_b**
   [Buras, Czarnecki, Misiak, Urban, 2002]
   Goal: Diagrams which b-quark loops $\Rightarrow$ Formally complete NLO calculation

The $b \to s\gamma$ amplitude becomes dependent on m_c only at the NLO, via 2-loop diagrams.
The question whether we should use:

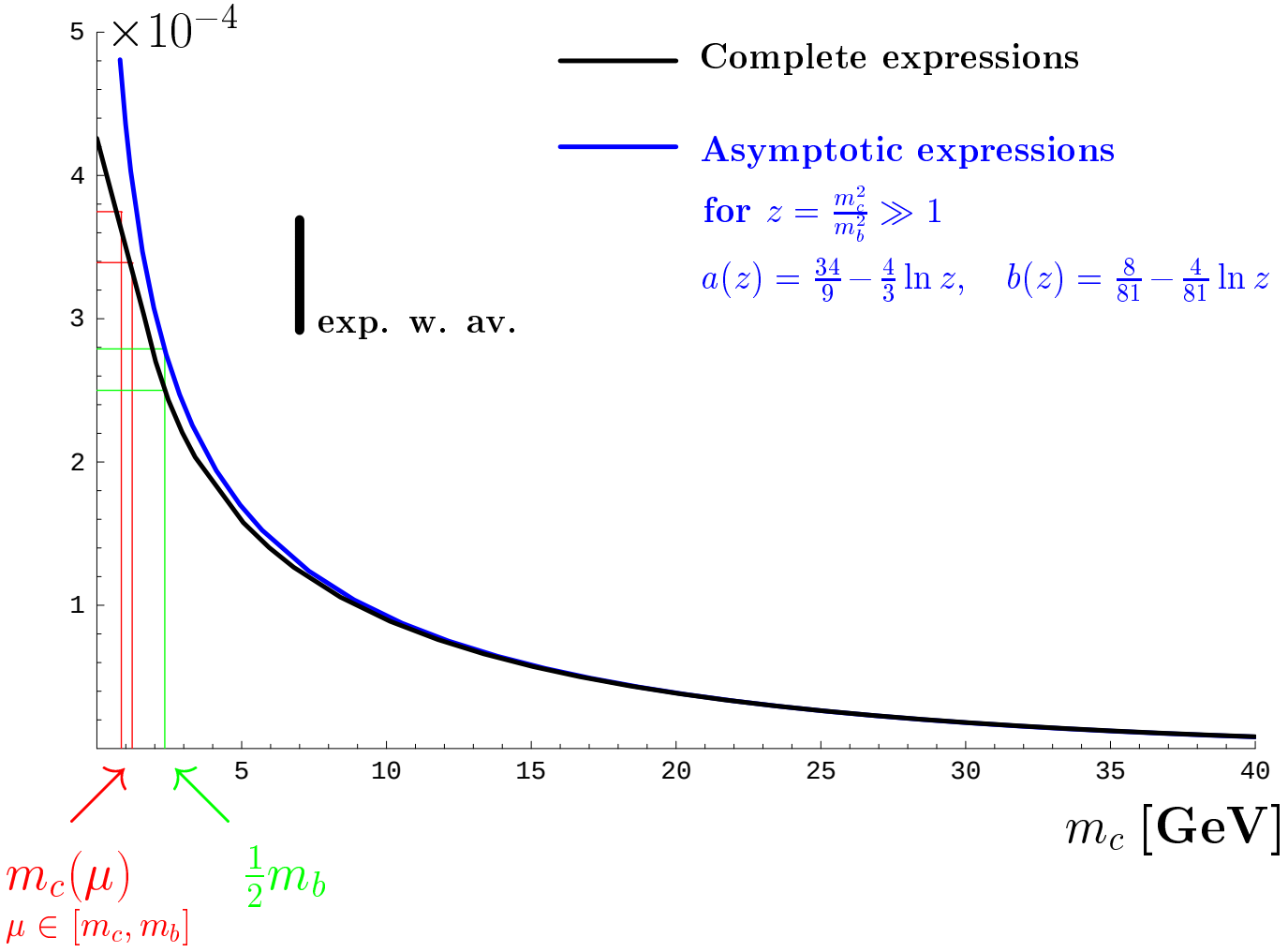
$$\frac{m_c^{\rm pole}}{m_b^{\rm pole}} = 0.29 \pm 0.02 \qquad \textbf{or} \qquad \frac{m_c^{\overline{\rm MS}}(\mu)}{m_b^{\rm pole}} = 0.22 \pm 0.04 \quad \mu \in [m_c, m_b]$$

can, in principle, be asked only at NNLO. However, changing m_c/m_b from 0.29 to 0.22 enhances BR$[\bar{B} \to X_s\gamma]$ by 10% ! Accuracy $\searrow$

Way out: NNLO calculation

Charm-mass dependence of

$\mathbf{BR}[\bar{B} \to X_s\gamma \quad (E_\gamma > 1.6\ \mathrm{GeV})]$



Summary:

1.

$$BR[\bar{B} \to X_s\gamma]^{\text{subtr. }\psi}_{E_\gamma > E_0} = \begin{cases} (3.57 \pm 0.30) \times 10^{-4}, & E_0 = 1.6 \text{ GeV} \\ 3.70 \times 10^{-4}, & E_0 = \frac{1}{20} m_b \end{cases}$$

$$\text{BR}[\bar{B} \to X_s\gamma]_{\text{exp}} = \left[3.40 \ \left(^{+0.42}_{-0.37}\right)\right] \times 10^{-4}$$

$\Rightarrow$ **Theory and experiment agree within** 1σ

2. The above agreement provides stringent constraints on new physics.

3. The dominant theoretical uncertainty is of perturbative origin. It is due to the renormalization-scheme dependence of m_c in two-loop contributions to the decay amplitude. Calculating the NNLO corrections would remove this problem and, in consequence, reduce the theoretical uncertainty by almost a factor of 2. The main challenge at the NNLO are UV-finite parts of 3-loop massive vertex integrals with non-vanishing external momenta.

4. The numerical behaviour of $\text{BR}[\bar{B} \to X_s\gamma]$ as a function of m_c suggests that the dominant charm mass dependence originates from distances much smaller than Λ_{QCD}. In such a case, the associated non-perturbative effects would be under control (and negligible).