Top Quark Pair Production and Decay at Linear Colliders: Signal vs. Off Resonance Background*

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* Based on work in collaboration with Adam Biernacik

Plan

- **Motivation.**
- **Outline of calculation.**
- `eett6f`**: a program for top quark pair production and decay into 6 fermions at linear colliders.**
- **Checks; comparisons against other results in the literature.**
- **Numerical results, mainly for** $e^+e^- \to b\nu_\mu\mu^+\bar{b}\mu^-\bar{\nu}_\mu$ **at lowest order of SM.**
- **Summary and outlook.**

Motivation

As the top quark is the heaviest particle ever observed, the measurement of its physical properties may give hints towards better understanding of the electroweak symmetry breaking mechanism and observed fermion mass hierarchy.

Should the effects of the physics beyond the SM be visible at the energy scale below 1 TeV, then top couplings may shaw deviations from the corresponding SM values.

High-precision measurements of the top quark properties and interactions are planned at TESLA and will most certainly belong to the research program of any future e^+e^- collider.

To disentangle the possible effects of physics beyond the SM, it is crucial to know the SM predictions for the top quark pair production and decay precisely, including radiative corrections.

The QCD and EW radiative corrections to the on-mass-shell top pair production

$$e^+e^- \to \bar{t}t$$

are known both in the threshold region and in the continuum. They are typically of $\mathcal{O}$(10%).

The corrections to the top decay into a W-boson and a b-quark, and to the W decay into a fermion pair are also known.

Motivation

The effects caused by the off-shellness of the $\bar{t}t$ pair may be important too, especially for measurements at c.m.s. energies much above the threshold.

Therefore, the 6 fermion reactions of the form

$$e^+e^- \to bf_1\bar{f}_1'\bar{b}f_2\bar{f}_2',$$

where $f_1 = \nu_\mu, \nu_\tau, u, c$, $f_2 = \mu^-, \tau^-, d, s$ and f_1', f_2' are the corresponding weak isospin partners, $f_1' = \mu^-, \tau^-, d, s$, $f_2' = \nu_\mu, \nu_\tau, u, c$, should be studied at least in the lowest order of SM, with a complete set of the Feynman diagrams.

In particular, results on

$$e^+e^- \to b\nu_\mu\mu^+\bar{b}\mu^-\bar{\nu}_\mu,$$

which is a pure EW reaction that receives contributions from 452 Feynman diagrams, will be shown.

The results will be compared with the double resonance approximation for a t- and $\bar{t}$-quark

$$e^+e^- \to \bar{t}^*t^* \to b\nu_\mu\mu^+\bar{b}\mu^-\bar{\nu}_\mu$$

with only two signal diagrams contributing.

As it is not feasible to calculate $\mathcal{O}(\alpha)$ radiative corrections to the complete set of the Feynman diagrams, it will be argued that it is reasonable to include higher order effects, at least the leading ones, just for the two signal diagrams.

Outline of calculation

The matrix elements are calculated with the helicity amplitude method described in

- **K. Kołodziej, M. Zrałek, Phys. Rev. D43 (1991) 3619;**
- **F. Jegerlehner, K. Kołodziej, Eur. Phys. J. C12 (2000) 77.**

Phase space integrations are performed with the Monte Carlo method.

The most relevant peaks of the matrix element squared related to

- **the Breit-Wigner shape of the W, Z, Higgs and top quark resonances,**
- **the exchange of a massless photon or gluon**

have to be mapped away.

As it is not possible to find out a single parametrization of the 14-dimensional phase space which would allow to cover the whole resonance structure of the integrand, it is necessary to utilize a multi-channel MC approach.

eett6f: a program

A computer program eett6f for calculating cross sections of $e^+e^- \to$ 6 fermions processes relevant for a $t\bar{t}$-pair production and decay at centre of mass energies typical for linear colliders has been written.

eett6f v. 1.0 allows for calculating both the total and differential cross sections at tree level of SM.

Some simple extensions of the SM have been already implemented in the program.

The program can be used as the Monte Carlo generator of unweighted events as well.

eett6f is a package written in FORTRAN 90.

It consists of 50 files including a makefile, all stored in one working directory.

The user should specify the physical input parameters in module inprms.f and select a number of options in the main program csee6f.f.

The options allow, among other, for calculation of the cross sections while switching on and off different subsets of the Feynman diagrams.

It is possible to calculate cross sections in two different narrow width approximations, for $t\bar{t}$-quarks, or $W^{\pm}$-bosons.

eett6f: a program

The program allows for taking into account both the electroweak and QCD lowest order contributions.

The constant widths of unstable particles, the massive electroweak vector bosons, the Higgs boson and the top quark, are introduced through the complex mass parameters:

$$M_V^2 = m_V^2 - i m_V \Gamma_V, \qquad V = W, Z,$$
$$M_H^2 = m_H^2 - i m_H \Gamma_H, \qquad M_t = m_t - i\Gamma_t/2,$$

which replace masses in the corresponding propagators, both in the s- and t-channel Feynman diagrams,

$$\Delta_F^{\mu\nu}(q) = \frac{-g^{\mu\nu} + q^\mu q^\nu / M_V^2}{q^2 - M_V^2},$$

$$\Delta_F(q) = \frac{1}{q^2 - M_H^2}, \qquad S_F(q) = \frac{\not{q} + M_t}{q^2 - M_t^2}.$$

Propagators of a photon and a gluon are taken in the Feynman gauge.

The EW mixing parameter may be defined either real or complex,

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} \quad \text{or} \quad \sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}.$$

As light fermion masses are not neglected, cross sections can be calculated without any kinematical cuts.

Checks

- **Matrix elements of $e^+e^- \to b\nu_\mu\mu^+\bar{b}\mu^-\bar{\nu}_\mu$ have been programmed independently by KK and Adam Biernacik.**
- **Matrix elements of different 'subprocesses': $e^+e^- \to t\bar{t}, bW^+\bar{b}W^-, bf_1\bar{f}_1'\bar{b}W^-, bW^+\bar{b}f_2\bar{f}_2'$ and $f_1\bar{f}_1'f_2\bar{f}_2'Z$ have been checked against MADGRAPH.**
- **Matrix elements of reactions with the single on-mass-shell top quark have been calculated in an arbitrary linear gauge.**
- **The multichannel phase space generation routine has been checked by comparing normalization of different channels against each other and testing energy-momentum conservation and on-mass-shell relations.**
- **Results obtained with the MC integration routine VEGAS have been reproduced with a more efficient own made MC integration routine carlos.**
- **A comparison against LUSIFER (S. Dittmaier, M. Roth). Lowest order total cross sections with cuts $(m_f = 0)$.**

$\sigma(\sqrt{s} = 500\ \mathrm{GeV})$ (fb)	eett6f	LUSIFER
$e^+e^- \to b\nu_\mu\mu^+\bar{b}\mu^-\bar{\nu}_\mu$	5.8019(33)	5.8091(49)
$e^+e^- \to b\nu_\mu\mu^+\bar{b}\tau^-\bar{\nu}_\tau$	5.8160(32)	5.7998(36)
$e^+e^- \to b\nu_\mu\mu^+\bar{b}d\bar{u}$	17.223(15)	17.171(24)

Checks

- **The invariant mass and angular distributions of the $bu\bar{d}$-quark triple of $e^+e^- \to bu\bar{d}b\mu^-\bar{\nu}_\mu$ at $\sqrt{s} = 500$ GeV of Dittmaier and Roth are nicely reproduced by `eett6f` within accuracy of plots.**

- **A comparison of lowest order SM total cross sections (in fb) of $e^+e^- \to b\nu_\mu\mu^+\bar{b}d\bar{u}$ against F. Yuasa, Y. Kurihara and S. Kawabata.**

$\sqrt{s}$	F. Yuasa, et. al.		Present Work		
(GeV)	$\sigma_{\text{all EW}}$	$\sigma_{t^*\bar{t}^*}$	$\sigma_{\text{all EW}}$	$\sigma_{t^*\bar{t}^*}$	$\sigma^{\text{no Higgs}}_{\text{all EW}}$
340	0.687(2)	0.4462(3)	0769(10)	0.4455(4)	0.689(2)
350	6.45(1)	6.187(4)	6.59(2)	6.175(4)	6.45(1)
360	14.97(2)	14.63(1)	15.03(5)	14.623(9)	14.97(3)
380	21.42(4)	21.00(1)	21.48(8)	20.99(1)	21.49(5)
500	22.32(4)	21.30(1)	22.55(4)	21.27(1)	22.32(5)

Numerical results

Lowest order SM total cross sections in fb of $e^+e^- \to b\nu_\mu\mu^+\bar{b}\mu^-\bar{\nu}_\mu$:

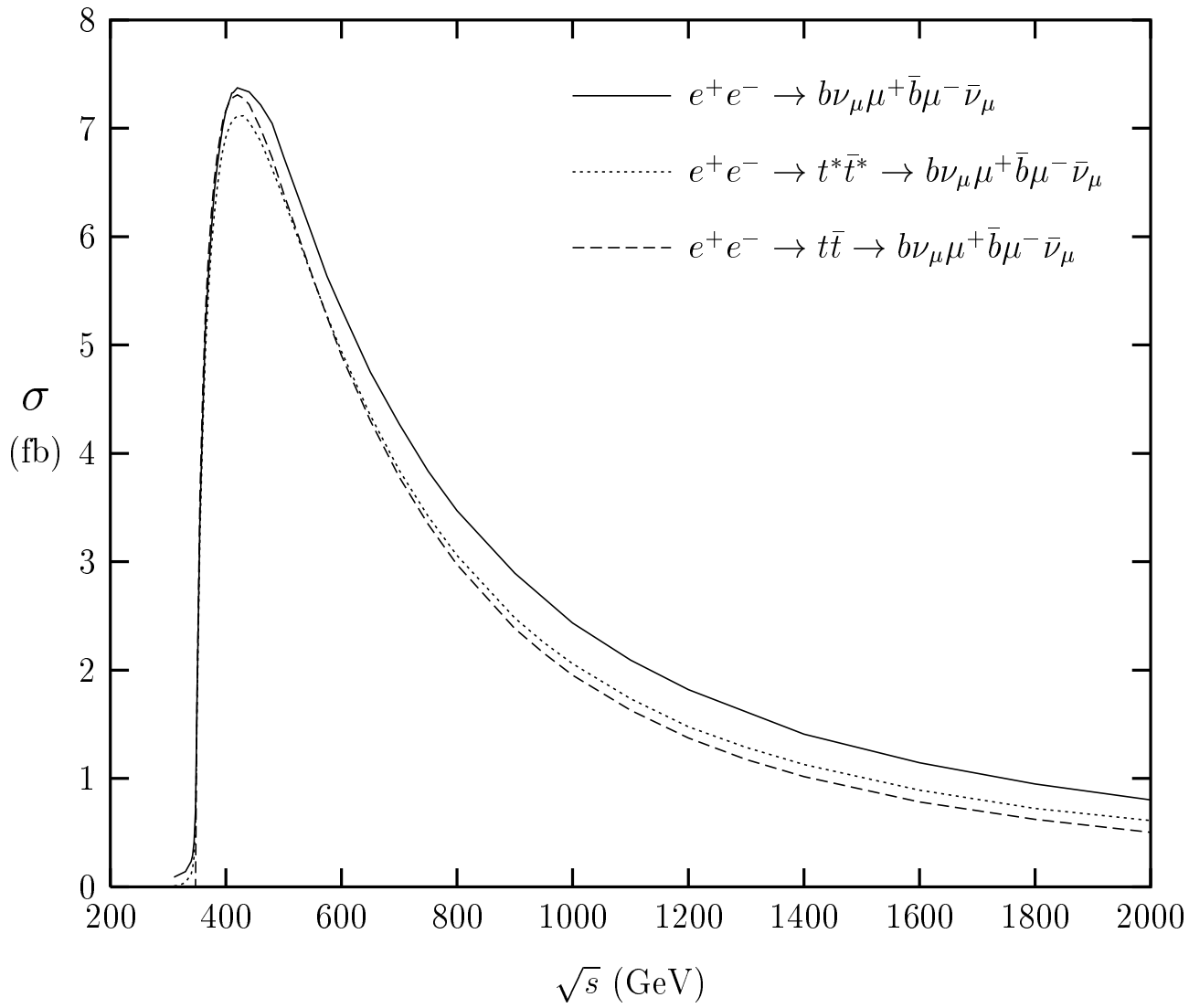
$\sqrt{s}$	σ	$\sigma_{\bar{t}^*t^*}$	$\sigma_{\bar{t}t}$
360	4.416(6)	4.262(1)	4.624(2)
500	6.705(6)	6.354(2)	6.400(7)
800	3.538(29)	3.058(2)	2.973(4)

Cross sections in fb of $e^+e^- \to b\nu_\mu\mu^+\bar{b}\mu^-\bar{\nu}_\mu$ for different values of the top quark width Γ_t:

$\sqrt{s}$	Γ_t (GeV)		
(GeV)	1.5	1.6	1.7
360	4.416(6)	3.862(7)	3.423(7)
500	6.705(6)	5.930(16)	5.272(12)
800	3.538(29)	3.134(40)	2.873(54)

Substantial dependence on the top width, $\sim 1/\Gamma_t^2$, may be a helpful in determining it.

Numerical results



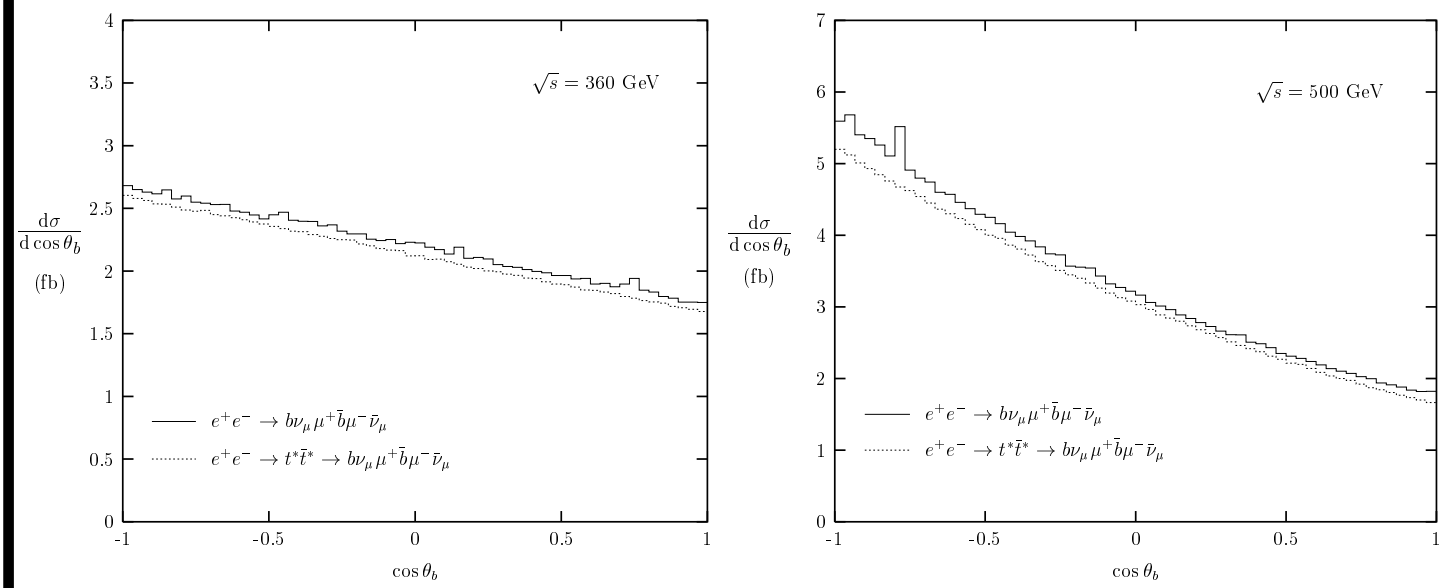
Total cross sections of $e^+e^- \to b\nu_\mu\mu^+\bar{b}\mu^-\bar{\nu}_\mu$ as functions of the c.m.s. energy.

Compare

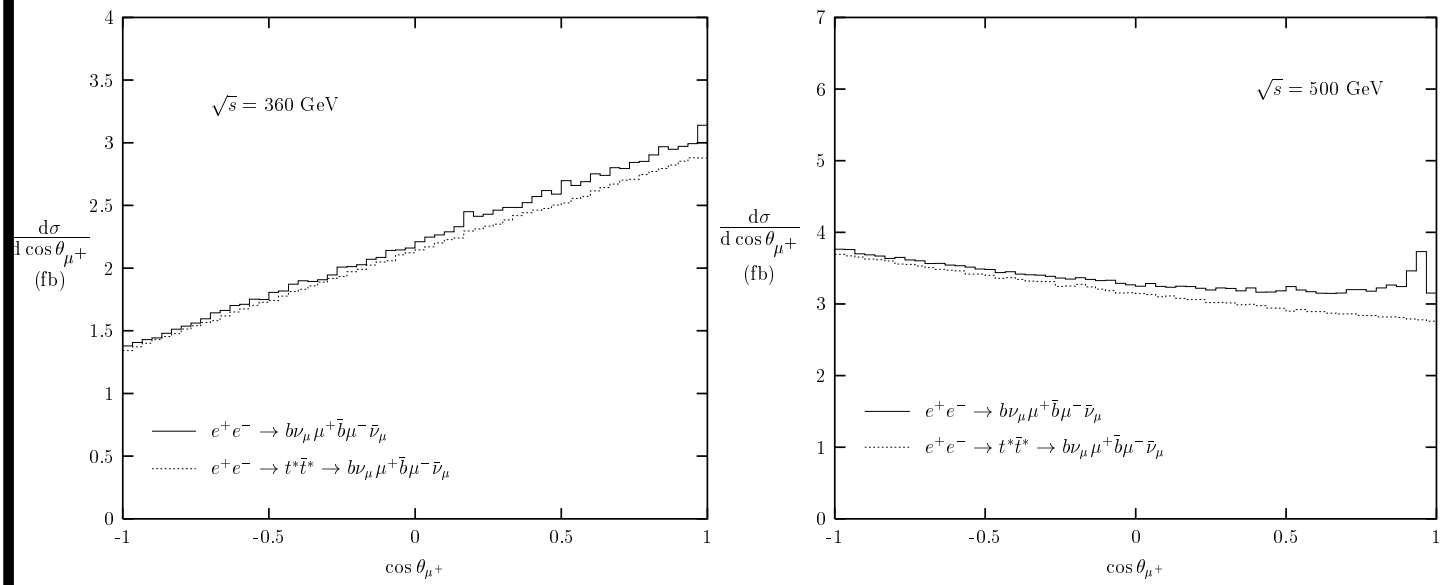
- **solid and dotted lines – signal vs. off resonance background**
- **dotted and dashed lines – pure off-shellness effects of the $t\bar{t}$-quark pair**

Numerical results

$$e^+e^-(\to \bar{t}^*t^*) \to b\nu_\mu\mu^+\bar{b}\mu^-\bar{\nu}_\mu$$



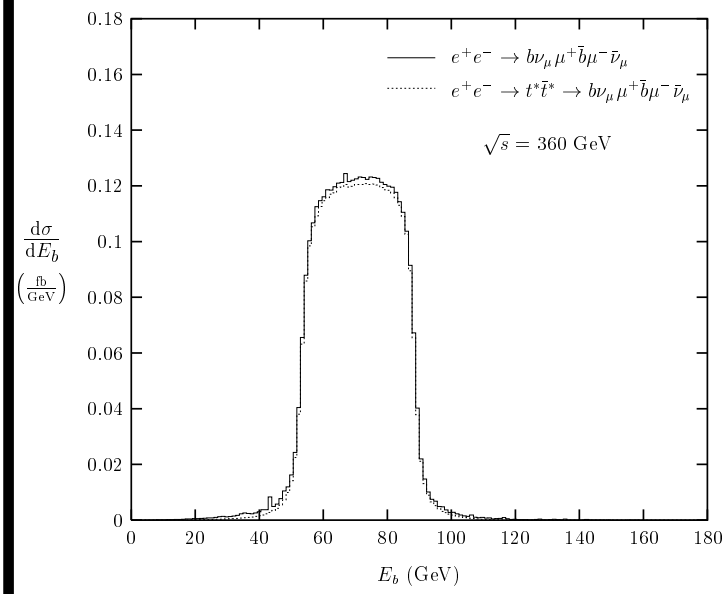
Angular distributions of a b-quark at $\sqrt{s} = 360$ GeV and $\sqrt{s} = 500$ GeV.

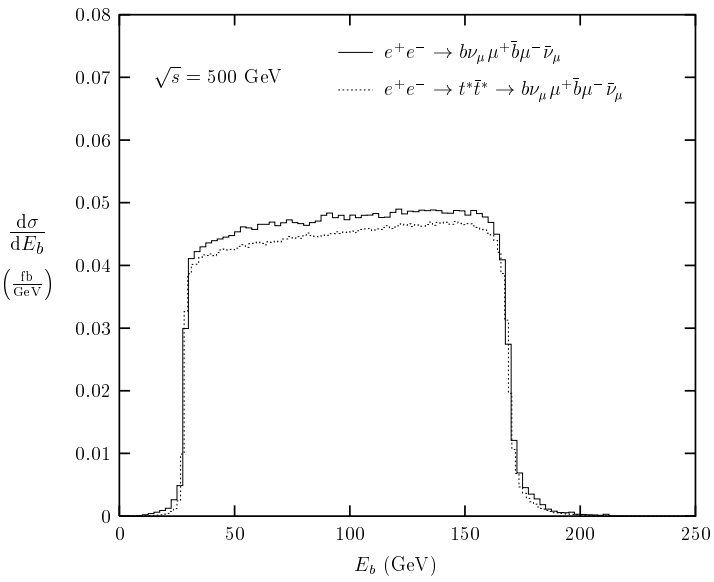


Angular distributions of a μ^+ at $\sqrt{s} = 360$ GeV and $\sqrt{s} = 500$ GeV.

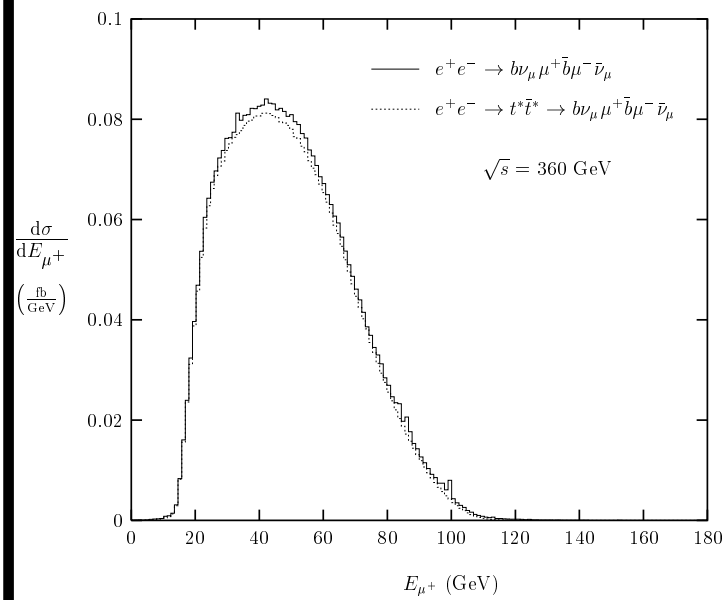
Numerical results

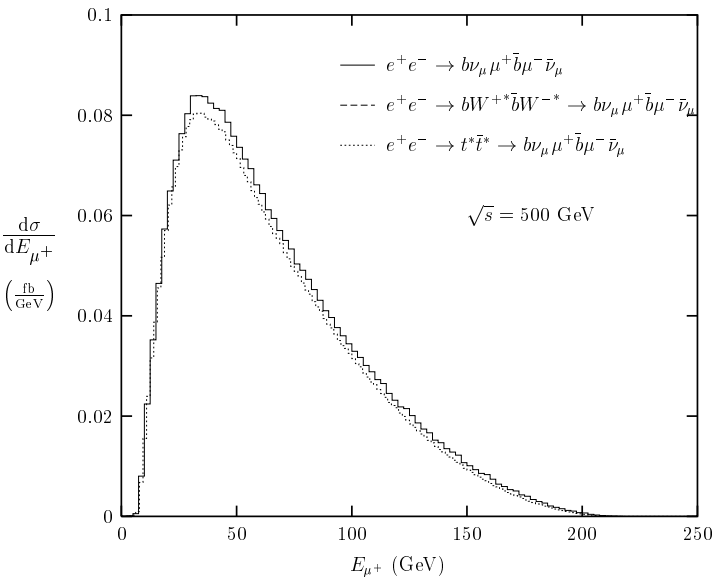
$$e^+e^-(\to \bar{t}^*t^*) \to b\nu_\mu\mu^+\bar{b}\mu^-\bar{\nu}_\mu$$





Energy distributions of a b-quark at $\sqrt{s} = 360$ GeV and $\sqrt{s} = 500$ GeV.





Energy distributions of a μ^+ at $\sqrt{s} = 360$ GeV and $\sqrt{s} = 500$ GeV.

Summary and outlook

- **Top quark pair production and decay into 6 fermions in e^+e^- annihilation at c.m.s. energies typical for linear colliders has been studied with a program `eett6f`.**
- **The two signal diagrams dominate total cross sections even at c.m.s. energies much above the $t\bar{t}$ threshold. The off resonance background may however affect the shape of some differential cross sections.**
- **All reactions, except for those that include $e^{\pm}$ in the final state, have been implemented in `eett6f`.**

  The t-channel contributions to those reactions can be suppressed by even a small angular cut in the direction of initial beams, see e.g.

  A. Biernacik, K. Ciećkiewicz, K. Kołodziej, Eur. Phys. J. C20 (2001) 233.
- **Some gluon contributions to $e^+e^- \to bu\bar{d}b\bar{d}\bar{u}$ are missing, but their implementation is underway.**
- **Implementation of anomalous couplings is being done, too.**
- **Inclusion of leading radiative effects to the two signal diagrams is planned in collaboration with the Zeuthen–Bielefeld group.**