Higher order QED corrections to deep inelastic scattering

Hiroyuki Kawamura

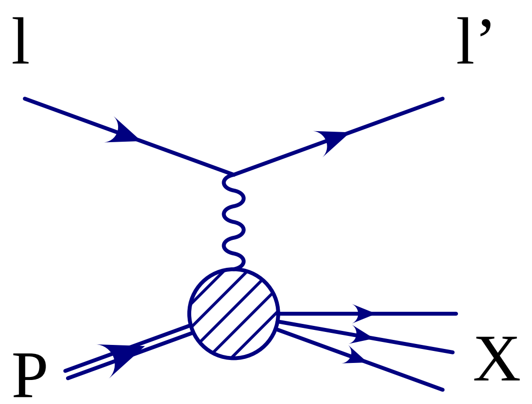
DESY

1. **Introduction**
2. **LLA to $O(\alpha^5)$**
3. $O\left(a^2 \log\left(\frac{Q^2}{m_e^2}\right)\right)$ **leptonic QED corrections to DIS**
   - 3.1. One loop result revisited
   - 3.2. Two loop contributions
4. **Conclusion**

Work in common with Johannes Blümlein

1. Introduction

Deep inelastic scattering :



· precise measurement of Λ_{QCD}

· precise extraction of quark and gluon distribution, $F_L, F_i^{c\bar{c}}$

Large corrections   QED, particularly leptonic corrections

$$\frac{d^2\sigma}{dydQ^2} = \frac{d^2\sigma^{(0)}}{dydQ^2} + \sum_{k=1}^{\infty} \frac{d^2\sigma^{(k)}}{dydQ^2}$$

Born: $$\frac{d^2\sigma^{(0)}}{dydQ^2} = \frac{2\pi\alpha^2}{yQ^4}\left[2y^2 x F_1(x,Q^2) + 2(1-y)F_2(x,Q^2)\right]$$

corrections: $$\frac{d^2\sigma^{(k)}}{dydQ^2} = \sum_{l=0}^{k} \left(\frac{\alpha}{2\pi}\right)^k \log^{k-l}\left(\frac{Q^2}{m_e^2}\right) C^{(k,l)}(y,Q^2)$$

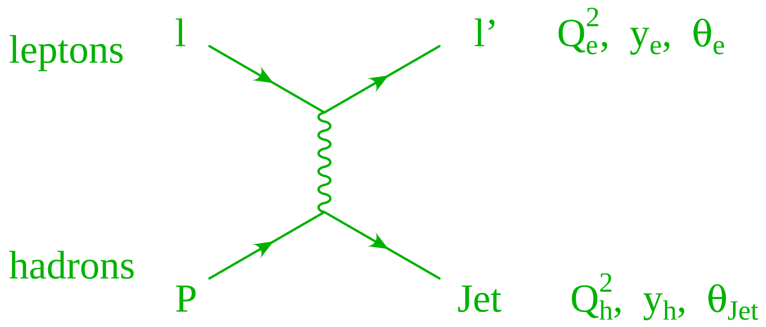
$l=0:$ $C^{(k,0)}(y,Q^2)$ leading logs. known to k=5 (NS,Singlet)

$k=1:$ all terms known

$k=2:$ only l=0 terms known

Main issue : Calculation of $O\left(\alpha^2 \log\left(\frac{Q^2}{m_e^2}\right)\right)$

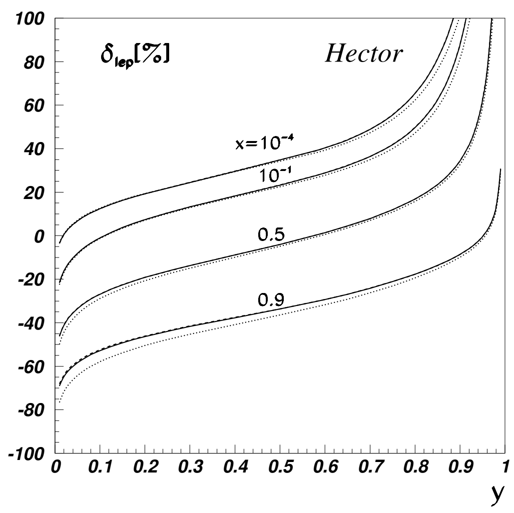
Radiative corrections to differential cross section strongly depend on the measurement of the kinematic variables, e.g. y, Q^2;



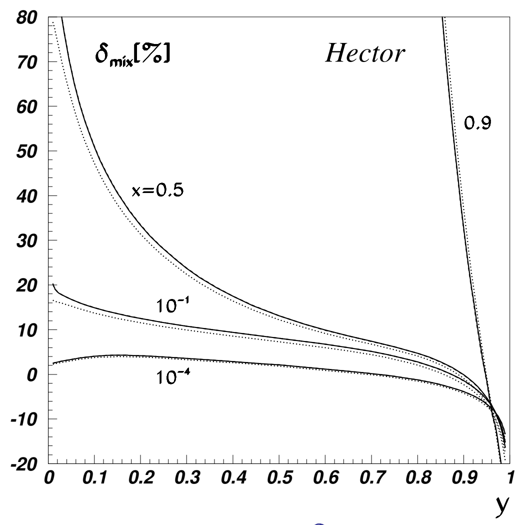
Exp. variables

1. leptonic variables	Q_e^2, y_e	
2. mixed variables	Q_e^2, y_h	
3. hadronic variables	Q_h^2, y_h	
4. JB variables	Q_{JB}^2, y_{JB}	J. Blümlein, 1994
5. double angle	θ_e, θ_{Jet}	Arbuzov et.al.
6. θ_e, y_{JB}	θ_e, y_{JB}	Comp.Phys.Comm.94(1996)128.
7. Σ method	Q_Σ, y_Σ	
8. eΣ method	$Q_l^2, y_{e\Sigma}$	

leptonic variables



mixed variables



$O(\alpha)$ corrections are large $\to$ $O(\alpha^2)$ needed

Literature

- $O(\alpha)$ calculations :

  Bardin et al. 1970ies, Bardin et al. 1989, Böhm and Spiesberger, 1988/89, Spiesberger: MC HERACLES , Bardin et al. Code HECTOR , Bardin et al. 1997 (polarized nucleons).

- LO results:

  Beenakker, Berends, van Neerven 1989, Fadin & Kuraev 1988, Blümlein 1989, 1994, Kripfganz, Möhring, Spiesberger, 1991

- LO 3rd order onwards:

  Jezabek 1992, Skrzypek 1992, Przybzcien 1993, Arbuzov 1999, Blümlein and Kawamura 2002

- Small z resummations:

  Blümlein, Riemersma and Vogt 1996; see also for QCD: Kirschner and Lipatov 1983, Bartels, Ermolaev, Ryskin 1996, Blümlein and Vogt 1995/1996, Kiyo, Kodaira, Tochimura 1997

... and many more

2. LLA to $O(\alpha^5)$

QED radiator:

$$
\begin{aligned}
D^e_{\rm NS}(a,z) &= D^{e^-}(a,z) - D^{e^+}(a,z), \\
D^e_{\Sigma}(a,z) &= D^{e^-}(a,z) + D^{e^+}(a,z), \qquad a = \frac{\alpha}{4\pi}
\end{aligned}
$$

Evolution eq.:

$$
\textbf{NS}: \quad \frac{\partial}{\partial \log Q^2} D^e_{NS}(a,z) = a P^0_{NS} \otimes D^e_{NS}(a,z),
$$

$$
\textbf{S}: \quad \frac{\partial}{\partial \log Q^2} \begin{pmatrix} D^e_{\Sigma}(a,z) \\ D^e_{\gamma}(a,z) \end{pmatrix} = a \mathbf{P}^0_S \otimes \begin{pmatrix} D^e_{\Sigma}(a,z) \\ D^e_{\gamma}(a,z) \end{pmatrix}
$$

♣ Non-singlet solution

$$
D^{e\pm}_{NS}(a,z) = \delta(1-z) + \sum_{k=1}^{\infty} \frac{P^{(k)}(z)}{k!} \left(-\frac{1}{\beta_0} \log\left(\frac{a}{a_0}\right)\right)
$$

$$
P^{(k)}(z) = \otimes^k_{l=1} P^{(0)}_{NS}(z)
$$

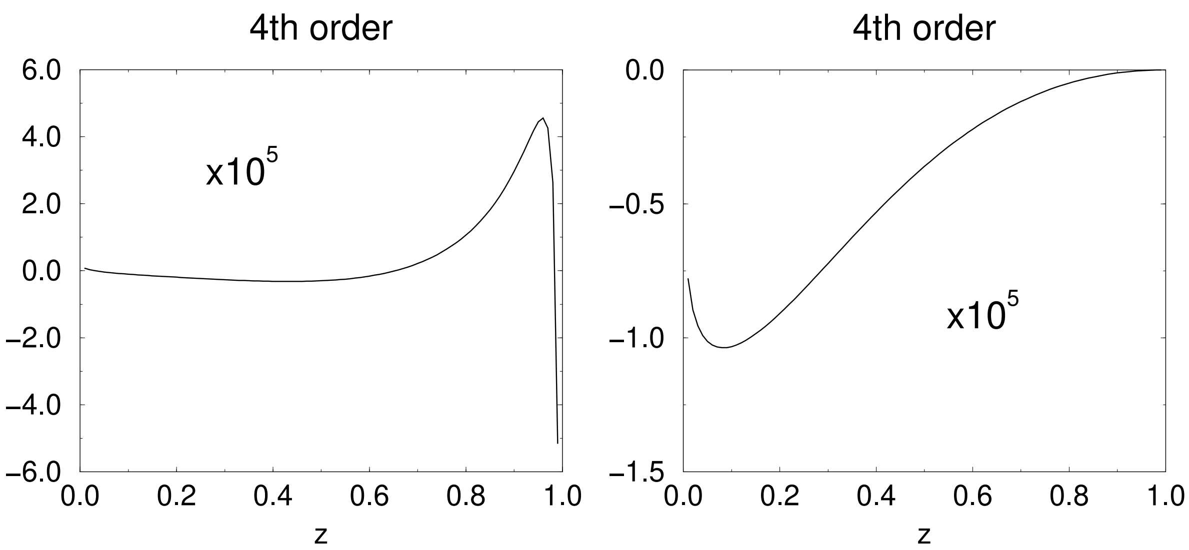
Improvement

- Soft exponentiation $(z \to 1)$
- small z exponentiation $(z \to 0)$

$$
D_{\rm NS}(z,\beta) = \left[\frac{\exp[\beta/2(3/4-\gamma_E)]}{\Gamma(1+\beta/2)} \frac{\beta}{2}(1-z)^{\beta/2-1} 2 \frac{\mathrm{I}_1\left((-\beta\ln(z))^{1/2}\right)}{[-\beta\ln(z)]^{1/2}} \times \sum_{n=0}^{\infty} \left(\frac{\beta}{2}\right)^n \Psi_n(z) \right]_+
$$

$$
\begin{aligned}
\Psi_0(z) &= 1 + z^2 \\
\Psi_1(z) &= -\frac{1}{2}\left[(1-z)^2 + z^2\ln(z)\right] \\
\Psi_2(z) &= \frac{1}{4}(1-z)\left[1 - z - z\ln(z) + (1+z)\mathrm{Li}_2(1-z)\right]
\end{aligned}
$$

$$
\begin{aligned}
\Psi_3(z) &= -\frac{1}{48}\{6(1-z^2)[2\mathrm{Li}_3(1-z)+\mathrm{Li}_2(1-z)]+5(z-1)^2 \\
&+(1+7z^2)[\ln(z)\mathrm{Li}_2(1-z)+2\mathrm{S}_{1,2}(1-z)] \\
&-\left(\frac{1}{2}+6z-\frac{13}{2}z^2\right)\ln(z)+\frac{1}{12}z^2\ln^3(z)\Big\} \\
\Psi_4(z) &= \frac{1}{96}\Big\{(1-z^2)\Big[24\mathrm{Li}_4(1-z)+12\mathrm{Li}_3(1-z)-\frac{5}{2}\mathrm{S}_{1,3}(1-z) \\
&-12\mathrm{S}_{2,2}(1-z)-\frac{3}{2}\ln(z)\mathrm{S}_{1,2}(1-z)-\frac{1}{4}\ln^2(z)\mathrm{Li}_2(1-z) \\
&+7\mathrm{Li}_2(1-z)]+4(1+z^2)\mathrm{Li}_2^2(1-z) \\
&+(1-8z+7z^2)[\ln(z)\mathrm{Li}_2(1-z)+2\mathrm{S}_{1,2}(1-z)] \\
&+2(1+7z^2)\ln(z)\mathrm{Li}_3(1-z)-\left(\frac{3}{4}+5z-\frac{23}{4}z^2\right)\ln(z) \\
&-\frac{1}{12}z(1-z)\ln^3(z)-\frac{1}{48}z^2\ln^4(z)+(1-z)^2\left[\frac{7}{2}+\frac{1}{8}\ln^2(z)\right]\Big\}
\end{aligned}
$$



♣ Singlet solution:

$$\mathbf{E}_s(a,z)=\mathbf{1}\delta(1-z)+\sum_{k=1}^{\infty}\mathbf{P}_0^{(k)}\left(-\frac{1}{\beta_0}\log\left(\frac{a}{a_0}\right)\right)^k$$

$$\Downarrow$$

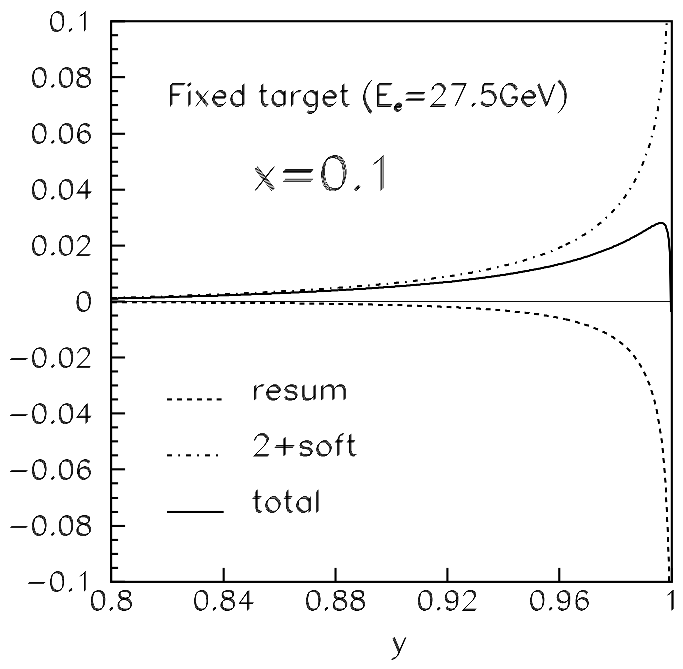
$O(\alpha^5)$ solutions are obtained for polarized case

- Resummation of $O((\alpha \log^2 z)^k)$

Non-singlet case　　　Blümlein, Riemersma & Vogt ('96)

splitting function in N space:

$$\mathbf{M}[P_{NS}](N,a) = \frac{N}{2}\left\{1 - \sqrt{1 + \frac{8a}{N^2}\left[1 - 2\sqrt{1 - \frac{8a}{N^2}}\right]}\right\}$$



Singlet case

$$\mathbf{P}(z,a)_{x\to 0} = \sum_{l=0}^{\infty} \mathbf{P}^{(l)}_{z\to 0} a^{(l+1)} \log^{2l}(z)$$

$$\begin{cases} \mathbf{F}_0(N,a) = 16\pi^2 \frac{a}{N}\mathbf{M}_0 - \frac{8a}{N^2}\mathbf{F}_8(N,a)\mathbf{G}_0 + \frac{1}{8\pi^2}\frac{1}{N}\mathbf{F}_0^2(N,a) \\ \mathbf{F}_8(N,a) = 4\pi^2\left(1 - \sqrt{1 - \frac{8a}{N^2}}\right)\mathbf{M}_8 \end{cases}$$

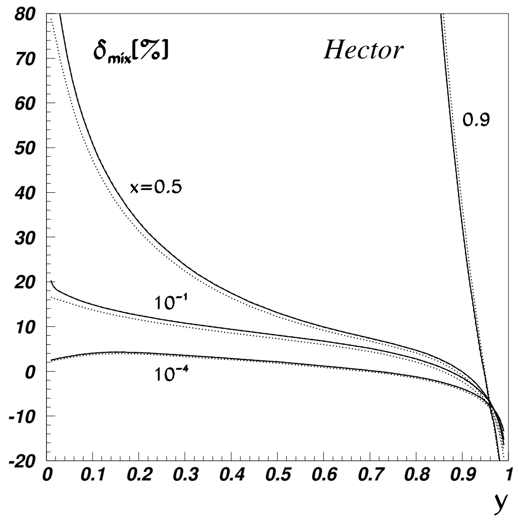
$$\Rightarrow P^{(0)} = 2\begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}, \quad P^{(1)} = 2\begin{pmatrix} -4 & -2 \\ 2 & -4 \end{pmatrix}, \quad \cdots$$

3. LLA to $O(a^2 \log\left(\frac{Q^2}{m_e^2}\right))$ corrections

Example: Mixed variables

Arbuzov et.al.

Comp.Phys.Comm.94(1996)128.



3.1. One loop result revisited

Kinematic separation in the case for $Q_h^2 \leq Q_l^2, \;\; Q_h^2 \geq Q_l^2$

$$\frac{d^2\sigma^{(1)}}{dydQ^2} = \frac{\alpha}{2\pi}\log\left(\frac{Q^2}{m_e^2}\right)\int_0^1 dz P_{ee}^0(z)\left[\theta(z-z_0^I)\frac{d^2\widetilde{\sigma}^{(0)}}{dydQ^2}J^I(z) - \frac{d^2\sigma^{(0)}}{dydQ^2}\right]$$
$$+\frac{\alpha}{2\pi}\log\left(\frac{Q^2}{m_e^2}\right)\int_0^1 dz P_{ee}^0(z)\left[\theta(z-z_0^F)\frac{d^2\widetilde{\sigma}^{(0)}}{dydQ^2}J^F(z) - \frac{d^2\sigma^{(0)}}{dydQ^2}\right]$$
$$+\frac{\alpha}{2\pi}C^{(1,1)}(y,Q^2)$$

$$\widetilde{F}(x,y,s) = F(\hat{x},\hat{y},\hat{s})$$

- Rescaling;

(i) $Q_h^2 < Q_l^2 \to$ ISR: $z = \frac{Q_h^2}{Q_l^2}, \quad \hat{y} = y_h/z, \quad \hat{Q}^2 = zQ_l^2, \quad \hat{x} = zx_m$
$J^I(z) = 1$

(ii)$Q_h^2 > Q_l^2 \to$ FSR: $z = \frac{Q_l^2}{Q_h^2}, \quad \hat{y} = y_h, \quad \hat{Q}^2 = Q_l^2/z, \quad \hat{x} = x_m/z$
$J^F(z) = 1/z,$

These rescaling preserve,

$$P_{ee}^0(z) = \left(\frac{1+z^2}{1-z}\right)$$

- Integration boundary:

(i) ISR

Avoid the Compton peak $\Rightarrow Q_h^2 \geq Q_0^2 \geq M_P^2$

$$\Longrightarrow z \geq z_0^I = max\left\{\frac{Q_0^2}{Q_l^2}, y_h\right\}$$

· Only leptonic collinear singularities
· Avoid Non-DIS structures

(ii) FSR

Kinematic boundary: $z \geq x_m \equiv \frac{Q_l^2}{y_h S}$

use frequently $Q^2 \equiv Q_l^2$

$$
\begin{aligned}
c^{(1,1,i)}(y_h,Q_l^2,z) = \frac{1}{z^2}\Bigg[\Bigg\{ & P_{ee}^0(z)\left[\ln\left(\frac{1-y_h/z}{1-z}\right)-\frac{1}{z}\ln\left(\frac{1-y_h}{1-z}\right)-1\right] \\
& +2\ln\left(\frac{z}{y_h}\right)+1-z\Bigg\}\frac{F_1(x_h,Q_h^2)}{S} \\
& +\Bigg\{(z-y_h)P_{ee}^0(z)\left[\ln\left(\frac{1-y_h/z}{1-z}\right)-\frac{1-y_h}{z^2(z-y_h)}\ln\left(\frac{1-y_h}{1-z}\right)-1\right] \\
& +\left(\frac{1}{z}+\frac{3}{2}+3z+2z^2\right)-y_h\left(\frac{1}{z^2}+\frac{2}{z}+4+3z\right) \\
& +\frac{y_h^2}{2}\left(\frac{1}{z^2}+\frac{2}{z}+2\right)\Bigg\}\frac{F_2(x_h,Q_h^2)}{y_hQ_l^2}\Bigg] \\
c^{(1,1,f)}(y_h,Q_l^2,z) = \Bigg[\Bigg\{ & P_{ee}^0(z)\left[\ln\left(\frac{1-y_h}{1-z}\right)-\frac{1}{z}\ln\left(\frac{1-y_hz}{1-z}\right)-1\right] \\
& +2\ln\left(\frac{1}{y_h}\right)+1-z\Bigg\}\frac{F_1(x_h,Q_h^2)}{S} \\
& +\Bigg\{z(1-y_h)P_{ee}^0(z)\left[\ln\left(\frac{1-y_h}{1-z}\right)-\frac{1-y_hz}{z^3(1-y_h)}\ln\left(\frac{1-y_hz}{1-z}\right)-1\right] \\
& +\left(\frac{1}{z}+\frac{3}{2}+3z+2z^2\right)-y_h\left(\frac{1}{z}+2+4z+3z^2\right) \\
& +\frac{y_h^2}{2}\left(1+2z+2z^2\right)\Bigg\}\frac{F_2(x_h,Q_h^2)}{y_hQ_l^2}\Bigg]
\end{aligned}
$$

$$
C^{(1,1)}(y_h,Q_l^2) = \frac{4\pi\alpha^2}{Q_l^2}\left[\int_{z_0^I}^1 dz c^{(1,1,i)}(y_h,Q_l^2,z)_\oplus + \int_{z_0^F}^1 dz c^{(1,1,f)}(y_h,Q_l^2,z)_\oplus\right]
$$

confirm Akhundov et. al. Fortschr. Phys. 44 (1996) 373.

3.2. Two loop contribution

massive calculation, electron mass to be kept$\Rightarrow$ double poles;

Method: use the RG equations for mass factorization and charge renormalization Berends, Burgers, van Neerven, 1988; e^+e^- Annih. ISR

RGE's for OME's and Wilson Coefficients: ISR and FSR

$$\begin{aligned}\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+\gamma^i_{ee}\right]\Gamma^i_{ee}+\gamma^i_{e\gamma}\Gamma^i_{\gamma e} &= 0\\ \left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+\gamma^f_{ee}\right]\Gamma^f_{ee}+\gamma^f_{\gamma e}\Gamma^f_{e\gamma} &= 0\end{aligned}$$

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}-\gamma^i_{ee}-\gamma^f_{ee}\right]\widehat{\sigma}_{ee}-\gamma^i_{\gamma e}\widehat{\sigma}_{e\gamma}-\gamma^f_{e\gamma}\widehat{\sigma}_{\gamma e}=0$$

$$\frac{da}{d\ln\mu^2}=-\sum_{k=0}^{\infty}\beta_k a^{k+2};\quad a=\alpha/(4\pi)$$

represent as series :

$$\Gamma^k_{ab}=\delta_{ab}+\sum_{l=1}^{\infty}a^l\sum_{m=0}^{l}ln^m(\mu^2/m_e^2)\Gamma^{k,l}_{ab,m}$$

$$\widehat{\sigma}_{ab}=\delta_{ab}+\sum_{l=1}^{\infty}a^l\sum_{m=0}^{l}ln^m(Q^2/\mu^2)\widehat{\sigma}^l_{ab,m}$$

Insert into (to $O(a^2)$) :

$$\sigma_{ee}=\Gamma^i_{ee}\otimes\widehat{\sigma}_{ee}\otimes\Gamma^f_{ee}+\Gamma^i_{\gamma e}\otimes\widehat{\sigma}_{e\gamma}\otimes\Gamma^f_{ee}+\Gamma^i_{ee}\otimes\widehat{\sigma}_{\gamma e}\otimes\Gamma^f_{e\gamma}$$

The logarithms match in the expansion and the final expression is independent of μ.

The final expression expands in $\alpha/2\pi$ and $\ln(Q^2/m_e^2)$.

2nd order cross section

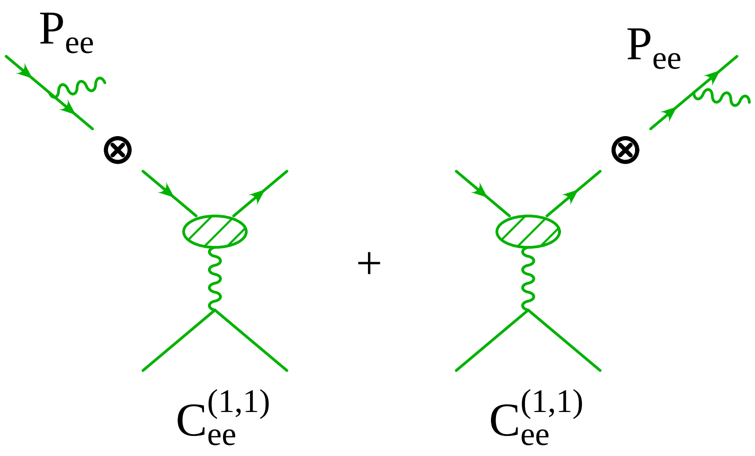
$$
\begin{aligned}
\frac{d^2\sigma^{0+\mathrm{RC}}}{dy_h dQ_l^2} = \frac{d^2\sigma^0}{dy_h dQ_l^2} \\
&+ \left(\frac{\alpha}{2\pi}\right) \left[\log\left(\frac{Q_l^2}{m_e^2}\right) C^{(1,0)} + C^{(1,1)} \right] \\
&+ \left(\frac{\alpha}{2\pi}\right)^2 \left[\log^2\left(\frac{Q_l^2}{m_e^2}\right) C^{(2,0)} + \log\left(\frac{Q_l^2}{m_e^2}\right) C^{(2,1)} + C^{(2,2)} \right]
\end{aligned}
$$

$C^{(2,0)}(y, Q^2)$: leading order term : 2nd order, ISR, FSR, interference

$C^{(2,1)}(y, Q^2)$: Several OME's and Wilson coefficients combine to cross sections

Individual Terms :

(A) $P_{ee}^0 \otimes C_{ee}^{(1,1)}$, $C_{ee}^{(1,1)} \otimes P_{ee}^0$

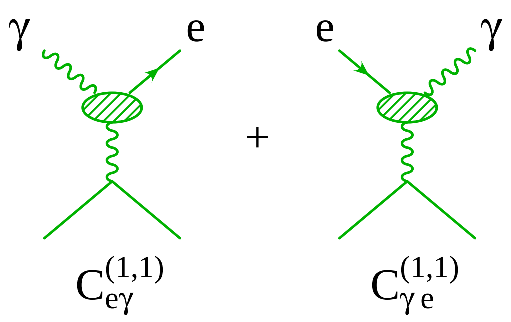


(A)

$$
\begin{aligned}
&C_A^{(2,1)}(y_h, Q_l^2) \\
&\quad = \int_0^1 dz P_{ee}^0(z) \ln\left(\frac{Q^2}{m_e^2}\right) \left[\theta(z - z_0^I)\widetilde{C}^{(1,1)}(y, Q^2) J^I(z) - C^{(1,1)}(y, Q^2)\right] \\
&\quad + \int_0^1 dz P_{ee}^0(z) \ln\left(\frac{Q^2}{m_e^2}\right) \left[\theta(z - z_0^F)\widetilde{C}^{(1,1)}(y, Q^2) J^F(z) - C^{(1,1)}(y, Q^2)\right]
\end{aligned}
$$

(B). $C_{e\gamma}^{(1,0)} \otimes P_{\gamma e}^{0}$, (C). $C_{\gamma e}^{(1,0)} \otimes P_{e\gamma}^{0}$

New subprocesses:



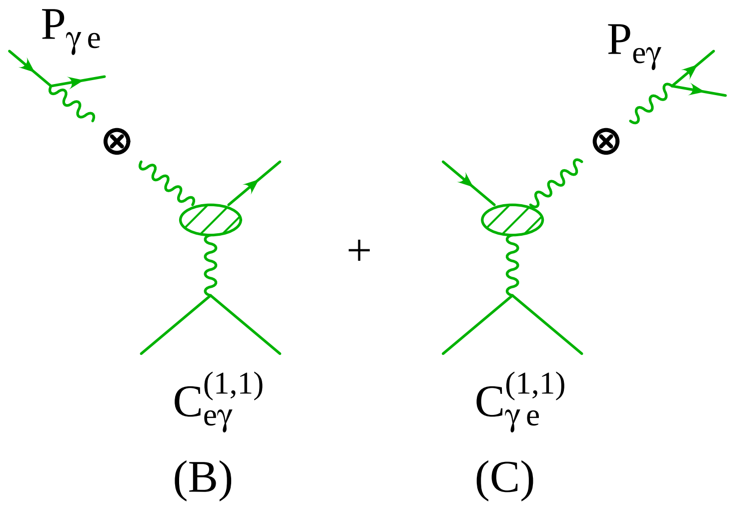
$$\frac{d^2\sigma_{e\gamma}^{(1)}}{dydQ^2} = \frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)C_{e\gamma}^{(1,0)}(y,Q^2) + \frac{\alpha}{2\pi}C_{e\gamma}^{(1,1)}(y,Q^2)$$

$$\frac{d^2\sigma_{\gamma e}^{(1)}}{dydQ^2} = \frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)C_{\gamma e}^{(1,0)}(y,Q^2) + \frac{\alpha}{2\pi}C_{\gamma e}^{(1,1)}(y,Q^2)$$

$$C_{e\gamma}^{(1,0)}(y,Q^2) = \int_{z_0^I}^{1} dz P_{e\gamma}^0(z)\frac{d^2\widetilde{\sigma}^0}{dydQ^2}J^I(z)$$

$$C_{\gamma e}^{(1,0)}(y,Q^2) = \int_{z_0^F}^{1} dz P_{\gamma e}^0(z)\frac{d^2\widetilde{\sigma}^0}{dydQ^2}J^F(z)$$

$O(\alpha^4)$ Results :



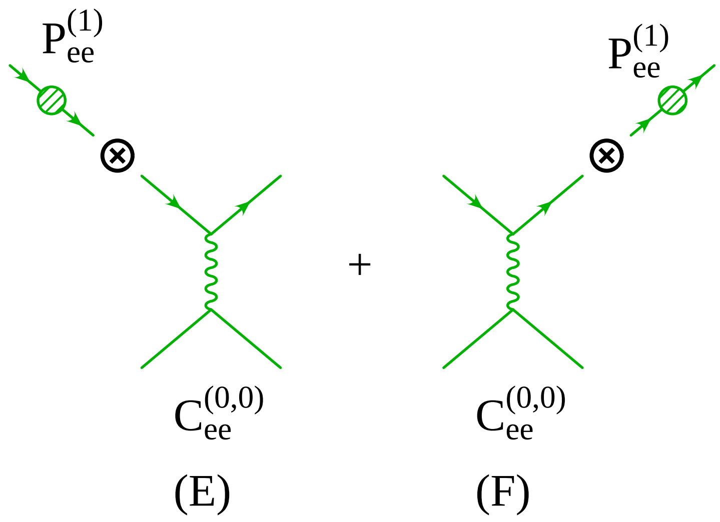
$$C_B^{(2,1)}(y, Q^2) = \int_{z_0^I}^1 dz P^0_{\gamma e}(z) \ln\left(\frac{Q^2}{m_e^2}\right) \widetilde{C}^{(1,1)}_{e\gamma}(y, Q^2) J^I(z)$$

$$C_C^{(2,1)}(y, Q^2) = \int_{z_0^F}^1 dz P^0_{e\gamma}(z) \ln\left(\frac{Q^2}{m_e^2}\right) \widetilde{C}^{(1,1)}_{\gamma e}(y, Q^2) J^F(z)$$

(D). Running Coupling Contribution

$$C_D^{(2,1)} = -\beta_0 C^{(1,1)}(y, Q^2); \qquad \beta_0 = -\frac{4}{3}$$

(E) & (F). NLO splitting functions



$$C_E^{(2,1)}(y,Q^2) = \int_0^1 dz P_{ee,NS}^{(1,S)}(z) \left[\theta(z-z_0^I)\frac{d^2\widetilde{\sigma}^0}{dydQ^2}J^I(z) - \frac{d^2\sigma^0}{dydQ^2}\right] + \int_{z_0^I}^1 dz P_{ee,PS}^{(1,S)}(z)\frac{d^2\widetilde{\sigma}^0}{dydQ^2}J^I(z)$$

$$C_F^{(2,1)}(y,Q^2) = \int_0^1 dz P_{ee,NS}^{(1,T)}(z) \left[\theta(z-z_0^F)\frac{d^2\widetilde{\sigma}^0}{dydQ^2}J^F(z) - \frac{d^2\sigma^0}{dydQ^2}\right] + \int_{z_0^F}^1 dz P_{ee,PS}^{(1,T)}(z)\frac{d^2\widetilde{\sigma}^0}{dydQ^2}J^F(z)$$

$P^{S,T}_{ee,\ OM}$ refer to $P^{S,T}_{ee,\ MS}$

$\overline{\mathrm{MS}}$ Splitting Functions:

$$
\begin{aligned}
P^{\mathrm{S}}_{ee,\mathrm{F}}(z) &= -2P^0_{ee}(z)\ln(z)\left[\ln(1-z)-\frac{3}{4}\right] \\
&\quad -\frac{1}{2}(1+z)\ln^2(z)-\frac{1}{2}(3+7z)\ln(z)-5(1-z) \\
P^{\mathrm{S}}_{ee,\mathrm{PS}}(z) &= 2\left[-(1+z)\ln^2(z)+\left(1+5z+\frac{8}{3}z^2\right)\ln(z)\right. \\
&\quad \left.+\frac{2}{9}\left(\frac{1-z}{z}\right)(10+z+28z^2)\right] \\
P^{\mathrm{T}}_{ee,\mathrm{F}}(z) &= 2P^0_{ee}(z)\ln(z)\left[\ln(1-z)-\ln(z)+\frac{3}{4}\right] \\
&\quad +\frac{1}{2}(1+x)\ln^2(z)-\frac{1}{2}(7+3z)\ln(z)-5(1-z) \\
P^{\mathrm{T}}_{ee,\mathrm{PS}}(z) &= 2\left[(1+z)\ln^2(z)-\left(5-9z-\frac{8}{3}z^2\right)\ln(z)\right. \\
&\quad \left.+\frac{4}{9}\left(\frac{1-z}{z}\right)(5+23z+14z^2)\right] \\
P^{\mathrm{S,T}}_{ee,\mathrm{N}_f}(z) &= -\frac{2}{3}\left\{P^0_{ee}(z)\left[\ln(z)+\frac{5}{3}\right]+2(1-z)\right\}
\end{aligned}
$$

Curci, Furmanski, Petronzio, 1980;

Furmanski and Petronzio, 1980;

Floratos, Kounnas, and Lacaze 1981

4. Conclusion

- The LO universal QED corrections were calculated in the non-singlet case and polarized singlet case to $O((\alpha \mathsf{L})^5)$ in analytic form. Exponentiation has been used for $z \to 0, 1$.
- The contributions of $O(\alpha \ln^2(z))$ were resummed to all orders.
- With the help of the RGE–decomposition of the higher order scattering QED cross section the leptonic corrections of $O(\alpha^2 \mathsf{L})$ were calculated for the double differential cross section $d^2\sigma/dydQ^2$ of deeply inelastic scattering in mixed variables including both the initial and final state contributions.
- The calculation was carried out in the on-shell scheme for finite electron mass. This method generalizes earlier investigations for ISR in e^+e^- annihilation and includes both space–and timelike splitting functions and Wilson coefficients, which are kinematically folded to double distributions. Partly they pair to scattering cross sections.