Full two-loop electroweak corrections to the pole masses of gauge bosons

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RADCOR 2002, Kloster Banz

- Pole Mass
- Expansion procedure and calculation
- $\overline{\mathrm{MS}}$ renormalization and two-loop results

Pole mass. General consideration.

The position of the pole s_P of the massive (gauge) boson propagator in a QFT is defined by equation:

$$\boxed{s_P - m^2 - \Pi(s_P, m^2, \cdots) = 0}$$

For gauge bosons Π is transversal part of the one-particle irreducible self-energy (depends on all SM parameters).

The pole s_P is

- gauge invariant
- infrared stable
- complex in general: $\mathrm{Im} s_p \neq 0$

We can define pole (on-shell) mass M and width Γ via

$$s_P = M^2 - \mathrm{i} M \Gamma$$

By iterative solution of pole formula up to 2-loops we have:

$$\boxed{\begin{aligned} s_P &= m^2 \\ &+ \Pi^{(1)}(m^2, m^2) \\ &+ \Pi^{(2)}(m^2, m^2) + \Pi^{(1)}(m^2, m^2)\Pi^{(1)\prime}(m^2, m^2). \end{aligned}}$$

$\Pi^{(L)}$ is the bare $(m = m_0)$ or $\overline{\mathrm{MS}}$- renormalized (m the $\overline{\mathrm{MS}}$-mass) L-loop contribution to Π, and the prime denotes the derivative with respect to p^2. In this way we need to evaluate propagator type diagrams and their derivatives at $p^2 = m^2$.

Pole mass and γ-Z mixing.

In neutral sector because of mixing two neutral bosons Z and γ form a 2×2 matrix propagator

$$D^{-1}(p^2) = \begin{pmatrix} p^2 - \Pi_{\gamma\gamma}(p^2) & \Pi_{\gamma Z}(p^2) \\ \Pi_{Z\gamma}(p^2) & p^2 - m_Z^2 - \Pi_{\gamma\gamma}(p^2) \end{pmatrix}$$

The equation for the pole is now modified

$$\boxed{s_P - m_Z^2 - \Pi_{ZZ}(s_P) - \frac{\Pi^2_{\gamma Z}(s_P)}{s_P - \Pi_{\gamma\gamma}(s_P)} = 0.}$$

- ♣ Mixing term $\Pi^2_{\gamma Z}$ starts contributing at two-loop
- ♠ Photon term $\Pi_{\gamma\gamma}$ only contributes beyond the two-loop

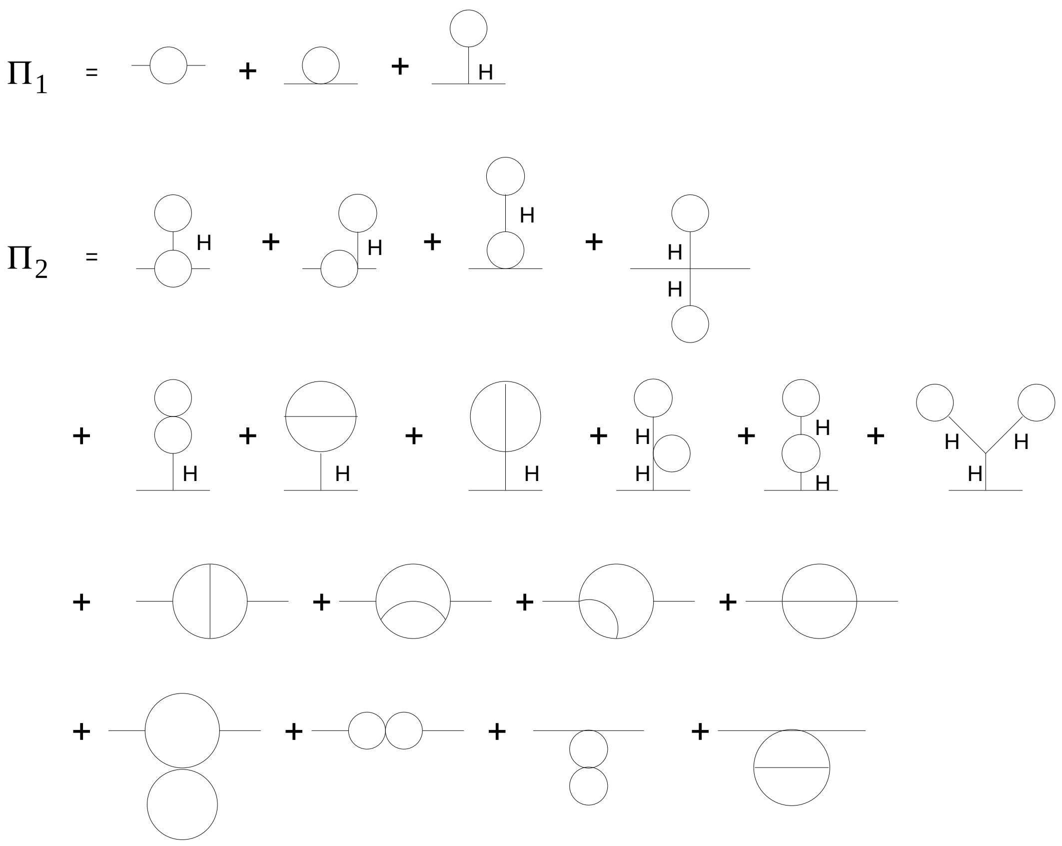
Solution (up to 2-loops):

$$\begin{aligned} s_Z = \; & m_Z^2 \\ & + \Pi^{(1)}_{ZZ}(m_Z^2) \\ & + \Pi^{(2)}_{ZZ}(m_Z^2) + \Pi^{(1)}_{ZZ}(m_Z^2)\Pi^{(1)}_{ZZ}{}'(m_Z^2) + \frac{\left[\Pi^{(1)}_{\gamma Z}(m_Z^2)\right]^2}{m_Z^2}. \end{aligned}$$

Diagrams and topologies

To be computed on-shell ($p^2 = m_Z^2$):

$$\Pi(p^2) = \Pi_1(p^2) + \Pi_2(p^2) + \ldots$$



Bosonic contribution				
Number of diagrams	linearR_ξ *gauge*		nonlinearR_ξ *gauge*	
one-loop :	~ 50			
two-loop :	$1PI$	$Total$	$1PI$	$Total$
Z	616	2348	410	1837
W	792	4084	537	2942
With one massive fermion family				
two-loop :	$1PI$	$Total$	$1PI$	$Total$
Z	802	4410	550	3631
W	990	7780	669	5604

Evaluation by expansion

In order to check the gauge invariance we perform all calculations in R_ξ gauge with three independent gauge parameters ξ_W, ξ_Z, ξ_γ. $\to$ Vector boson propagator reads

$$D_{\mu\nu}(p) = \frac{i}{p^2 - m_Z^2}\left(-g_{\mu\nu} + (1-\xi)\frac{p_\mu p_\nu}{p^2 - \xi m_Z^2}\right)$$

Then there are several scales: m_V, $\sqrt{\xi_V} m_V$, m_H, m_t

We perform expansion in 3 steps:

1. Taylor (naive) expansion in $(\xi_V - 1)$: i.e., propagator of the vector bosons look like

$$D^V_{\mu\nu}(p) \sim -g_{\mu\nu} + (1-\xi_V)\frac{p_\mu p_\nu}{p^2 - m_V^2} - (1-\xi_V)^2 \frac{m_V^2 p_\mu p_\nu}{(p^2 - m_V^2)^2}$$

2. Expansion in small parameter

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} < 0.25.$$

   After expansion in $\sin^2\theta_W$: the diagrams without Higgs are the two-loop on-shell diagrams can be evaluated analytically by FORM package **ONSHELL2**.
   J. Fleischer & M.Yu. Kalmykov '00
   Diagrams with massless fermion loop demand special consideration.

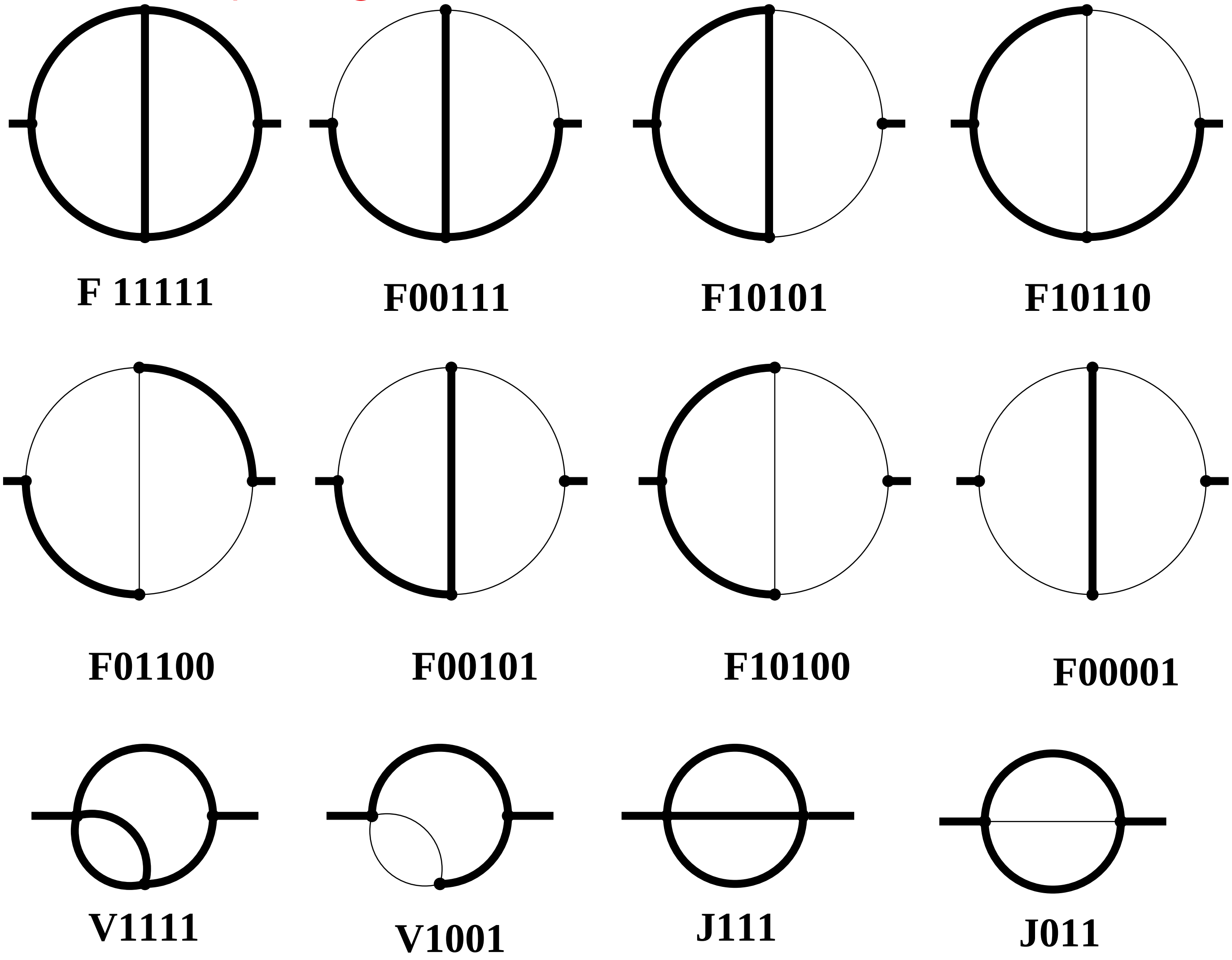
3. Large mass expansion in $m_Z^2/m_t^2 < 0.3; m_Z^2/m_H^2 < 0.64$
   Package **TLAMM** L.V. Avdeev et al., '97

ONSHELL2

www-zeuthen.desy.de/~kalmykov/onshell2/onshell2.html

J. Fleischer, M.Yu. Kalmykov & A. Kotikov '99
A.I. Davydychev & M.Yu. Kalmykov '01
J. Fleischer & M.Yu. Kalmykov '00

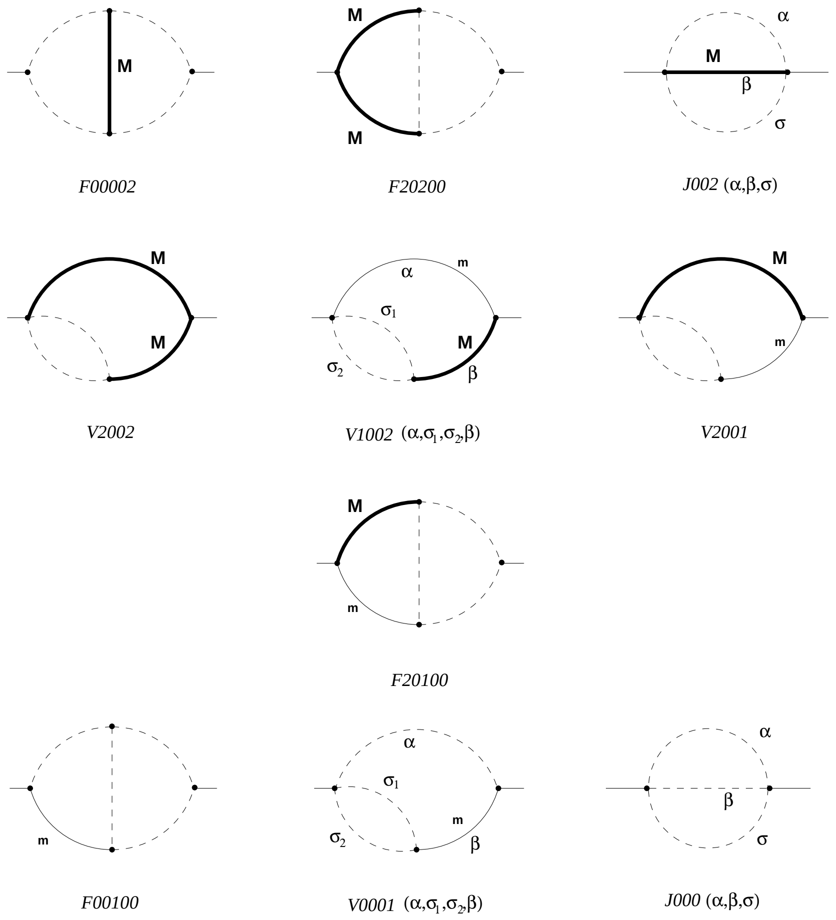
All two-loop on-shell self-energy diagrams with one mass, with arbitrary power of propagators are reduced to 13 scalars integrals plus bubble-type integrals and product of the one-loop integrals.



Bold and thin lines correspond to the mass and massless propagators, respectively.

Massless fermion contribution

In contrast to the bosonic corrections where all diagrams can be expanded from the very beginning in the small parameter $\sin^2\theta_W$ the individual diagrams with massless fermion loop develop the thresholds singularity which behave like $\ln^j \sin^2\theta_W$. To control these terms we need the exact analytical result without expansion. Using Tarasov's recurrence relations we reduce all diagrams with massless fermion loop to a minimal set of master-integrals



F00002 F20200 J002 (α,β,σ)
V2002 V1002 (α,σ1,σ2,β) V2001
F20100
F00100 V0001 (α,σ1,σ2,β) J000 (α,β,σ)

The analytical results for these diagrams are presented by Scharf, Tausk, '94 We performed an independent analytical calculations of these master integrals.

After summing of all contribution the singular terms, $\ln^j \sin^2\theta_W$ are canceled.

Master-integrals

E.E. Boos & A.I. Davydychev, '91
J. Fleischer, A. Kotikov & O.Veretin '99
T. van Ritbergen & R. G. Stuart '00
A.I. Davydychev & M.Yu. Kalmykov '01

$$
\begin{aligned}
m^2 F_{20100} = \sum_{n=1}^{\infty} \binom{2n}{n} \frac{x^n}{n} \Bigg\{ & -\frac{1}{4}\ln^2 x + \frac{1}{2}\ln(-x)\ln x \\
& + \left[2S_1 - 2\bar{S}_1 + \frac{1}{n}\right] \ln x + \left[-S_1 + \bar{S}_1 - \frac{1}{n}\right] \ln(-x) \\
& -\zeta_2 - \frac{3}{2}V_2 - \frac{3}{2}S_2 + 3\bar{S}_2 - 3S_1^2 + 6S_1\bar{S}_1 - 3\bar{S}_1^2 \\
& -4\frac{S_1}{n} + 4\frac{\bar{S}_1}{n} - \frac{3}{2}\frac{1}{n^2} - 2\frac{1}{\binom{2n}{n}}\frac{1}{n^2} \Bigg\}
\end{aligned}
$$

with

$$
x = \frac{m^2}{M^2}\,, \qquad S_a = \sum_{j=1}^{n-1} \frac{1}{j^a}\,, \qquad \bar{S}_a = \sum_{j=1}^{2n-1} \frac{1}{j^a}\,,
$$

$$
\begin{aligned}
\sum_{k=1}^{\infty} \binom{2k}{k} \frac{x^k}{k^4} = & -8\mathrm{S}_{1,2}(-\chi)\ln(1+\chi) - \mathrm{Li}_4(-\chi) \\
& +4\mathrm{Li}_3(-\chi)\ln(1+\chi) - 4\mathrm{Li}_2(-\chi)\ln^2(1+\chi) \\
& -\frac{2}{3}\ln^4(1+\chi) + 4\mathrm{S}_{2,2}(-\chi) - 8\mathrm{S}_{1,3}(-\chi)
\end{aligned}
$$

where

$$
\chi = \frac{1-\sqrt{1-4x}}{1+\sqrt{1-4x}}\,, \qquad x = \frac{\chi}{(1+\chi)^2},
$$

Renormalization.

We have s_P parametrized in terms of *bare* parameters

$$s_P = f(e_0, m_{Z,0}, m_{W,0}, m_{H,0}).$$

Renormalization of charges and masses:

$$\begin{aligned} e_0 &= \mu^\varepsilon \, e \, Z_e \\ m_0^2 &= m^2 \, Z_m \end{aligned}$$

We need:
- $+$ Z_e, Z_H, Z_t up to 1-loop
- $+$ Z_W, Z_Z up to 2-loops

We don't need:
- $-$ field renormalization
- $-$ renormalization of ghost sector
- $-$ gauge parameters renormalization

The charge $\overline{\mathrm{MS}}$ renormalization constant can be calculated in the unbroken theory

$$Z_e = 1 + \frac{e^2}{16\pi^2 \varepsilon} \left(-\frac{7}{2} + \frac{16}{9} n_F \right) \; ,$$

where e, m_Z, m_W, m_H are $\overline{\mathrm{MS}}$ parameters, n_F is a number of fermion families ($n_F = 3$).

Mass renormalization.

$$m_0^2 = m^2 \left(1 + \frac{g^2}{16\pi^2} \frac{Z^{(1,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,2)}}{\varepsilon^2} \right)$$

The mass renormalization constants cannot be calculated from the unbroken theory. The calculation of the Feynman diagrams in the Standard Model is required.

One-loop results:

$$Z_H^{(1,1)} = -\frac{3}{2} - \frac{3}{4}\frac{m_Z^2}{m_W^2} + \frac{3}{4}\frac{m_H^2}{m_W^2} + \sum_{\text{fermion}} \frac{1}{2}\frac{m_f^2}{m_W^2}$$

$$Z_W^{(1,1)} = -\frac{3}{4}\frac{m_H^2}{m_W^2} - 3\frac{m_W^2}{m_H^2} - \frac{3}{2}\frac{m_Z^4}{m_H^2 m_W^2} + \frac{3}{4}\frac{m_Z^2}{m_W^2} - \frac{17}{3} + \sum_{\text{fermion}} \left(2\frac{m_f^4}{m_W^2 m_H^2} - \frac{1}{2}\frac{m_f^2}{m_W^2} \right) + \frac{4}{3} n_F$$

$$Z_Z^{(1,1)} = -\frac{3}{4}\frac{m_H^2}{m_W^2} - 3\frac{m_W^2}{m_H^2} - \frac{3}{2}\frac{m_Z^4}{m_W^2 m_H^2} + \frac{11}{12}\frac{m_Z^2}{m_W^2} - 7\frac{m_W^2}{m_Z^2} + \frac{7}{6} + \sum_{\text{fermion}} \left(2\frac{m_f^4}{m_W^2 m_H^2} - \frac{1}{2}\frac{m_f^2}{m_W^2} \right) + \frac{1}{9} n_F \left[20\frac{m_Z^2}{m_W^2} + 32\frac{m_W^2}{m_Z^2} - 40 \right]$$

$$Z_{top}^{(1,1)} = \frac{2}{3} - \frac{2}{3}\frac{m_Z^2}{m_W^2} - \frac{3}{2}\frac{m_Z^4}{m_W^2 m_H^2} - \frac{3}{4}\frac{m_H^2}{m_W^2} - 3\frac{m_W^2}{m_H^2} + \frac{3}{4}\frac{m_t^2 - m_b^2}{m_W^2} + 2\sum_{\text{lepton}} \frac{m_l^4}{m_W^2 m_H^2} + 6\sum_{\text{quark}} \frac{m_q^4}{m_W^2 m_H^2} \, ,$$

2-loop renormalization constants in $\overline{\mathrm{MS}}$.

$$
\begin{aligned}
Z_W^{(2,1)} \quad = \quad & \frac{63}{64}\frac{m_H^4}{m_W^4} - \frac{3}{4}\frac{m_H^2}{m_W^2} - \frac{3}{8}\frac{m_H^2 m_Z^2}{m_W^4} - \frac{53}{3} + \frac{109}{18}n_F \\
& - \left[\frac{301}{192} + \frac{25}{36}n_F\right]\frac{m_Z^4}{m_W^4} + \left[\frac{17}{12} + \frac{14}{9}n_F\right]\frac{m_Z^2}{m_W^2} \\
& + \left[\frac{31}{12} - \frac{80}{9}n_F\right]\frac{m_Z^4}{m_H^2 m_W^2} - \left[\frac{17}{2} - \frac{64}{9}n_F\right]\frac{m_Z^2}{m_H^2} \\
& - \left[\frac{176}{3} - \frac{16}{3}n_F\right]\frac{m_W^2}{m_H^2} + \left[\frac{59}{24} + \frac{40}{9}n_F\right]\frac{m_Z^6}{m_H^2 m_W^4} \\
& - \frac{15}{2}\frac{m_t^6}{m_W^4 m_H^2} + \frac{4}{3}\frac{m_t^4 m_Z^2}{m_W^4 m_H^2} - \frac{21}{32}\frac{m_t^4}{m_W^4} - \frac{4}{3}\frac{m_t^4}{m_W^2 m_H^2} \\
& - \frac{85}{96}\frac{m_t^2 m_Z^2}{m_W^4} + \frac{19}{4}\frac{m_t^2 m_Z^4}{m_W^4 m_H^2} + \frac{9}{8}\frac{m_t^2 m_H^2}{m_W^4} \\
& - 20\frac{m_t^2 m_Z^2}{m_W^2 m_H^2} - \frac{43}{48}\frac{m_t^2}{m_W^2} + \frac{35}{2}\frac{m_t^2}{m_H^2}
\end{aligned}
$$

- $Z_Z^{(2,1)}$ has similar structure.

- To restore gauge invariance of renormalization constants the tadpole contribution should be taken into account.

- The terms proportional to m_H^4 are generated by pure bosonic contributions.

- The terms proportional to higher power of m_H^2 and m_t^2 are generated by tadpole diagrams.

Renormalization group analysis

We can check two-loop coefficients $Z^{(2,2)}$ and $Z^{(2,1)}$ in mass renormalization formula

$$
\begin{aligned}
m_0^2 &= m^2 \left(1 + \frac{g^2}{16\pi^2} \frac{Z^{(1,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,2)}}{\varepsilon^2} \right) \\
&= m^2 \left(1 + \frac{z^{(1)}}{\varepsilon} + \frac{z^{(2)}}{\varepsilon^2} + \ldots \right)
\end{aligned}
$$

using renormalization group.

Introduce anomalous mass dimension

$$
\gamma = \mu^2 \frac{d}{d\mu^2} \log m^2 .
$$

From

$$
0 = \mu^2 \frac{d}{d\mu^2} \log m_0^2
$$

we have

$$
\begin{aligned}
\gamma &= \frac{1}{2} \sum_j g_j \frac{\partial}{\partial g_j} z^{(1)}, \\
\left(\gamma + \sum_i \beta_{g_i} \frac{\partial}{\partial g_i} + \sum_i \gamma_i m_i^2 \frac{\partial}{\partial m_i^2} \right) z^{(n)} &= \frac{1}{2} \sum_j g_j \frac{\partial}{\partial g_j} z^{(n+1)},
\end{aligned}
$$

where

$$
z^{(n)} = \sum_{k=1} \left(\frac{g^2}{16\pi^2} \right)^k Z^{(n,k)} .
$$

Pole structure

Check of $1/\varepsilon^2$ pole:

From previous formula we can derive

$$2Z^{(2,2)} = \left(Z^{(1,1)}\right)^2 + 2\frac{16\pi^2}{g^3}\beta_g^{(1)} Z^{(1,1)} + \sum_i Z_{m_i}^{(1,1)} m_i^2 \frac{\partial}{\partial m_i^2} Z^{(1,1)}$$

Check $1/\varepsilon$ pole:

Use relation between charges

$$e^2 = g^2\left(1 - \frac{m_W^2}{m_Z^2}\right) \quad \text{and} \quad \frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}.$$

Differentiating the former formulae w.r.t. $\log\mu^2$ we derive

$$\left(\frac{g^2}{16\pi^2}\right)^2 \left(Z_Z^{(2,1)} - Z_W^{(2,1)}\right) c^2 = g^2\frac{\beta_{g'}^{(2)}}{g'^3}s^4 - \frac{\beta_g^{(2)}}{g}s^2c^2.$$

β-functions for g and g' are calculated from symmetric phase theory

D.R.T. Jones '82

M.E. Machacek & M.T. Vaughn '83

$$\begin{aligned}
\beta_{g'} &= \left(\frac{1}{12} + \frac{10}{9}n_F\right)\frac{g'^3}{16\pi^2} - \frac{17}{24}\frac{m_t^2}{m_W^2}\frac{g'^3 g^2}{(16\pi^2)^2} \\
&+ \left(\frac{1}{4} + \frac{95}{54}n_F\right)\frac{g'^5}{(16\pi^2)^2} + \left(\frac{3}{4} + \frac{1}{2}n_F\right)\frac{g'^3 g^2}{(16\pi^2)^2} \\
\beta_g &= \left(-\frac{43}{12} + \frac{2}{3}n_F\right)\frac{g^3}{16\pi^2} - \frac{3}{8}\frac{m_t^2}{m_W^2}\frac{g^5}{(16\pi^2)^2} \\
&+ \left(-\frac{259}{12} + \frac{49}{6}n_F\right)\frac{g^5}{(16\pi^2)^2} + \left(\frac{1}{4} + \frac{n_F}{6}\right)\frac{g^3 g'^2}{(16\pi^2)^2}
\end{aligned}$$

$\overline{\mathrm{MS}}$-Renormalization

The bare relation is

$$s_V = m_{0,V}^2 + \Pi_{0,V}^{(1)} + \Pi_{0,V}^{(1)}{}'$$

$\overline{\mathrm{MS}}$-renormalized relation can be written as

$$s_V = m_V^2(\mu) + \left\{ \Pi_{0,V}^{(1)} \right\}_{MS} + \left\{ \Pi_{0,V}^{(2)} + \Pi_{0,V}^{(1)} \Pi_{0,V}^{(1)}{}' \right\}_{MS}$$

and

$$\Pi_{0,V}^{(j)} = m_{0,V}^2 \left(\frac{g_0^2}{16\pi^2} \right)^j X_{0,V}^{(j)}$$

$$\left\{ \Pi_{0,V}^{(1)} \right\}_{MS} = m_V^2(\mu) \frac{e^2}{16\pi^2 \sin^2\theta_W} \lim_{\varepsilon \to 0} \left(\frac{1}{\varepsilon} Z_V^{(1,1)} + X_{0,V}^{(1)} \right)$$

$$\left\{ \Pi_{0,V}^{(2)} + \Pi_{0,V}^{(1)} \Pi_{0,V}^{(1)}{}' \right\}_{MS} = \lim_{\varepsilon \to 0} \Bigg(\Pi_{0,V}^{(2)} + \Pi_{0,V}^{(1)} \Pi_{0,V}^{(1)}{}'$$
$$+ \frac{m_V^2(\mu)}{\varepsilon} \left(\frac{e^2}{16\pi^2 \sin^2\theta_W} \right)^2 \left[Z_V^{(1,1)} + \Delta g^2 + \sum_j Z_{m_j^2}^{(1,1)} \frac{\partial}{\partial m_j^2} \right] X_{0,V}^{(1)}$$
$$+ m_V^2(\mu) \left(\frac{e^2}{16\pi^2 \sin^2\theta_W} \right)^2 \left[\frac{1}{\varepsilon} Z_V^{(2,1)} + \frac{1}{\varepsilon^2} Z_V^{(2,2)} \right] \Bigg)$$

where the sum runs over all species of particles $j = Z,\ W,\ H,\ t$ and

$$\Delta g^2 = \frac{\cos^2\theta_W}{\sin^2\theta_W} \left(Z_W^{(1,1)} - Z_Z^{(1,1)} \right) + \sin^2\theta_W \left(-7 + \frac{32}{9} n_F \right)$$

Results

$$\frac{M_V^2}{m_V^2} = 1 + \frac{e^2}{16\pi s^2} X_{1,V} + \left(\frac{e^2}{16\pi s^2}\right)^2 X_{2,V},$$

$$X_{2,V} = \sum_{i=0}^{5} A_i^V s^{2i} \,,$$

$$A_i^V = \sum_{j,k} A_{i,j,k}^V(m_H^2, m_t^2, \mu^2) \left(\frac{m_V^2}{m_H^2}\right)^j \left(\frac{m_V^2}{m_t^2}\right)^k \,, V = Z, W$$

- All parameters in $\overline{\mathrm{MS}}$- scheme.
- Expansion in powers and $\log$'s (i.e. is asymptotic expansion not naive Taylor expansion).
- Expansion coefficients $A_{i,j}^V(m_H^2, m_t^2, \mu^2)$ includes some transcendental constants, logarithms and polylogarithm function.
- www-zeuthen.desy.de/~kalmykov/pole/pole.html
- Corrections of order m_H^4/m_V^4 are generated by bosonic diagrams and $m_t^6/(m_H^2 m_V^4)$ and $m_t^4/(m_H^2 m_W^2)$ by fermion loop.

Position of the pole s_p of gauge bosons Z, W is a gauge invariant quantity up to two-loop.
Stuart '91, Sirlin'91, $\cdots$ Kniehl & Sirlin '98

After summing all diagrams with massless fermion loop all singularities of type $\ln^j \sin^2\theta_W$ canceled $\longrightarrow$ infrared finiteness of the massless fermion contribution to the pole mass.
After exact UV renormalization the pole mass is a finite expression. $\longrightarrow$ infrared finiteness of two-loop EW corrections to the pole mass.

Gambino & Grassi '00

RG invariance of the pole s_p also requires the inclusion of tadpole.

One counterterms is sufficiently to renormalize real and imaginary part of the pole s_p

Kniehl & Sirlin '02

$\overline{\mathrm{MS}}$ mass in terms of on-shell mass

Inverse of our "master formula" express all $\overline{\mathrm{MS}}$ parameters in terms of on-shell ones:

$$
\begin{aligned}
m_V^2 \quad = \quad & M_V^2 - \left\{ \Pi_V^{(1)} \right\}_{MS} - \left\{ \Pi_V^{(2)} + \Pi_V^{(1)} \Pi_V^{(1)\prime} \right\}_{MS} \\
- \quad & \sum_j (\Delta m_j^2)^{(1)} \frac{\partial}{\partial m_j^2} \left\{ \Pi_V^{(1)} \right\}_{MS} - (\Delta e)^{(1)} \frac{\partial}{\partial e} \left\{ \Pi_V^{(1)} \right\}_{MS} \Bigg|_{m_j^2 = M_j^2}^{e = e_{\mathrm{OS}}} ,
\end{aligned}
$$

where the sum runs over all species of particles $j = Z,\ W,\ H\ t$ and

$$
(\Delta m_j^2)^{(1)} = -\mathrm{Re} \left\{ \Pi_V^{(1)} \right\}_{MS} \Bigg|_{m_j^2 = M_j^2,\, e = e_{\mathrm{OS}}}
$$

stands for the self-energy of the jth particle at $p^2 = m_j^2$ in the $\overline{\mathrm{MS}}$scheme and parameters replaced by the on-shell ones. Note that in the above relation we had to perform a change from the $\overline{\mathrm{MS}}$ to the on-shell scheme also for the electric charge.

Unphysical terms, proportional to m_H^4 and m_t^6 drop out in $\sin^2 \theta_W$

$$
\begin{aligned}
\sin^2 \theta_W \quad = \quad & 1 - \frac{m_W^2}{m_Z^2} \left(1 - \frac{M_W^2}{M_Z^2} \right) \times \\
& \left[1 - \frac{\cos^2 \theta_W}{\sin^2 \theta_W} \left((\delta_W^{(1)} - \delta_Z^{(1)})(1 - \delta_Z^{(1)}) + \delta_W^{(2)} - \delta_Z^{(2)} \right) \right]
\end{aligned}
$$

where $m_V^2 / M_V^2 = 1 + \delta_V^{(1)} + \delta_V^{(2)}$