Unique Factorization in QFT

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i) Lie algebra of Feynman graphs
- -> forest formula
- -> renormalization group
- -> pre-Lie structure

ii) ζ- functions $\Leftrightarrow$ Dyson-Schwinger equations

$$\zeta(t_i, s) = \Phi_s(1) + \Phi_s\left[B_+^P(\bar{\zeta}(t_i))\right]$$

$$\bar{\zeta}(t_i) = 1 + t\, B_+^P\left[\bar{\zeta}(t_i)\right]$$

iii) gauge symmetry

 $\vee$  $= 0$

$\Leftrightarrow$ Ward Id.

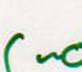
Then $\Gamma_1 * \Gamma_2 - \Gamma_2 * \Gamma_1$ is a Lie bracket.

$$\sum_{\substack{i,j,k\\ \text{cyclic}}} [[\Gamma_i, \Gamma_j], \Gamma_k] = 0$$

$$\Rightarrow \quad \Delta[\Gamma] = \Gamma \otimes 1 + 1 \otimes \Gamma + \sum_{\gamma \subset \Gamma} \gamma \otimes \Gamma/\gamma$$

$$S_R^\phi * \phi \, (\Gamma) = \underbrace{S_R^\phi(\Gamma)}_{\text{counterterm}} + \underbrace{\sum_{\gamma \subset \Gamma} S_R^\phi(\gamma)\, \phi(\Gamma/\gamma)}_{\bar{R}}$$

$$\Delta[\Gamma] = \Gamma \otimes 1 + (\mathrm{id} \otimes B_+^{\gamma,i})\, \Delta[X], \quad \Gamma = B_+^{\gamma,i}(X)$$

$\gamma =$  $\quad \Gamma =$  $\quad B_+$  i ()

 D-S

Interlude Riemann ζ

$$\zeta(s) = \prod_p \frac{1}{1-p^s} = \sum_n \frac{1}{n^s}$$

"Feynman graphs"  p_α prime

$$B_+^p(T) = $$  $$\cdot \frac{1}{\deg(T)+1}$$

$\boxed{\text{algebraic structure unchanged!}}$

$$\bar{\zeta}(t) = 1 + t \sum_p B_+^p(\bar{\zeta}(t))$$

$$= 1 + t \sum_p \bullet + t^2 \frac{1}{2} \sum_{p_1,p_2}$$ $$+ \dots$$

Feynman rules: $\phi_s(1) = 1 \quad \phi_s(T) = X(T)^{-s}$ $\quad \prod p_i$

$$\tilde{\zeta}(t,s) = \phi_s(1) + \phi_s(B_+^p(\bar{\zeta}(t)))$$

$\zeta(1,s) = \zeta(s)$ Riemann zeta.

$$X = 1 + \sum_{\gamma} B_+^{\gamma}\left(X^{|\gamma|}\right) g^{|\gamma|}$$

← skeletons

symmetrized insertions

Then $X = \prod_{\gamma}^{\vee} \dfrac{1}{1-\gamma}$

RING $\mathcal{R}$

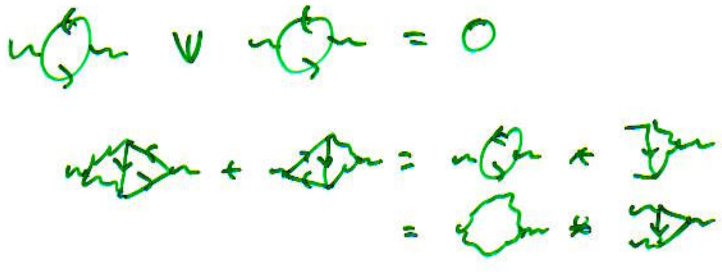
$A * B + B * A = 0$

A, B graphs

$\overset{?}{\Rightarrow} A = 0 \vee B = 0$

ϕ^3_6 yes ϕ^4_4 yes

$$\boxed{\begin{array}{ll} \text{QED} & \text{no} \\ \text{YM} & \text{no} \end{array}}$$



Then in the ring $\mathcal{R}$, the relation

defines a new ring $\bar{\mathcal{R}}$ in which the Ward Ids hold.

[goes through under the Feynman rules]