# Supersymmetric Sudakov corrections

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Introduction

High energy effective approach

Resummation of MSSM-Sudakov logarithms

Applications to processes at a LC

Conclusions

* work done in collaboration with M. Beccaria, F.M. Renard and C. Verzegnassi

# Introduction

Standard Model:

Precision NLC physics ($<$1%) to disentangle new physics.

At 1 Loop, EW DL's 10-20 %, at 2 Loops few %. Subleading could be significant too.

$\longrightarrow$ Need a complete leading & subl. log analysis through 2 Loops!

In addition:

Theoretical understanding of IR-structure of broken gauge theories (2 Mass scales!).

Minimal Supersymmetric Standard Model:

Novel contributions at TeV energies if $m_s \leq 500$GeV:

No new double logarithms (DL) (only spin 1)

No new angular dependent logarithmic corrections (only spin 1)

New universal logarithms of gauge and Yukawa type appear. In addition: new RG corrections.

Yukawa-terms depend strongly on parameters of the MSSM $(\alpha, \beta)$

Theoretical and phenomenological understanding: Higher order MSSM-Sudakov corrections

## High energy effective approach

Consider massive(M) boson at rest: $k^\nu = (M,0,0,0)$
Polarization vector is lin.comb. of:

$$e_1 \equiv (0,1,0,0) \quad , \quad e_1 \equiv (0,0,1,0) \quad , \quad e_3 \equiv (0,0,0,1)$$

Boost along 3-axis, pol. vect. still satisfy:

$$e_\nu k^\nu = 0 \quad , \quad e^2 = -1$$

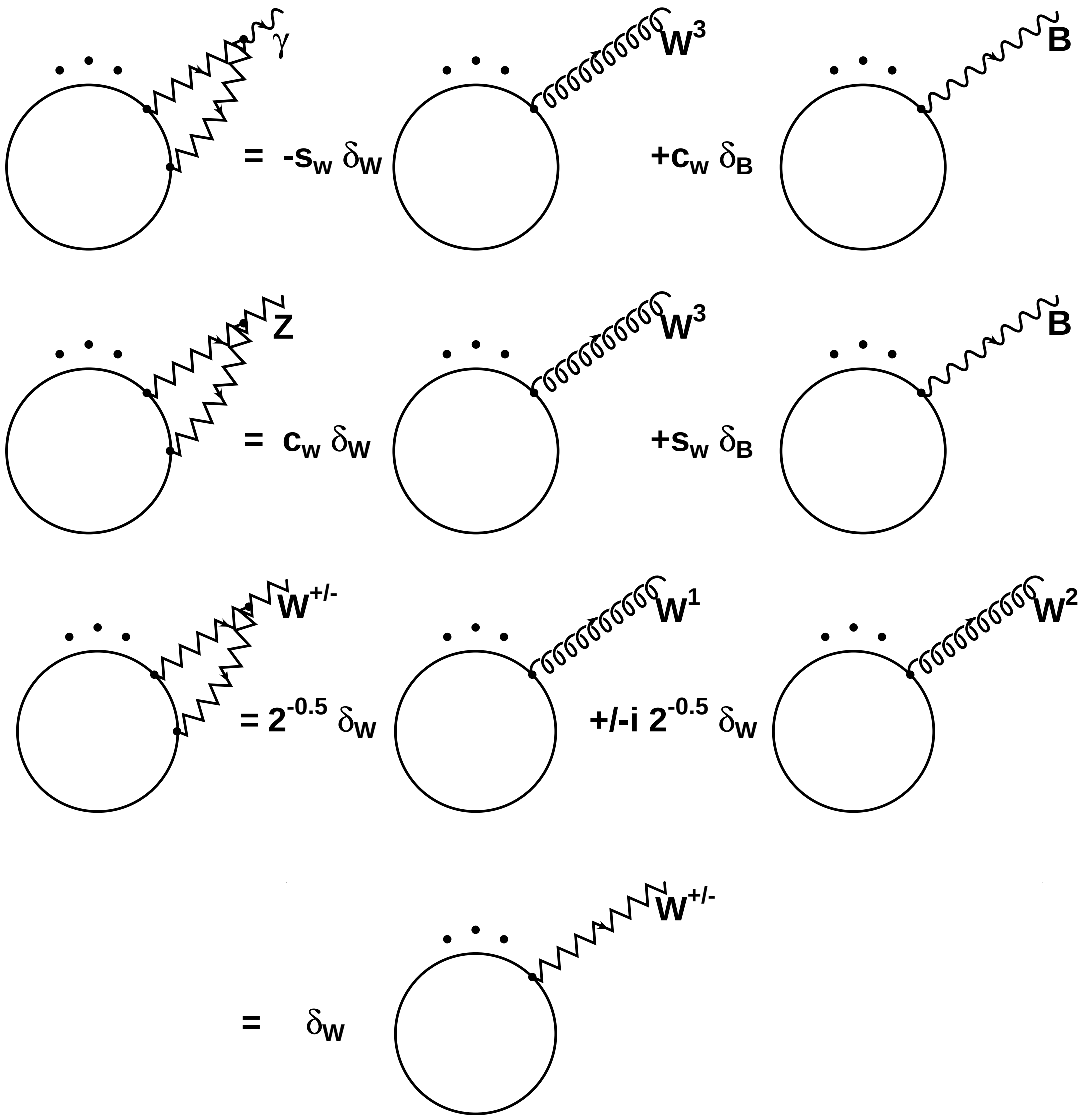
$\longrightarrow$ 2 transverse $(e_1, e_2)$ and 1 longitudinal:

$$e_L^\nu(k) = (k/M, 0, 0, E_k/M) = k^\nu/M + \mathcal{O}(M/E_k)$$

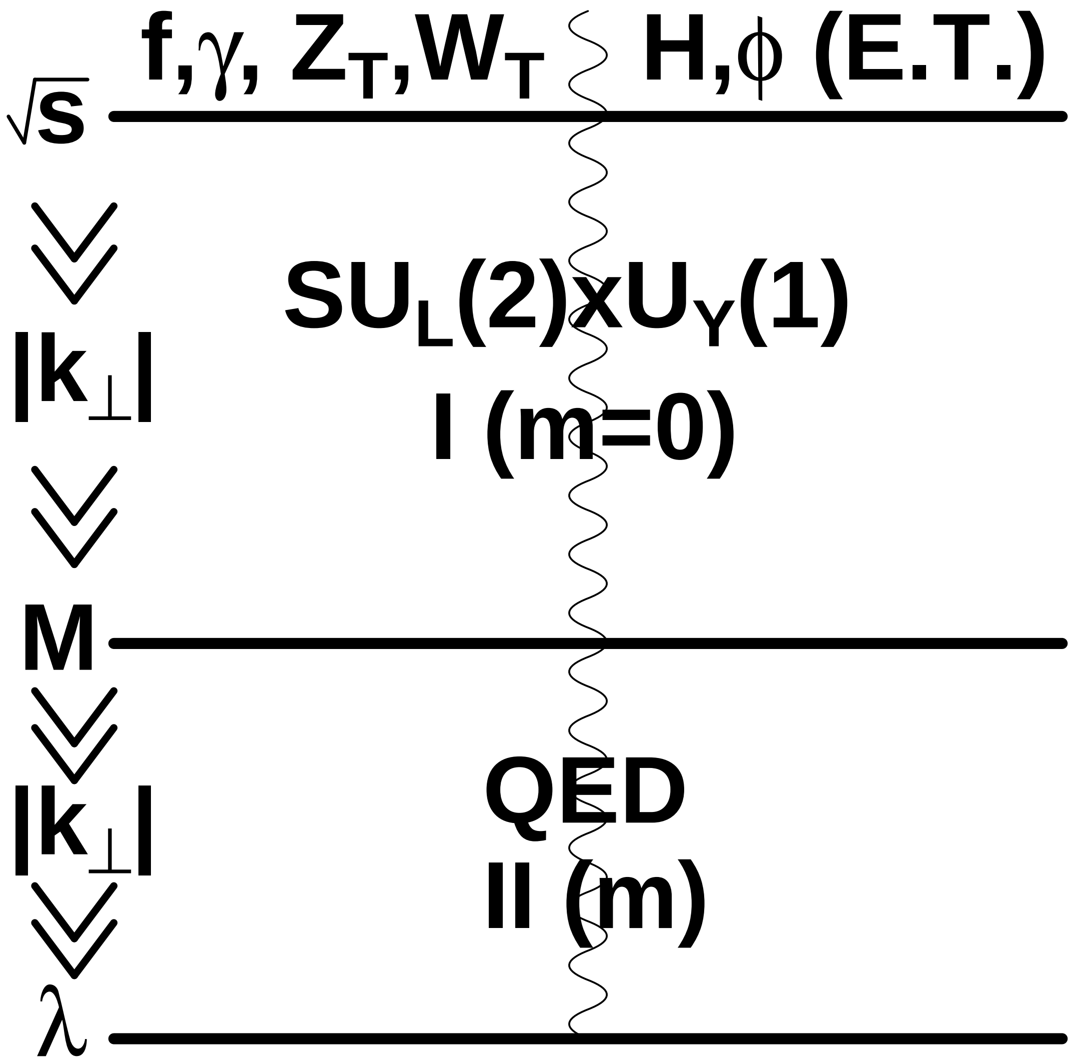
Transverse d.o.f. correspond to massless theory!
Use massless virtual "QCD" results at high energies:

Additional complications: mixing & broken sym.

$$\begin{aligned} W_\nu^\pm &= \frac{1}{\sqrt{2}}\left(W_\nu^1 \pm i W_\nu^2\right) \\ Z_\nu &= \cos\theta_{\mathrm{W}} W_\nu^3 + \sin\theta_{\mathrm{W}} B_\nu \\ A_\nu &= -\sin\theta_{\mathrm{W}} W_\nu^3 + \cos\theta_{\mathrm{W}} B_\nu \end{aligned}$$




Only corrections to external $W^{\pm}$ factorize with respect to the physical amplitude!




Effective regime in region I) is described by the fields in the symmetric basis:

$$\mathcal{L}_{\text{symm}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{H}} + \mathcal{L}_{\text{F}} + \mathcal{L}_{\text{fix}} + \mathcal{L}_{\text{FP}}$$

QED corrections in region II) through matching.

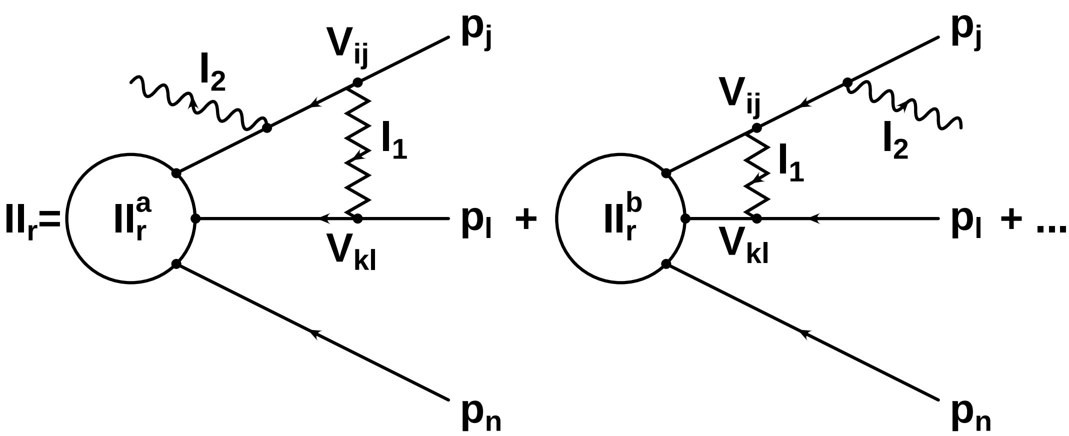
Physical principles :

* $SU_{\mathsf{L}}(2) \times U_Y(1)$ Gauge Invariance

* $\sqrt{s} \gg M,\ \lambda = M$ : symmetric limit

* $\sqrt{s} = M,\ \lambda \ll M$ : QED limit

* Mass-Gap only relevant on external legs :




* Real soft QED ($\Delta E \leq M$) emission
leads to virtual soft $\mathcal{O}\left(\log \frac{M}{\lambda}\right)$ exponentiation

The symmetric Lagrangian contains:

$$\mathcal{L}_{\mathsf{YM}} = -\frac{1}{4}\left(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\varepsilon^{abc}W_\mu^b W_\nu^c\right)^2$$
$$-\frac{1}{4}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right)^2$$

$$\mathcal{L}_{\mathsf{H}} = \left(D_\mu\Phi\right)^\dagger\left(D^\mu\Phi\right)\,, D_\mu = \partial_\mu - igT^a W_\mu^a + ig'\frac{Y}{2}B_\mu$$

with

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}, \phi^0(x) = \frac{1}{\sqrt{2}}\left(H(x) + i\chi(x)\right)$$

and

$$\mathcal{L}_{\mathsf{F}} = \sum_j \left(\overline{L}_j i\gamma^\mu D_\mu L_j\right) + \sum_j \left(\overline{R}_j i\gamma^\mu D_\mu R_j\right)$$
$$-\sum_{j,l}\left(\overline{L}_j G_{jl} R_l \Phi + h.c.\right)$$

$$\mathcal{L}_{\mathsf{fix}} = -\frac{1}{2}\left[\left(F^W\right)^2 + \left(F^B\right)^2\right]$$

where

$$F^W = \frac{1}{\sqrt{\xi_W}}\partial^\mu W_\mu^a \;\;,\;\; F^B = \frac{1}{\sqrt{\xi_B}}\partial^\mu B_\mu$$

and

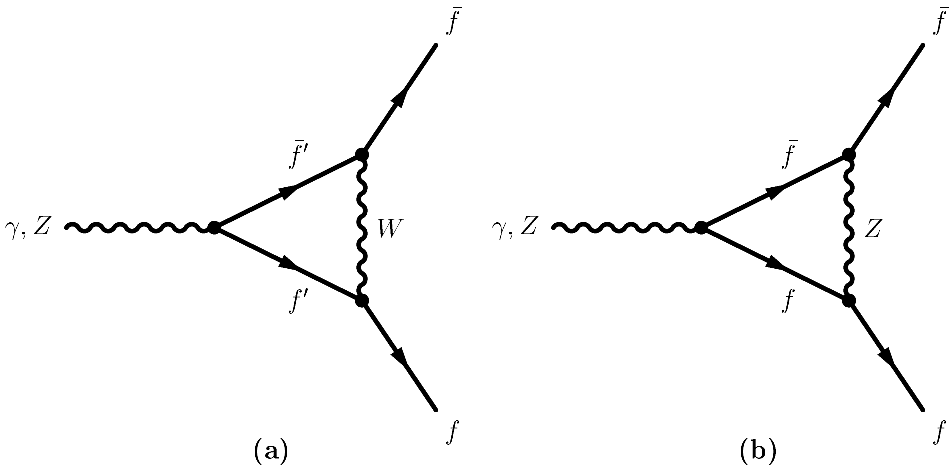
$$\mathcal{L}_{\mathsf{FP}} = \overline{u}^\alpha(x)\frac{\delta F^\alpha}{\delta\theta^\beta(x)}u^\beta(x)$$

To SL accuracy, treat $\chi, \phi^\pm$ as physical!

One loop structure:

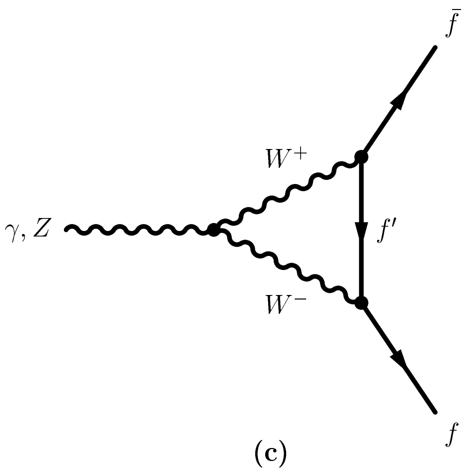
UV logarithms related to the Renormalization Group: $W, Z, \gamma$ self energies.

Sudakov logarithms due to single $W, Z$ exchange.



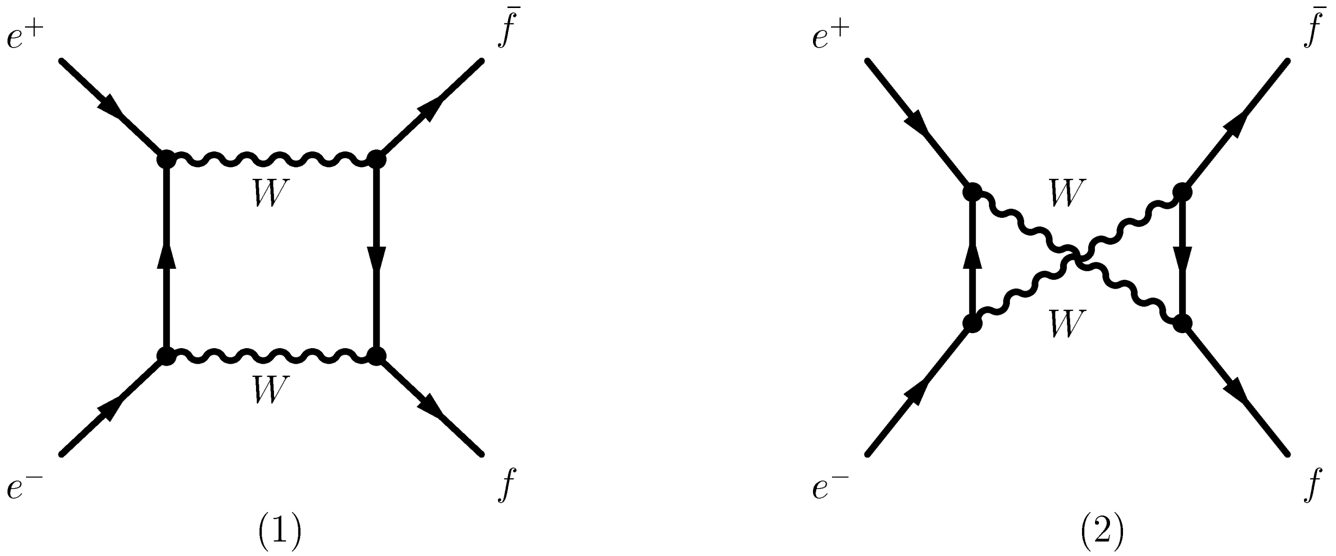

(a) (b)

Sudakov logarithms due to double $W$ exchange.



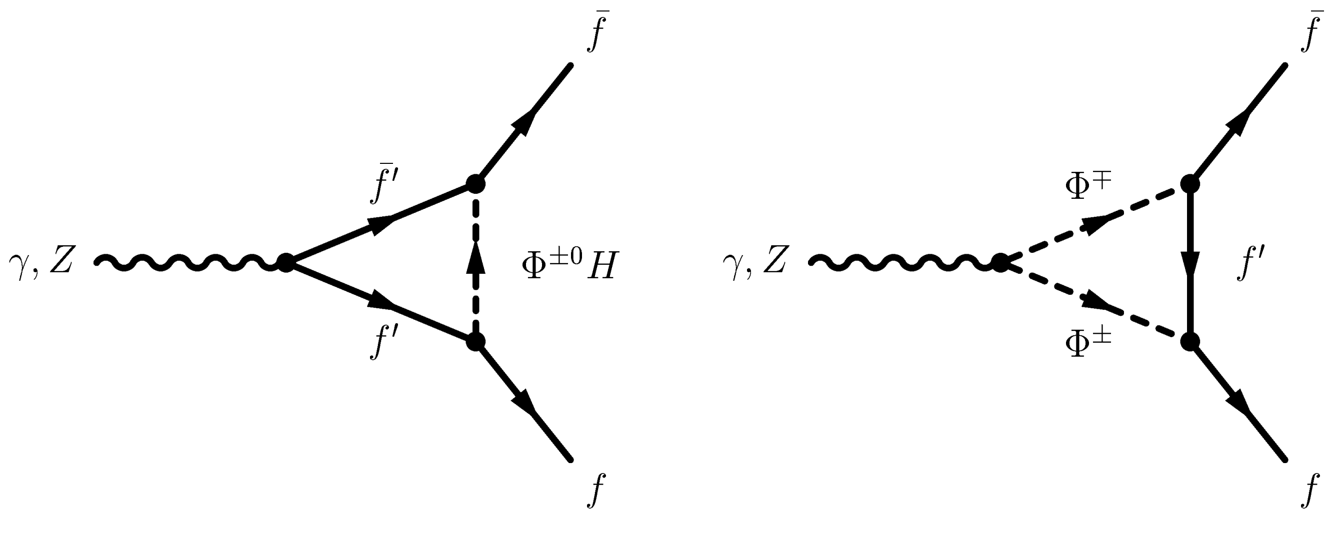

(c)

Sudakov logarithms due to $W, Z$ boxes:




**(d)**

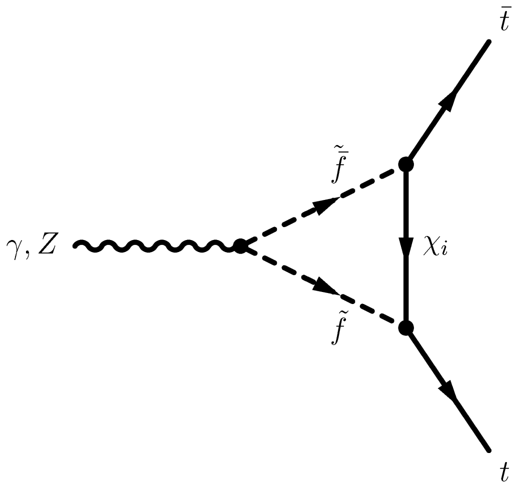
and due to Goldstone bosons and Higgs exchange:

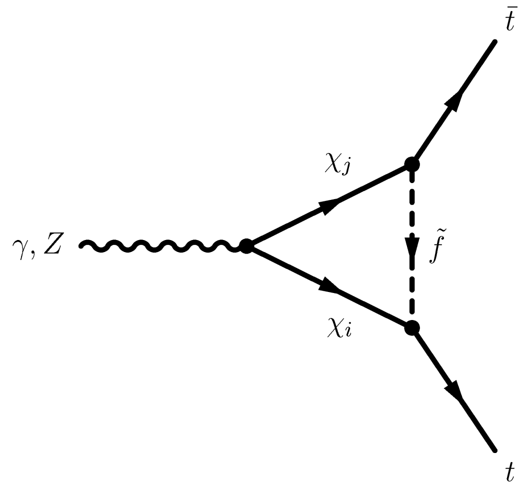



**(f)**

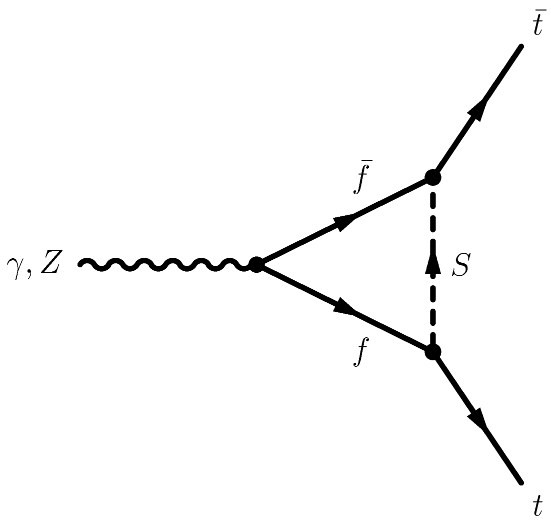
Extra Terms in the MSSM:

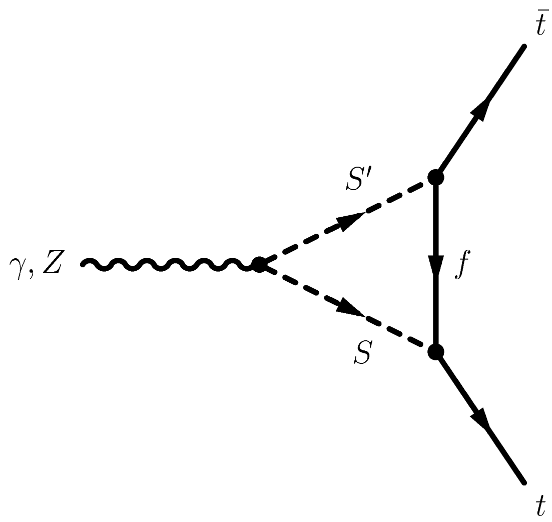
Logarithms from vertices with gaugino exchange




Logarithms of Yukawa kind from vertices with Goldstone-Higgs exchange ($H^0, h^0, A^0, H^{\pm}, \Phi^0, \Phi^{\pm}$)



In SM:

One loop corrections exponentiate to SL accuracy, for angular dependent terms in operator form:

$$\mathcal{M}_{SL}^{i_1,\ldots,i_n}(\{p_k\};\{m_l\};M,\lambda) =$$
$$\exp\left\{I^{(1)}_{\text{QED}} - I^{(1)}_{\text{QED}}(\lambda \leftrightarrow M)\right\} \exp\left\{I^{(1)}_{\text{symm}}\right\}$$
$$\mathcal{M}_{\text{Born}}^{i_1,\ldots,i'_k,\ldots,i'_l,\ldots,i_n}(\{p_k\};\{m_n\})$$

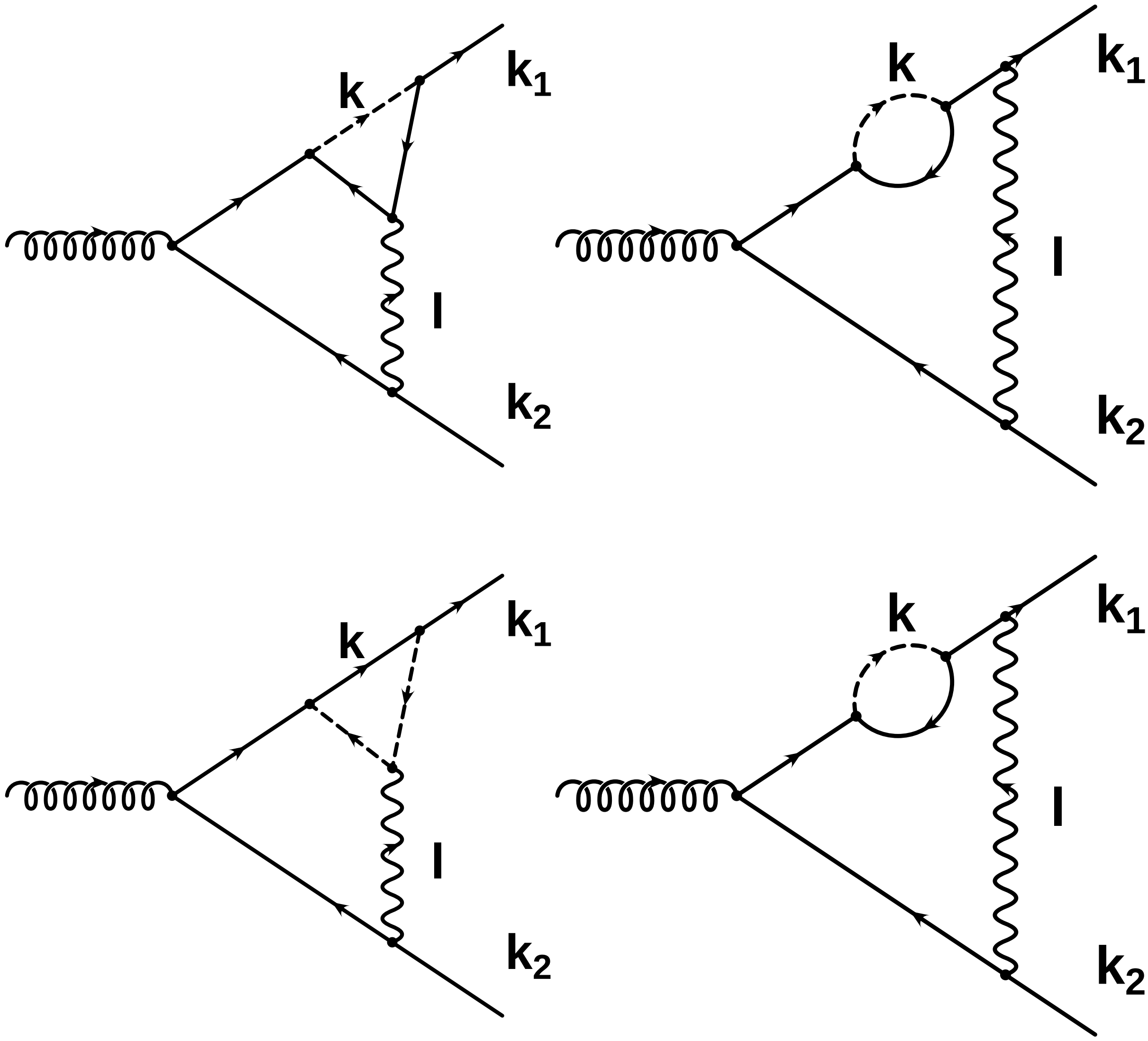
with

$$I^{(1)}_{\text{symm}}\left(\{p_k\},\{m_l\},M,\lambda\right) = \frac{e^2}{16\pi^2}\sum_{k=1}^{n}\left\{-\frac{1}{2}\left[C^{\text{ew}}_{i'_k,i_k}\log^2\frac{s}{M^2}\right.\right.$$
$$\left.+\delta_{i'_k,i_k}Q_k^2\left(2\log\frac{s}{M^2}\log\frac{M^2}{\lambda^2}+\log^2\frac{M^2}{\lambda^2}-\log^2\frac{m_k^2}{\lambda^2}\right)\right]+$$
$$\left.\delta^{\text{SL}}_{i'_k,i_k}\log\frac{s}{M^2}+2\sum_{l<k}^{n}\sum_{V_a=A,Z,W^\pm} I^{V_a}_{i'_k,i_k} I^{\overline{V}_a}_{i'_l,i_l}\log\frac{s}{M^2}\log\frac{2p_lp_k}{s}\right\}$$

confirmed at 2 Loops (Denner, Melles, Pozzorini)

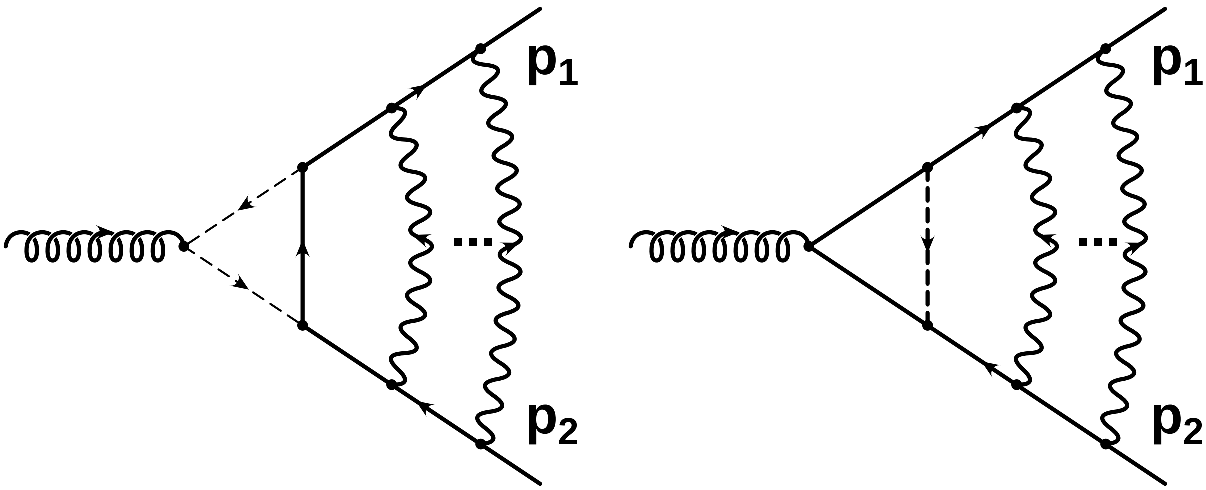
In MSSM:

Novel terms at two loop level:




To SL accuracy: Total sum cancels!

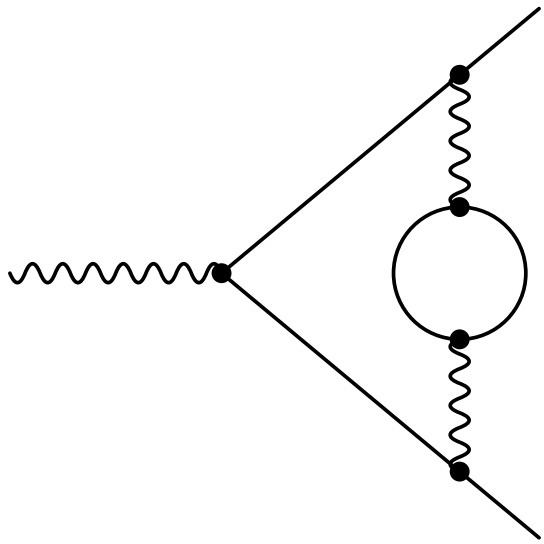
Only remaining corrections:




Application of non-Abelian Gribov theorem:
MSSM-Sudakov logarithms exponentiate
to SL accuracy

Note: DL and angular logarithms same as SM!

Higher order RG-SL corrections:



A folding of DL loop with RG correction!
In unbroken phase, we can use:

$$\widetilde{W}^{RG}_{i_V}\left(s,\mu^2\right) =$$

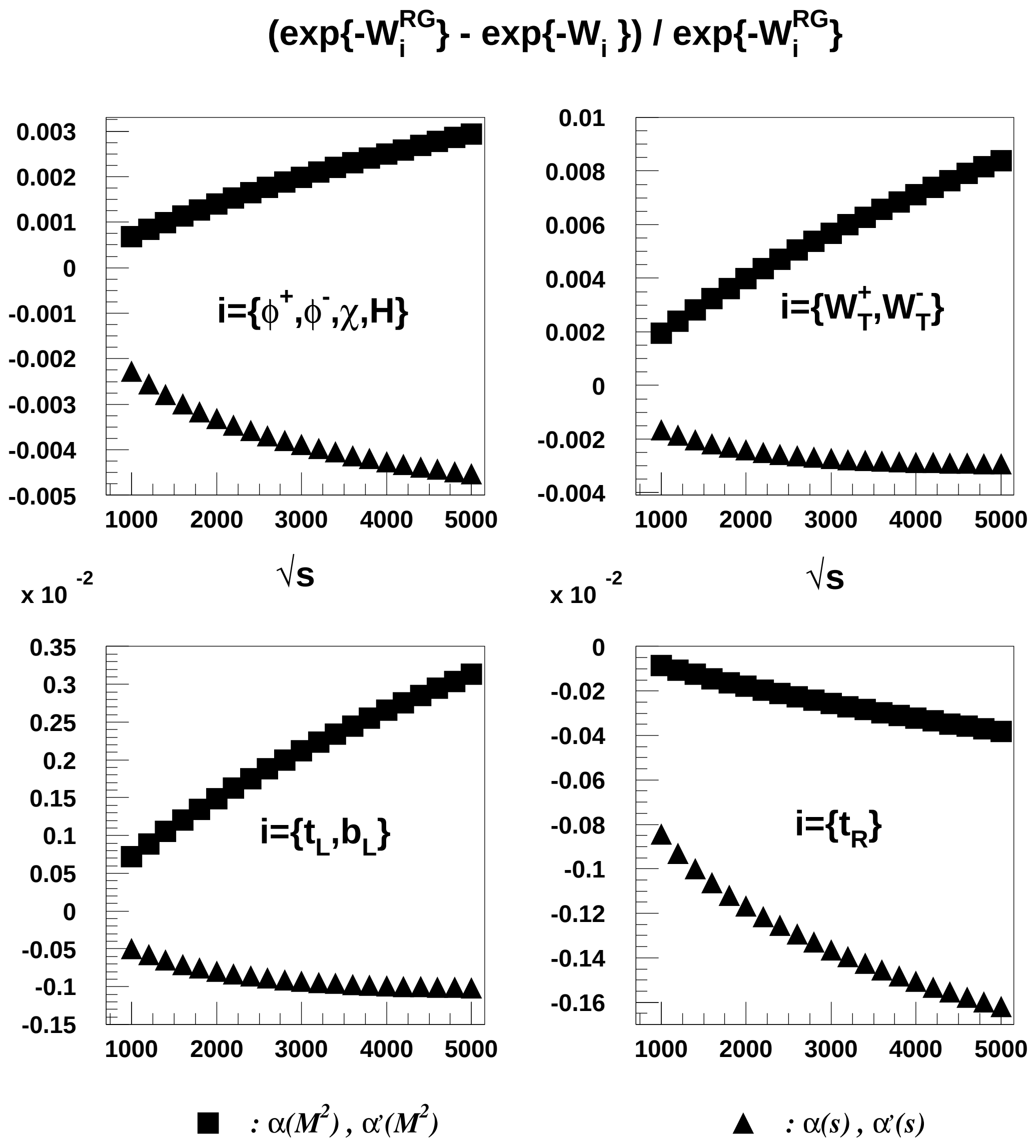
$$\frac{1}{2\pi}\int_{\mu^2}^{s}\frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2}\int_{\boldsymbol{k}_\perp^2/s}^{1}\frac{dv}{v}\left\{\frac{T_i(T_i+1)\alpha(\mu^2)}{1+c\ \log\frac{\boldsymbol{k}_\perp^2}{\mu^2}}+\frac{(Y_i^2/4)\alpha'(\mu^2)}{1+c'\ \log\frac{\boldsymbol{k}_\perp^2}{\mu^2}}\right\} =$$

$$\frac{\alpha(\mu^2)T_i(T_i+1)}{2\pi}\left\{\frac{1}{c}\log\frac{s}{\mu^2}\left(\log\frac{\alpha(\mu^2)}{\alpha(s)}-1\right)+\frac{1}{c^2}\log\frac{\alpha(\mu^2)}{\alpha(s)}\right\}$$

$$+\frac{\alpha'(\mu^2)Y_i^2}{8\pi}\left\{\frac{1}{c'}\log\frac{s}{\mu^2}\left(\log\frac{\alpha'(\mu^2)}{\alpha'(s)}-1\right)+\frac{1}{c'^2}\log\frac{\alpha'(\mu^2)}{\alpha'(s)}\right\}$$

where $\alpha(\mu^2) = g^2(\mu^2)/4\pi$, $\alpha'(\mu^2) = g'^2(\mu^2)/4\pi$, $c = \alpha(\mu^2)\beta_0/\pi$ and analogously, $c' = \alpha'(\mu^2)\beta_0'/\pi$.

The size of the effect depends on s:



Explicit MSSM-high energy results:

chiral fermion/sfermion:

$$\delta^{\rm SL}_{f_{L,R}} = \delta^{\rm SL}_{\tilde{f}_{L,R}} = \left[ \frac{I_f(I_f+1)}{s_W^2} + \frac{Y_f^2}{4c_W^2} \right]_{L,R}$$
$$-\frac{1}{4s_W^2 M^2} \left[ (\hat{m}_t^2 + \hat{m}_b^2)(\delta_{f,t_L} + \delta_{f,b_L}) \right.$$
$$\left. +2\hat{m}_t^2 \delta_{f,t_R} + 2\hat{m}_b^2 \delta_{f,b_R} \right]$$

with $\hat{m}_t^2 = m_t^2(1+\cot^2\beta), \hat{m}_b^2 = m_b^2(1+\tan^2\beta)$

charged Higgs bosons :

$$\delta^{\rm SL}_{H^\pm} = \left[ \frac{I_H(I_H+1)}{s_W^2} + \frac{Y_H^2}{4c_W^2} \right]$$
$$-\frac{3}{4s_W^2 M^2}(m_t^2 \cot^2\beta + m_b^2 \tan^2\beta)$$

charginos :

$$\delta^{\mathrm{SL}}_{\chi_i^+,\chi_j^+} = \tilde{\beta}_0(Z^+_{1i}Z^+_{1j}P_L + Z^{-*}_{1i}Z^-_{1j}P_R) + \frac{1+2c_W^2}{4s_W^2c_W^2}\,(Z^+_{2i}Z^+_{2j}P_L + Z^-_{2i}Z^-_{2j}P_R) - \frac{3}{4s_W^2M^2}(\widehat{m}_t^2\;Z^+_{2i}Z^+_{2j}P_L + \widehat{m}_b^2\;Z^-_{2i}Z^-_{2j}P_R)$$

$Z^{\pm}_{1i}$ : charged gaugino components

$Z^{\pm}_{2i}$ : charged higgsino components.

with $\widehat{m}_t^2 = m_t^2(1+\cot^2\beta), \widehat{m}_b^2 = m_b^2(1+\tan^2\beta)$

Plus: process dependent angular corrections

$$
\begin{aligned}
&d\sigma^{\mathsf{SL}}_{e^+_\alpha e^-_\alpha \longrightarrow \overline{f}_\beta f_\beta} = d\sigma^{\mathsf{Born}}_{e^+_\alpha e^-_\alpha \longrightarrow \overline{f}_\beta f_\beta} \times \\
&\exp\left\{ -\frac{g^2(m_s^2) I_{e^-_\alpha}(I_{e^-_\alpha}+1)}{8\pi^2}\left[\log^2\frac{q^2}{M^2} - \frac{1}{3}\tilde{\beta}_0 \frac{g^2(m_s^2)}{4\pi^2}\log^3\frac{q^2}{m_s^2}\right]\right. \\
&-\frac{{g'}^2(m_s^2) Y^2_{e^-_\alpha}}{32\pi^2}\left[\log^2\frac{q^2}{M^2} - \frac{1}{3}\tilde{\beta}'_0 \frac{{g'}^2(m_s^2)}{4\pi^2}\log^3\frac{q^2}{m_s^2}\right] \\
&+\left(\frac{g^2(m_s^2)}{8\pi^2} I_{e^-_\alpha}(I_{e^-_\alpha}+1) + \frac{{g'}^2(m_{\tilde{f}}^2)}{8\pi^2}\frac{Y^2_{e^-_\alpha}}{4}\right) 2\log\frac{q^2}{M^2} \\
&-\frac{g^2(m_s^2) I_{f_\beta}(I_{f_\beta}+1)}{8\pi^2}\left[\log^2\frac{q^2}{M^2} - \frac{1}{3}\tilde{\beta}_0 \frac{g^2(m_s^2)}{4\pi^2}\log^3\frac{q^2}{m_s^2}\right] \\
&-\frac{{g'}^2(m_s^2) Y^2_{f_\beta}}{32\pi^2}\left[\log^2\frac{q^2}{M^2} - \frac{1}{3}\tilde{\beta}'_0 \frac{{g'}^2(m_s^2)}{4\pi^2}\log^3\frac{q^2}{m_s^2}\right] \\
&+\left(\frac{g^2(m_s^2)}{8\pi^2} I_{f_\beta}(I_{f_\beta}+1) + \frac{{g'}^2(m_{\tilde{f}}^2)}{8\pi^2}\frac{Y^2_{f_\beta}}{4}\right) 2\log\frac{q^2}{M^2} \\
&-\frac{g^2(m_s^2)}{8\pi^2}\left(\frac{1+\delta_{\beta,\mathsf{R}}}{2}\frac{\widehat{m}_f^2}{M^2} + \delta_{\beta,\mathsf{L}}\frac{\widehat{m}_{f'}^2}{2M^2}\right)\log\frac{q^2}{m_s^2} \\
&-\frac{g^2(m_s^2)}{8\pi^2}\log\frac{q^2}{M^2}\left[\left(\tan^2\theta_{\mathsf{w}} Y_{e^-_\alpha} Y_{f_\beta} + 4 I^3_{e^-_\alpha} I^3_{f_\beta}\right)\log\frac{t}{u}\right. \\
&\left.\left.+\frac{\delta_{\alpha,L}\delta_{\beta,L}}{\tan^2\theta_{\mathsf{w}} Y_{e^-_\alpha} Y_{f_\beta}/4 + I^3_{e^-_\alpha} I^3_{f_\beta}}\left(\delta_{d,f}\log\frac{-t}{q^2} - \delta_{u,f}\log\frac{-u}{q^2}\right)\right]\right\}
\end{aligned}
$$

where we use: $\lambda = M$

$\widehat{m}_f = m_t/\sin\beta$ if $f = t$ and

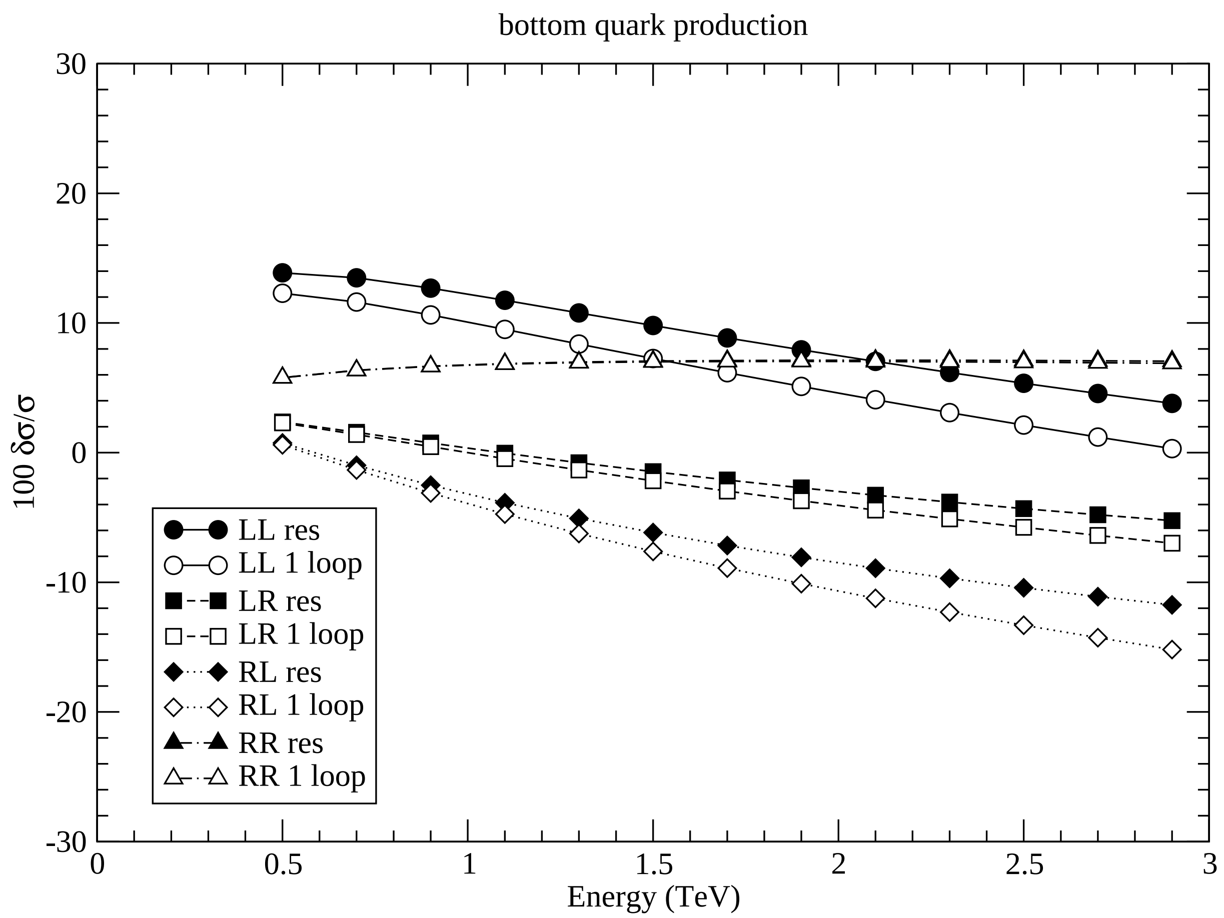
$\widehat{m}_f = m_b/\cos\beta$ if $f = b$.

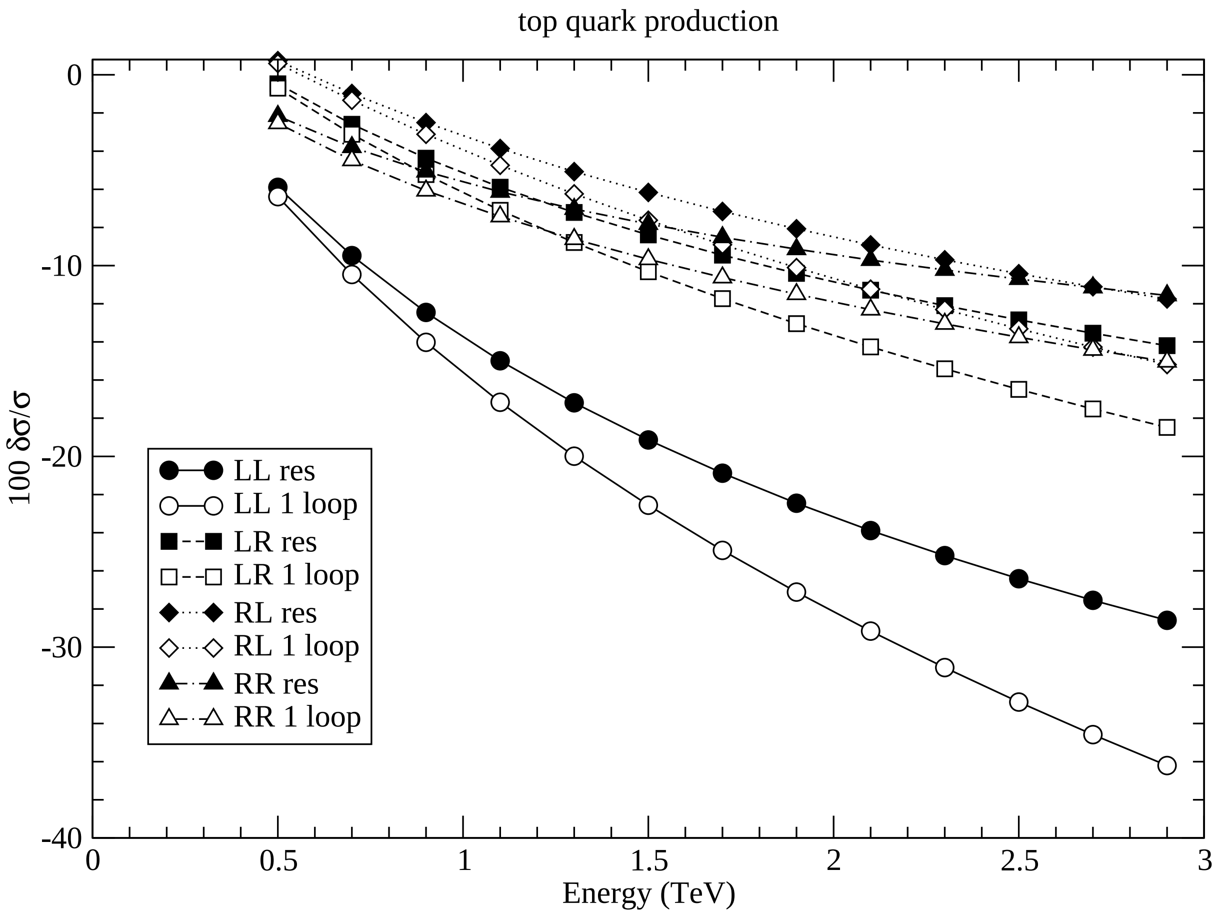
$$\begin{aligned}
\tilde{\beta}_0 &= \frac{3}{4}C_A - \frac{n_g}{2} - \frac{n_h}{8} \ , \ \tilde{\beta}'_0 = -\frac{5}{6}n_g - \frac{n_h}{8} \\
g^2(q^2) &= \frac{g^2(m_s^2)}{1+\tilde{\beta}_0 \frac{g^2(m_s^2)}{4\pi^2}\ln\frac{q^2}{m_s^2}} \ , \ {g'}^2(q^2) = \frac{{g'}^2(m_s^2)}{1+\tilde{\beta}'_0 \frac{{g'}^2(m_s^2)}{4\pi^2}\ln\frac{q^2}{m_s^2}}
\end{aligned}$$

where $C_A = 2$, $n_g = 3$ and $n_h = 2$.

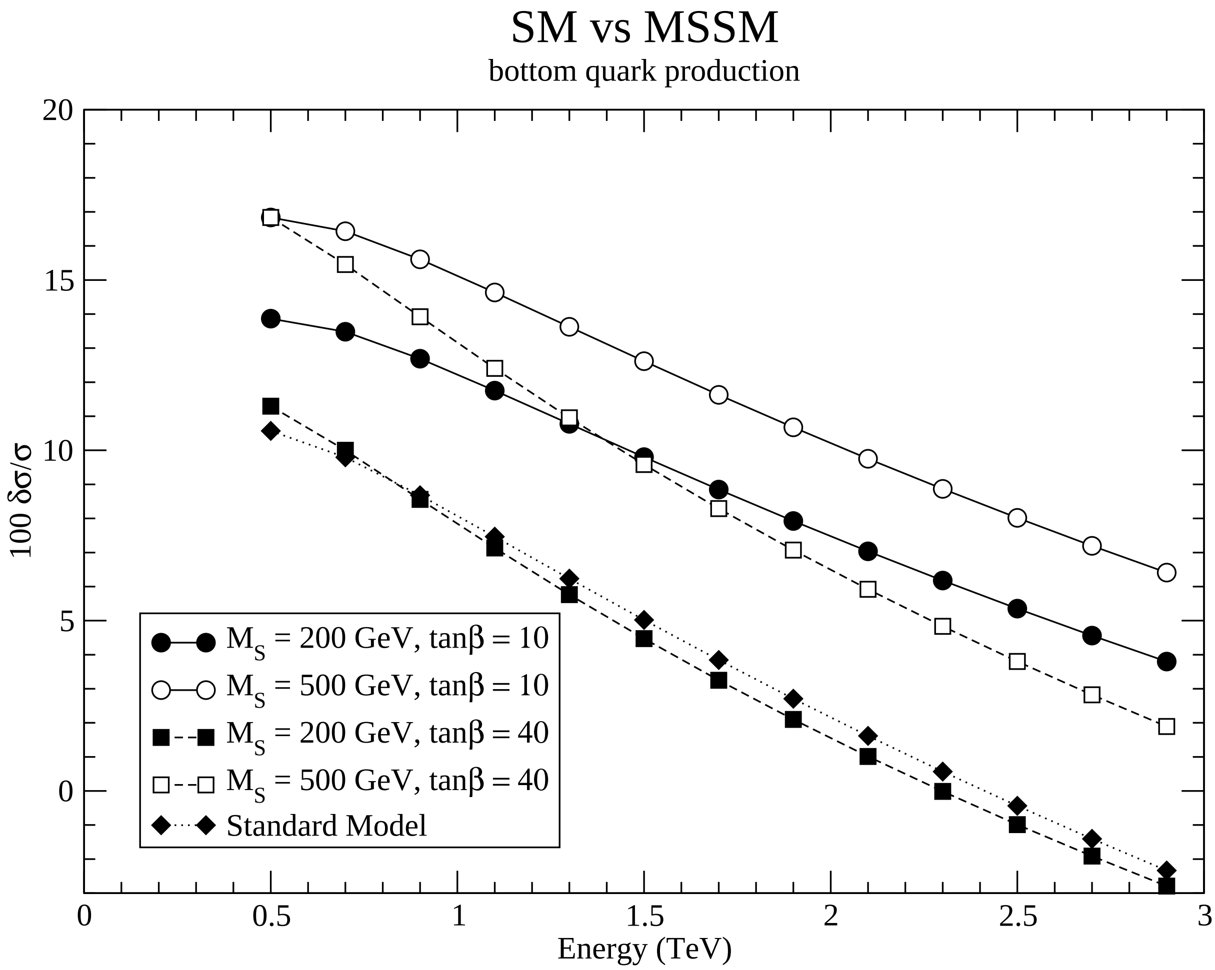
Analogous results for: $H^+H^-$ and $\tilde{f}\tilde{\bar{f}}$

Numerical results for various LC-processes:



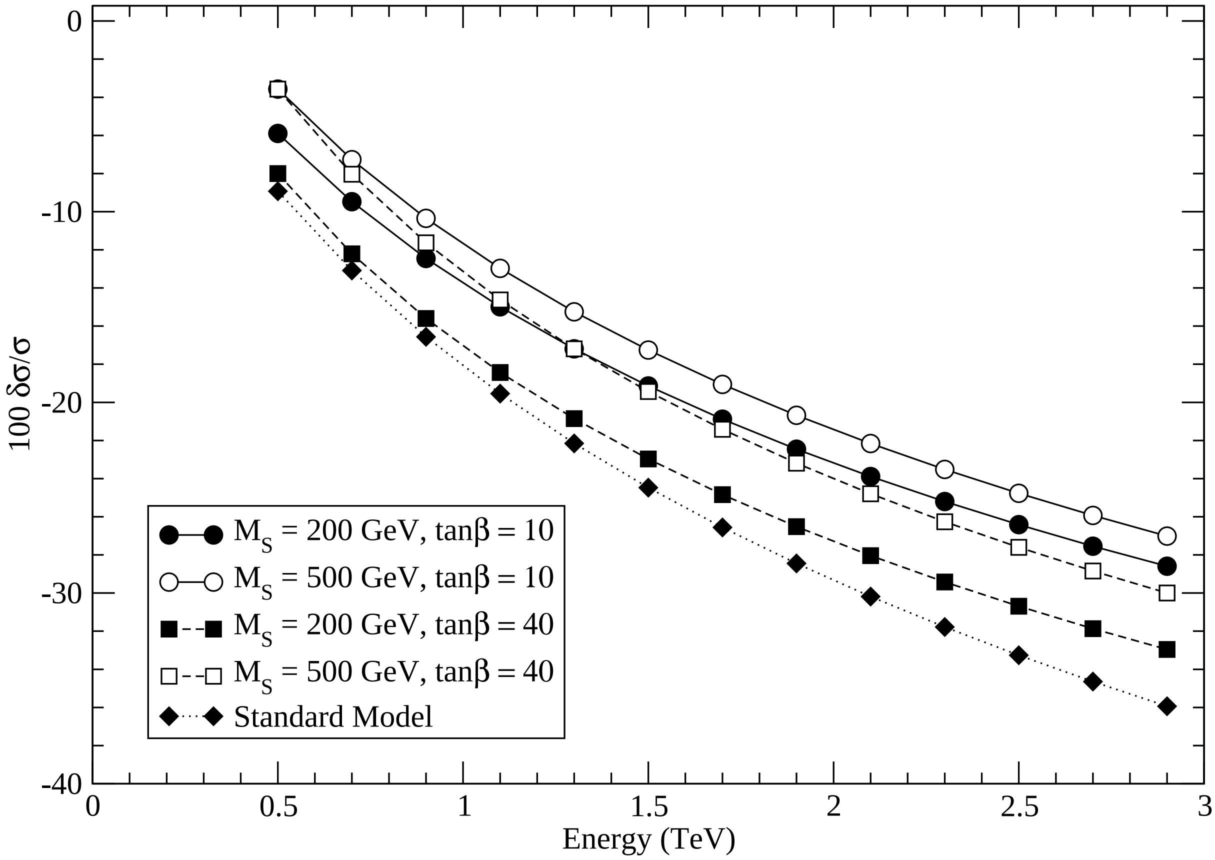


top quark production
0
-10
-20
-30
-40
100 δσ/σ
0
0.5
1
1.5
2
2.5
3
Energy (TeV)
LL res
LL 1 loop
LR res
LR 1 loop
RL res
RL 1 loop
RR res
RR 1 loop

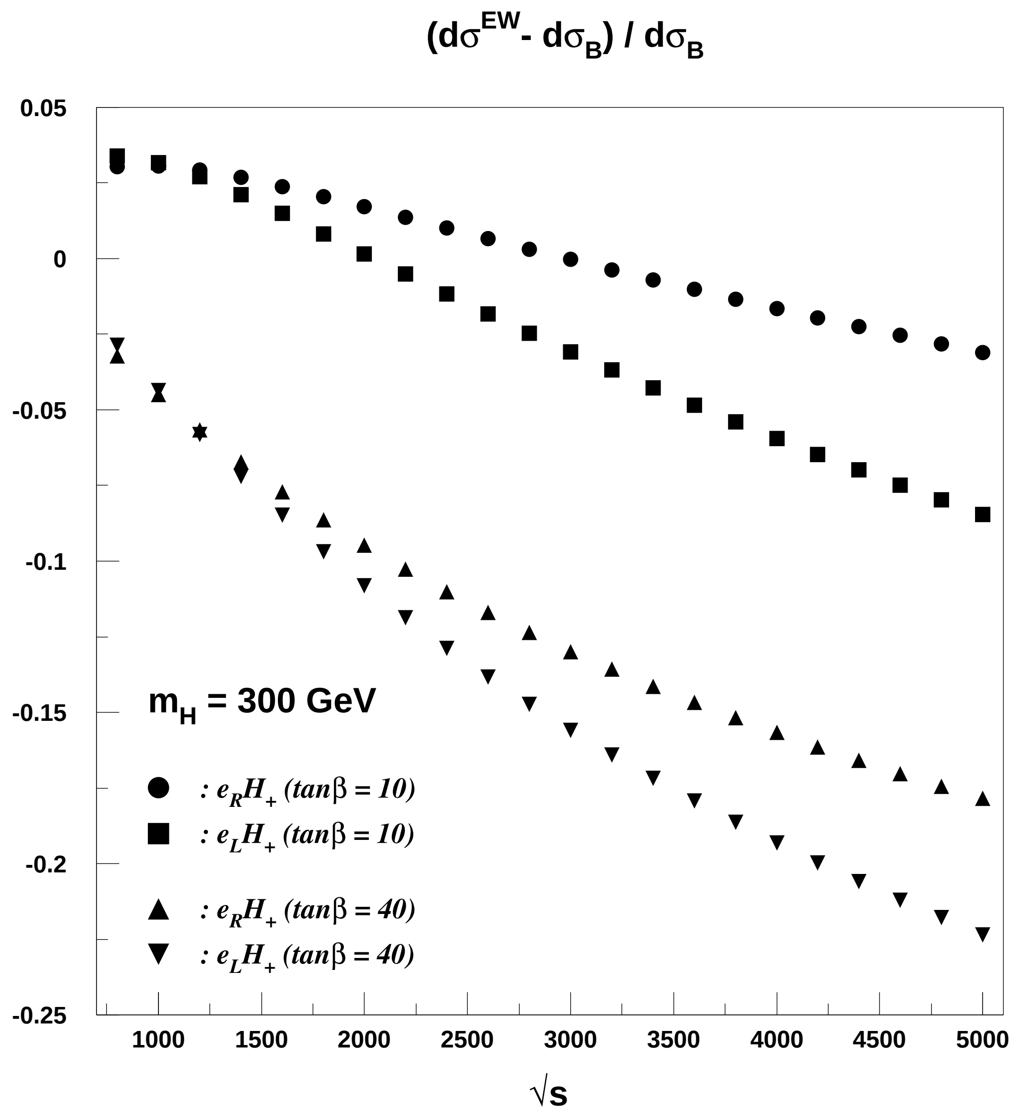



Comparison for left-handed case

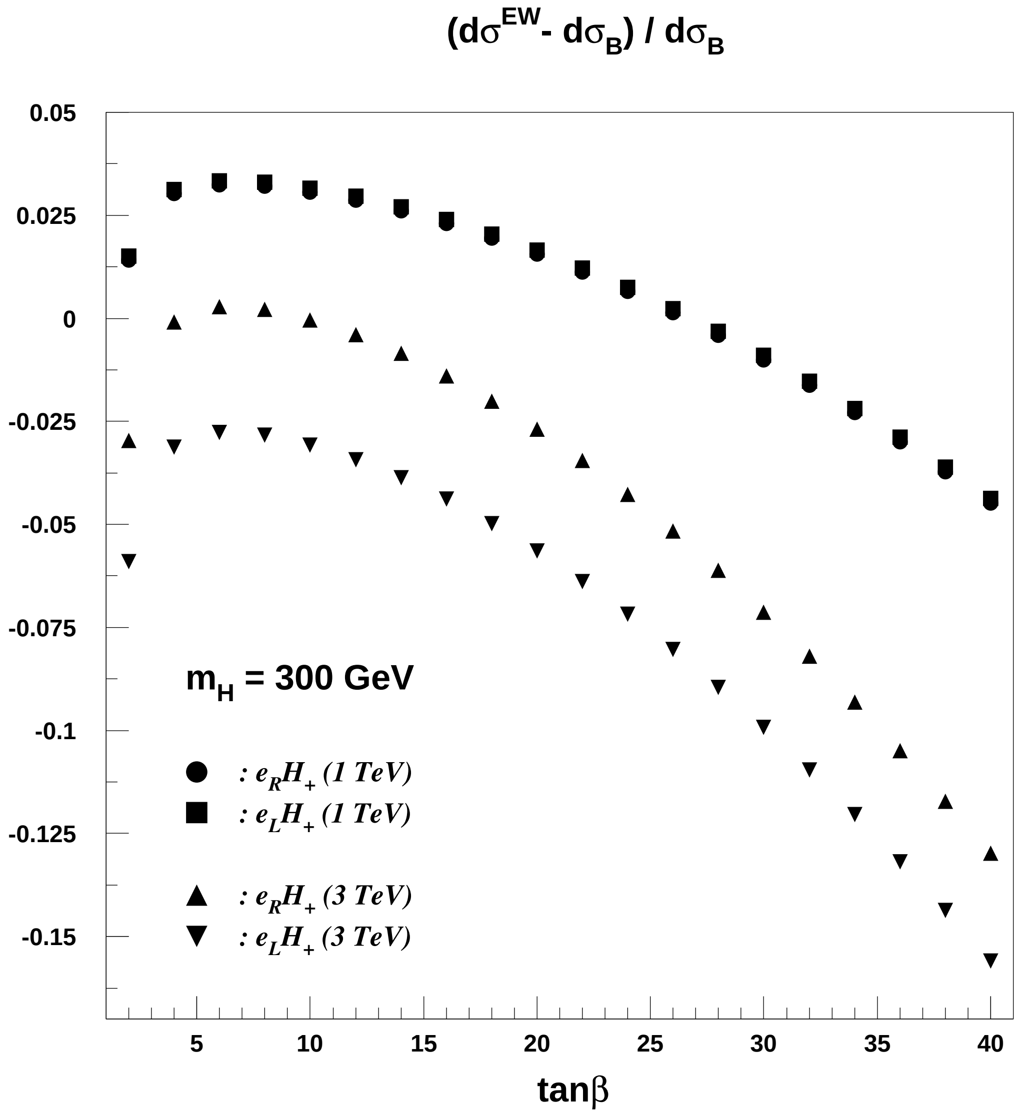
top quark production




Comparison for left-handed case




Fixed $\theta = 90^o$




Fixed $\theta = 90^o$

## Monte Carlo Study of $\tan\beta$

$$\epsilon(q^2) = \frac{\alpha}{4\pi}\ F(\tan\beta)\ln\frac{q^2}{M_S^2} + G + \mathcal{O}\left(\frac{M^2}{q^2}\right).$$

$G$ depends on all mass ratios of virtual particles.
$\rightarrow$ Set of $N$ independent measurements at c.m. energies

$$\sqrt{q_1^2}, \sqrt{q_2^2}, \ldots, \sqrt{q_N^2}$$

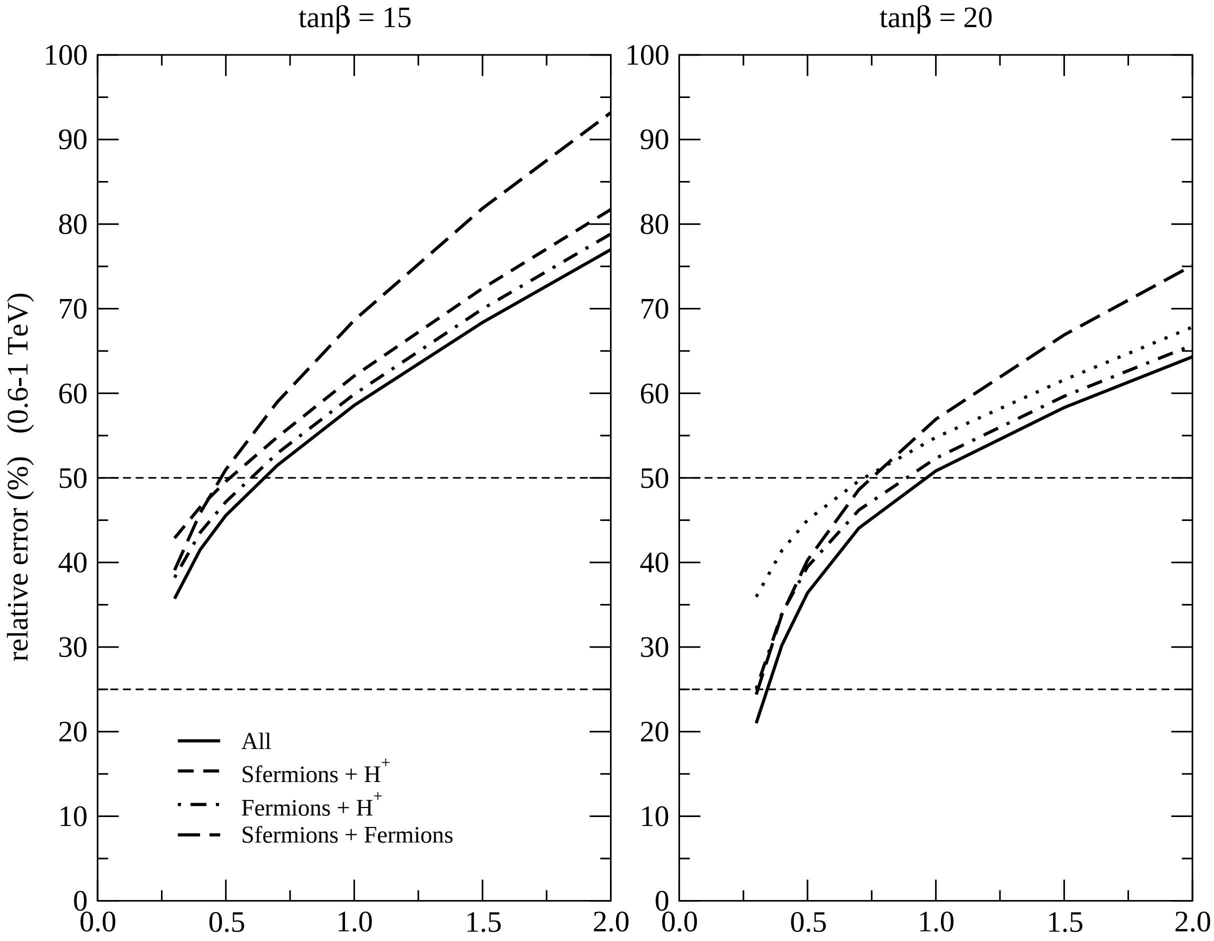
For each observable

$$\delta_i = \epsilon(q_i^2) - \epsilon(q_1^2),$$

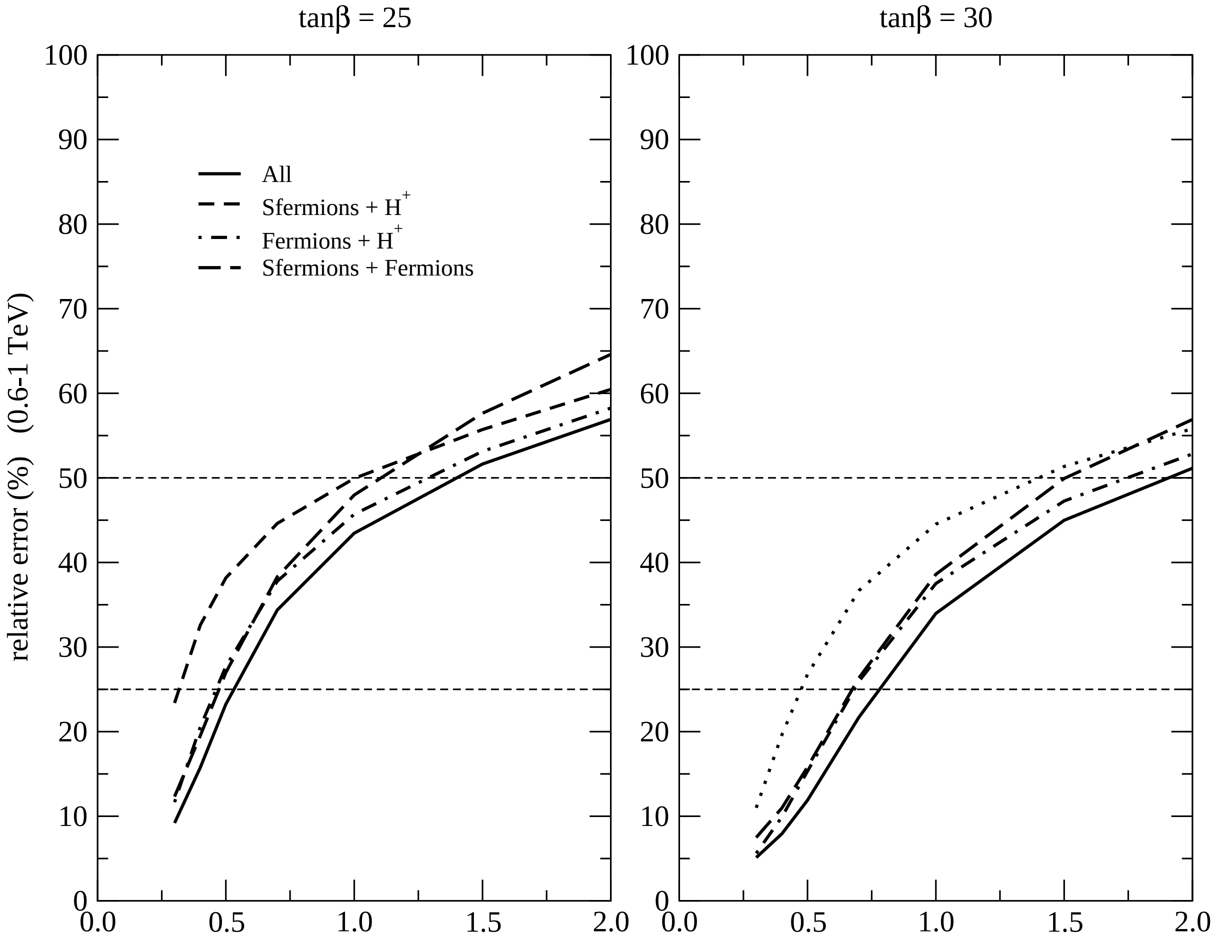
does not contain $G$ (SL) and takes the simple form

$$\delta_i = F(\tan\beta^*)\ \ln\frac{q_i^2}{q_1^2},$$

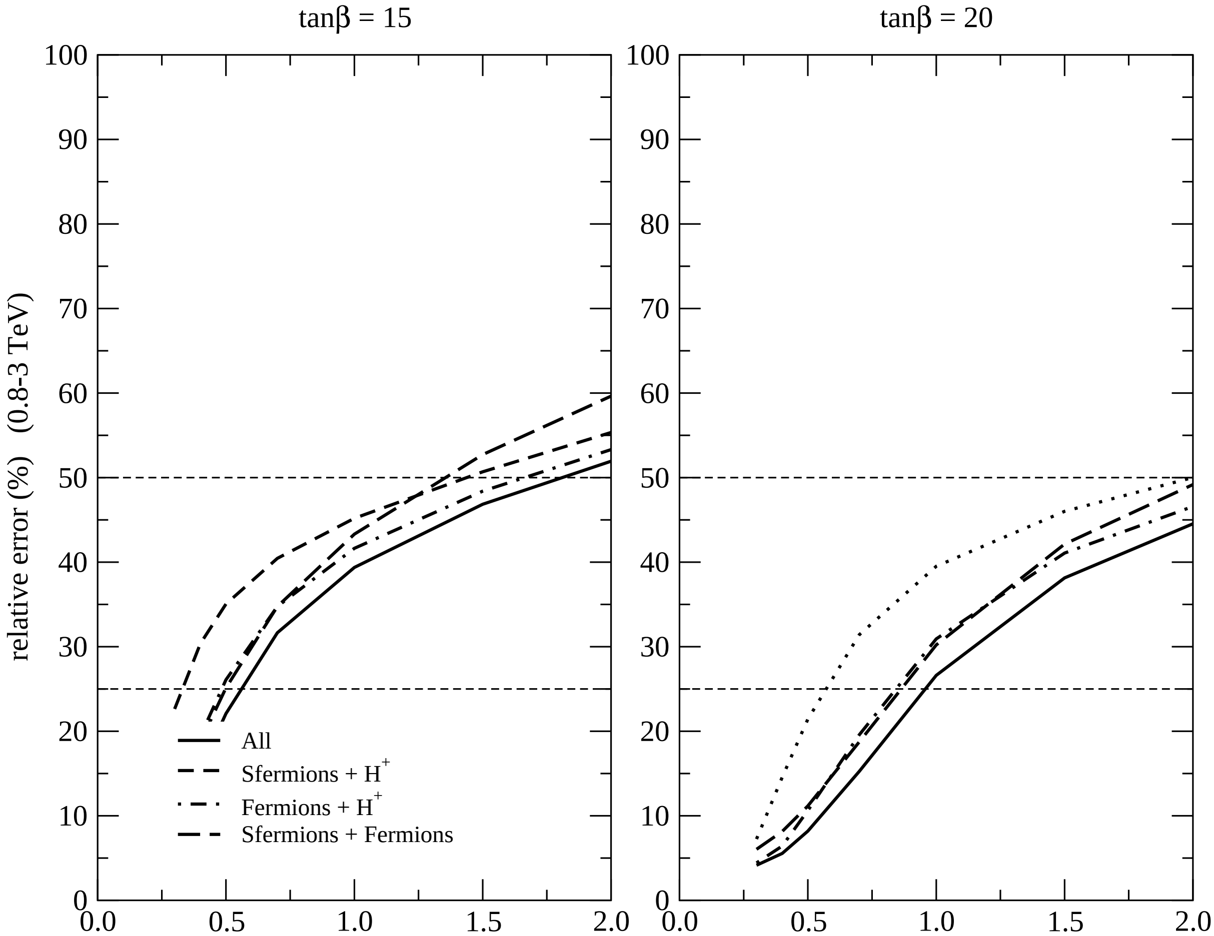
where $\tan\beta^*$ is the *true* unknown value that describes the experimental measurements.
$\rightarrow$ minimize $\chi^2$: Measure of $\tan\beta$



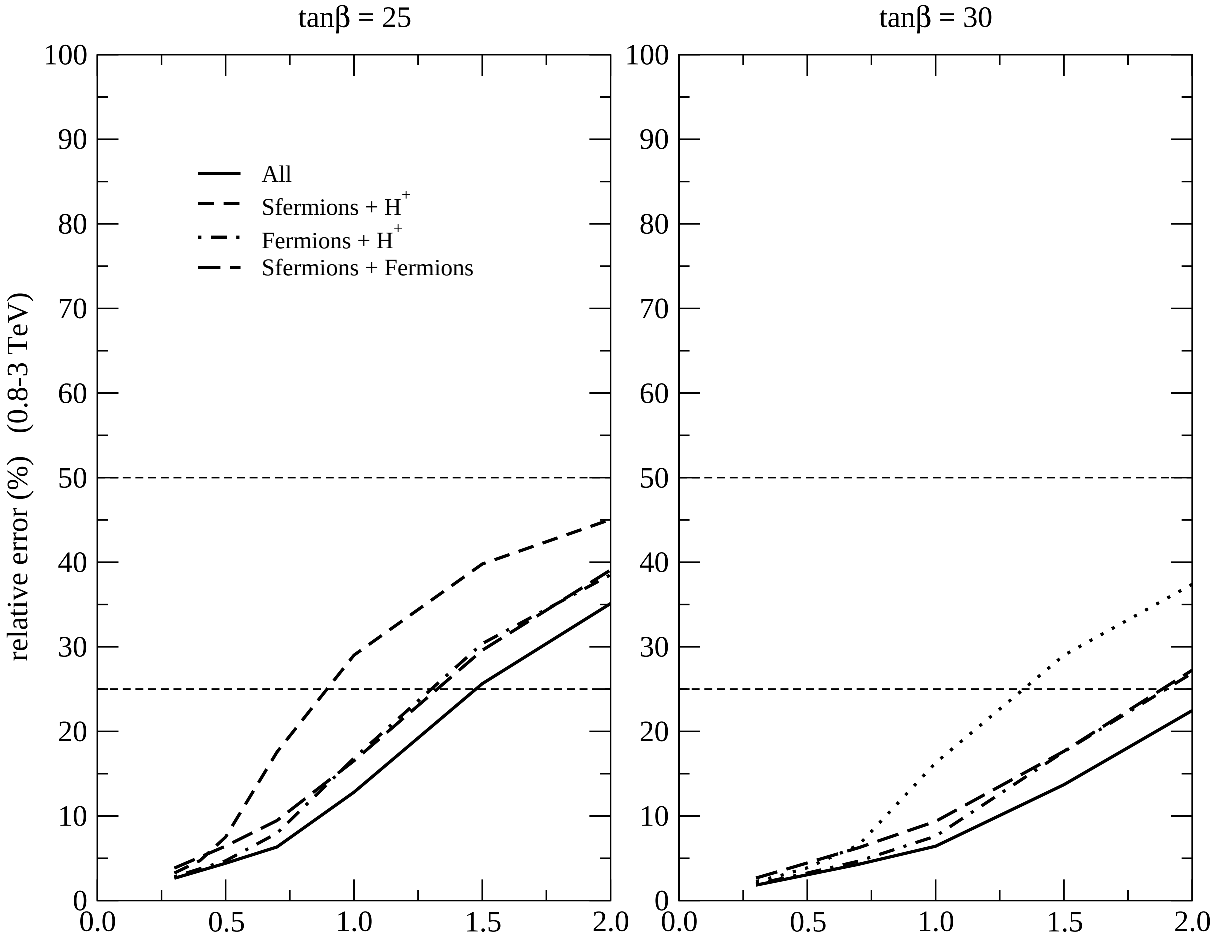
tanβ = 15
tanβ = 20
relative error (%) (0.6-1 TeV)
100
90
80
70
60
50
40
30
20
10
0
0.0
0.5
1.0
1.5
2.0
All
Sfermions + H+
Fermions + H+
Sfermions + Fermions



tanβ = 25
tanβ = 30
relative error (%) (0.6-1 TeV)
All
Sfermions + $H^+$
Fermions + $H^+$
Sfermions + Fermions
100
90
80
70
60
50
40
30
20
10
0
0.0
0.5
1.0
1.5
2.0



tanβ = 15
tanβ = 20
relative error (%) (0.8-3 TeV)
All
Sfermions + $H^+$
Fermions + $H^+$
Sfermions + Fermions
0
10
20
30
40
50
60
70
80
90
100
0.0
0.5
1.0
1.5
2.0



tanβ = 25
tanβ = 30
relative error (%)   (0.8-3 TeV)
All
Sfermions + H+
Fermions + H+
Sfermions + Fermions
0
10
20
30
40
50
60
70
80
90
100
0.0
0.5
1.0
1.5
2.0

## Conclusions

SL-Sudakov logarithms in the MSSM are calculated to all orders using fields of the effective high energy theory with matching at the weak scale.

They *factorize* and *exponentiate* (with reasonable experimental cuts, mixing). For angular dependent corrections in *operator* form!

Agreement at one loop up to NLO , to two loops for DL and angular SL terms with calculation using physical fields!

The method is universal for all processes at a future collider (LC, LHC).

$\tan\beta$ can be determined to better than 50% for $\tan\beta > 15$ at LC in light susy-scenario if $\delta\sigma \leq 1\%$, better than 25% at CLIC.