NNLO Higgs production at Tevatron and LHC

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RADCOR / Loops and Legs (2002)

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- Introduction and Motivation
- Techniques
- Results

Introduction and Motivation

- Next big milestone in particle physics:

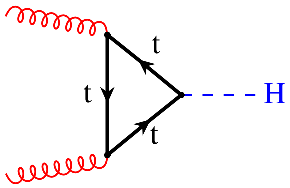
  discovery of a Higgs boson

- if it exists: Tevatron or LHC will see it

- to find the $\mathcal{EXOTIC}$, understand the STANDARD:

  need reliable Standard Model predictions

- here: SM Higgs production



NLO is up to 100%

$\rightarrow$ need NNLO!

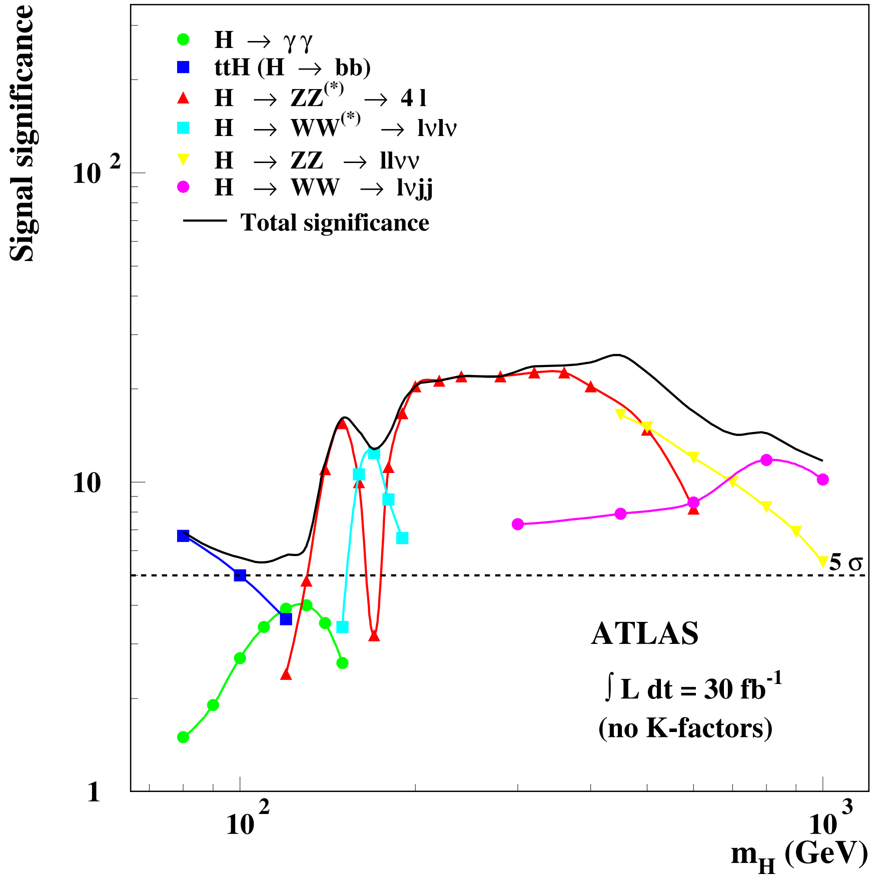
Higgs searches at hadron colliders

- two distinct mass ranges: above/below $\approx 135\,\mathrm{GeV}$
  - heavy Higgs: $H \to WW^{(*)}$ allowed

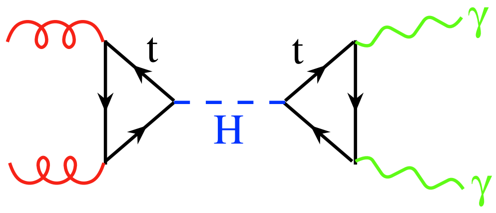
    e.g., $gg \to H \to WW^{(*)} \to l\nu l\nu$
  - light Higgs: $H \to WW^{(*)}$ forbidden $\Rightarrow$ $H \to b\bar{b}$ dominant

    but $gg \to H \to b\bar{b}$ impossible!
    - $gg \to H \to \gamma\gamma$: extremely difficult, but necessary!
    - $gg \to t\bar{t}H \to 4b's + \ldots$
    - vector boson fusion: $pp \to H$+2 forward jets
    - $\ldots$

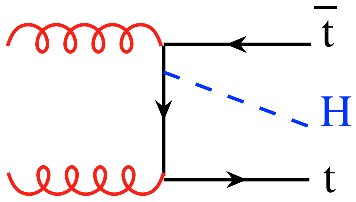
Discovery modes



$H \rightarrow \gamma\gamma$:



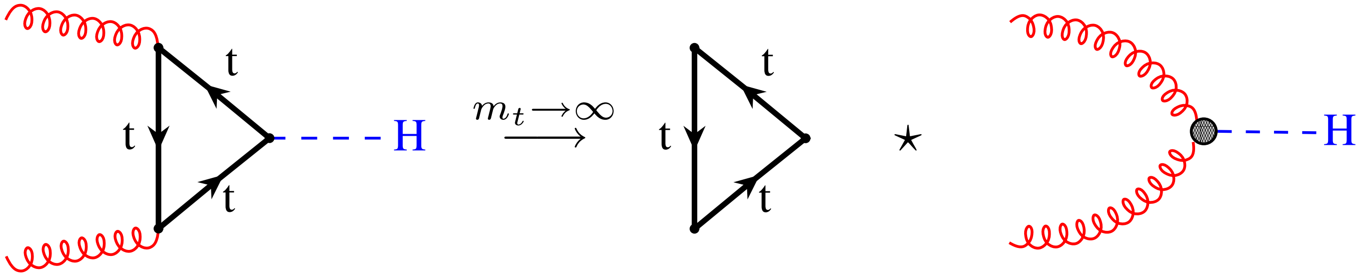
$t\bar{t}H$:



Higher orders in $gg \to H$

- LO: 1-loop [Ellis, Gaillard, Nanopoulos, Sachrajda ('77)]

- higher orders: use effective Lagrangian:



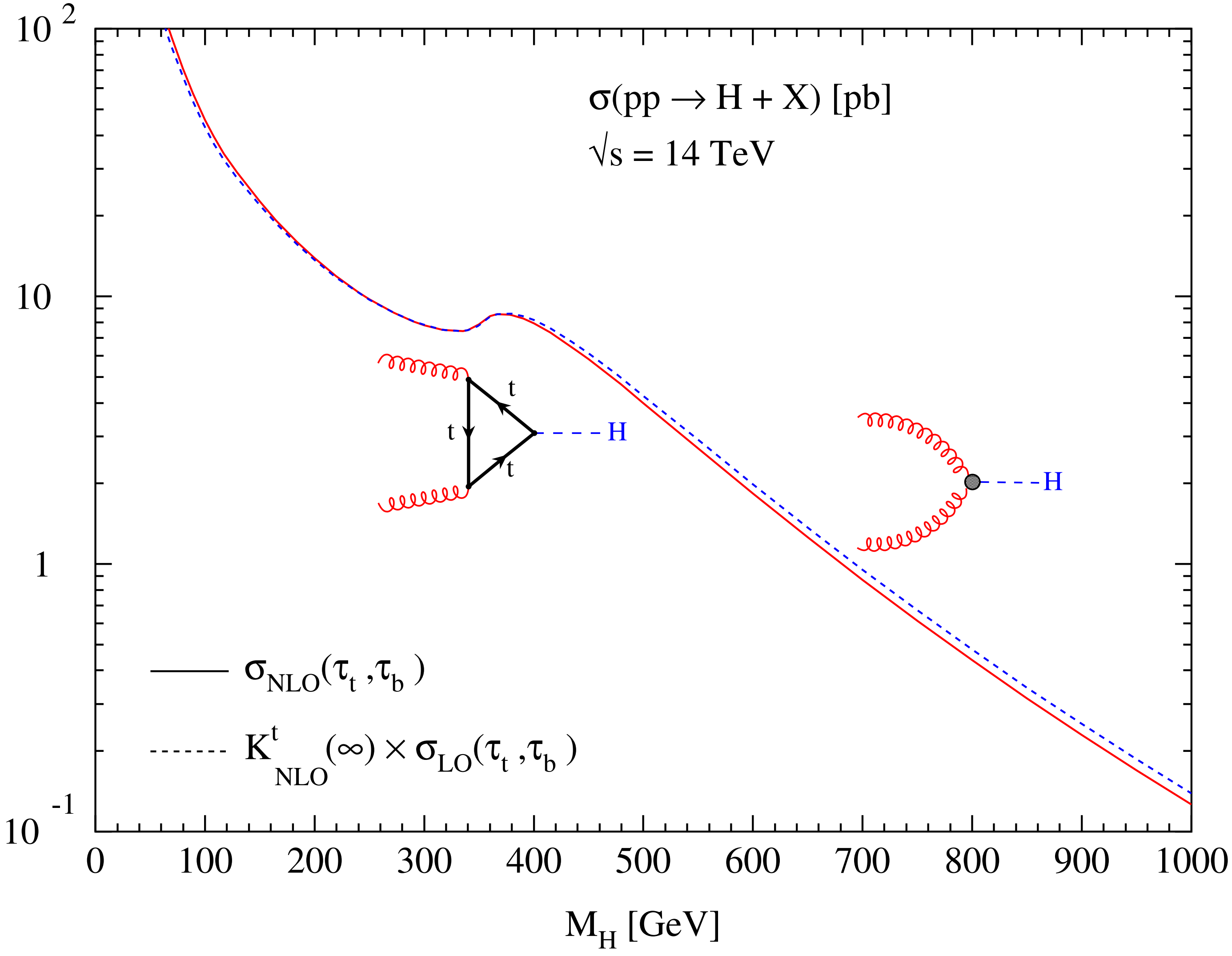
$$\mathcal{L}_{\text{eff}} \quad = \quad C(\alpha_s) \quad \star \quad G^a_{\mu\nu} G^{a,\mu\nu} \, H$$

- $C(\alpha_s)$: universal, known to $\mathcal{O}(\alpha_s^4)$, i.e. NNN(!)LO
  [Chetyrkin, Kniehl, Steinhauser '98]

- NLO: effective Lagrangian works extremely well!

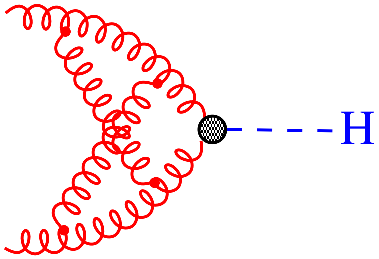
  [S. Dawson '91], [S. Dawson, Kauffman '92]
  [Djouadi, Spira, Zerwas '91], [Graudenz, Spira, Zerwas '93]

NLO effective vs. exact

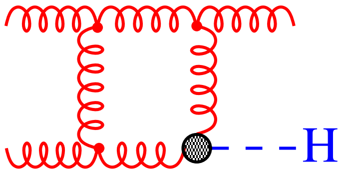


taken from [Spira '97]

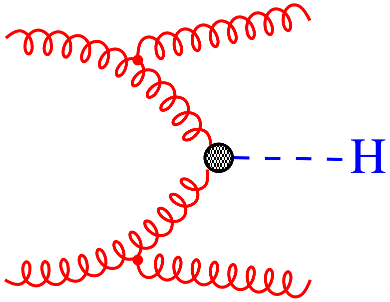
NNLO

- 

: 2-loop,
$\propto \delta(1-x)$, $\qquad x = M_H^2/\hat{s}$

- 

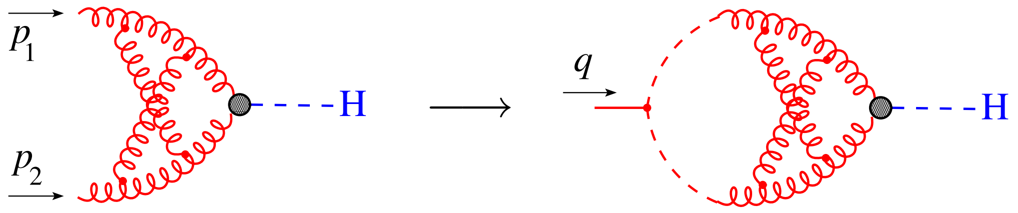
: 1-loop, two-particle phase space
use known techniques

- 

: tree-level, 3-particle phase space
brute force? – tough!
need new techniques!

NNLO: Virtual Corrections

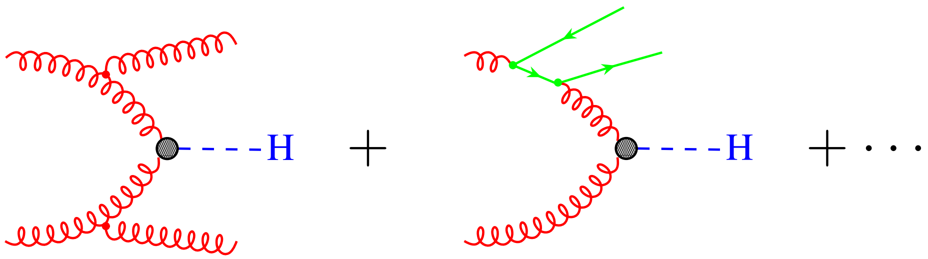
- map 2-loop vertex diagrams to 3-loop propagators:
  [Baikov, Smirnov '00]



- evaluate analytically: "Integration-by-parts"
  [Chetyrkin, Tkachov '81]
- algebraic implementation: `MINCER / FORM`
  [Larin, Tkachov, Vermaseren '91]
- result [R.H. '00]:

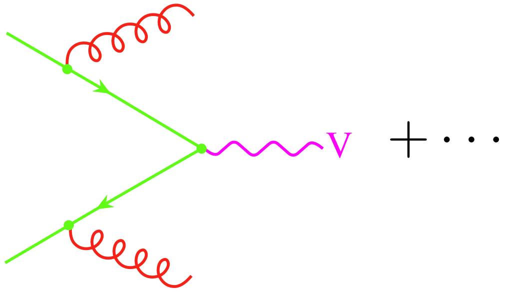
$$\sigma^{(2)}_{\rm virt} = \delta(1-x)\left\{\frac{9}{8\epsilon^4} + \cdots + \epsilon^0\left(\frac{5861}{288} + \text{“}\zeta\text{'s”}\right) + n_l\left[\cdots\right]\right\}$$

Double Real Radiation



- very complicated phase space integrals

- similar calculation: NNLO Drell-Yan:
  [Hamberg, v.Neerven, Matsuura '91]



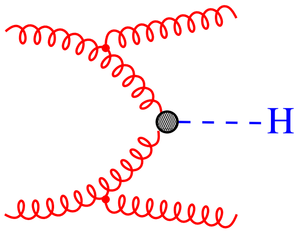
  but:

  – integrals unpublished

  – calculation unchecked

First step: Soft Limit

[R.H., W.Kilgore '01]
[Catani, de Florian, Grazzini '01]



- simplest *finite* partial result

$$\hat{\sigma}^{\text{soft}}(x) = a\,\delta(1-x) + \sum_{n\geq 0} \mathcal{D}_n(x)\,,$$

$$x = M_H^2/\hat{s}\,, \qquad \mathcal{D}_n(x) = \left(\frac{\ln^n(1-x)}{1-x}\right)_+$$

- $\delta(1-x) \quad \Rightarrow \quad \hat{s} = M_H^2 \quad \Rightarrow$ real gluons are *soft*

- $\mathcal{D}_n(x)$–terms are for free:

$$(1-x)^{-1+\epsilon} = \frac{1}{\epsilon}\delta(1-x) + \sum_{n=0}^{\infty} \frac{\epsilon^n}{n!}\mathcal{D}_n(x)$$

Systematic Expansion of $\hat{\boldsymbol{\sigma}}_{ij}(\boldsymbol{x})$

[R.H., W.Kilgore '02]

$$\hat{\sigma}(x) = \hat{\sigma}^{\text{soft}} + \sum c_{ln}\,(1-x)^l \ln^n(1-x)$$

- most general expression: e.g.,

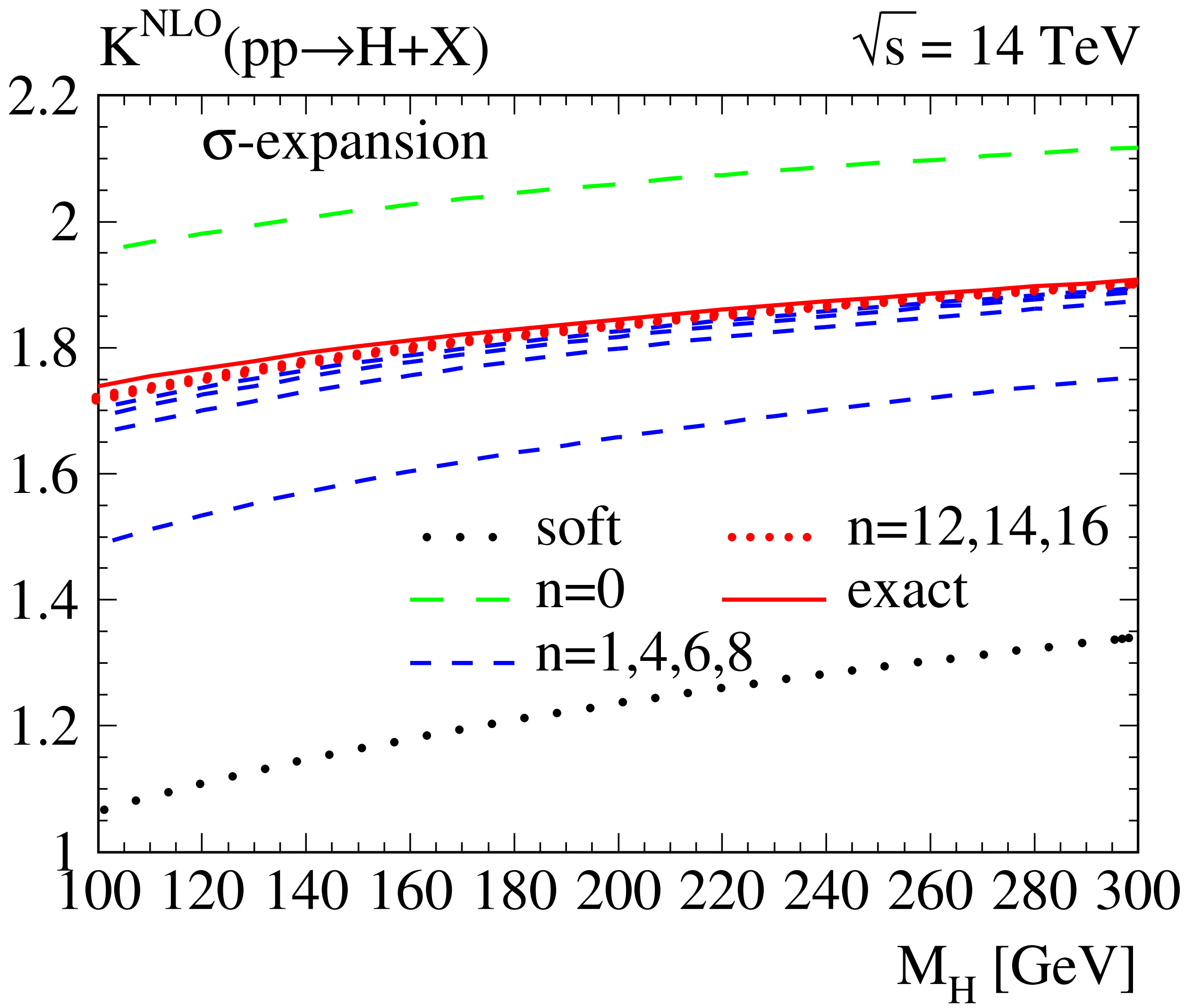
$$\text{Li}_2(x) = \frac{\pi^2}{6} + (1-x)\left(\ln(1-x) - 1\right) + (1-x)^2\left(\frac{1}{2}\ln(1-x) - \frac{1}{4}\right) + (1-x)^3\left(\frac{1}{3}\ln(1-x) - \frac{1}{9}\right) + \ldots$$

- *note:* $\ln^3(1-x)$ (i.e., c_{03}) derived earlier!

[Krämer, Laenen, Spira '98]

$\Rightarrow$ "soft + collinear" approximation

Convergence at NLO



Evaluation of higher orders in $(1-x)$

- phase space: express in terms of $s, \tilde{t}, \tilde{u} \Leftrightarrow x, y, z$ and ϕ

$$d\Phi_3^{(d)} \propto (1-x)^{3-4\epsilon} \int d\phi$$
$$\times \int dy \int dz \, [1-(1-x)y]^{-1+\epsilon} \, y^{1-2\epsilon}(1-y)^{1-2\epsilon} z^{-\epsilon}(1-z)^{-\epsilon}$$

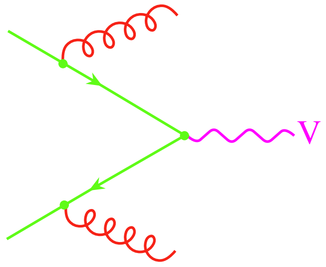
- invariants: e.g.,

$$(p_1 - p_H)^2 = -s(1-x)y \frac{x + (1-x)(1-y)(1-z)}{1-(1-x)y(1-z)}$$

- integrand is smooth within (finite!) integration region $\Rightarrow$ expand everything in $(1-x)$!
- phase space integration simplifies greatly!

Test: NNLO Drell-Yan

- NNLO Drell-Yan process:

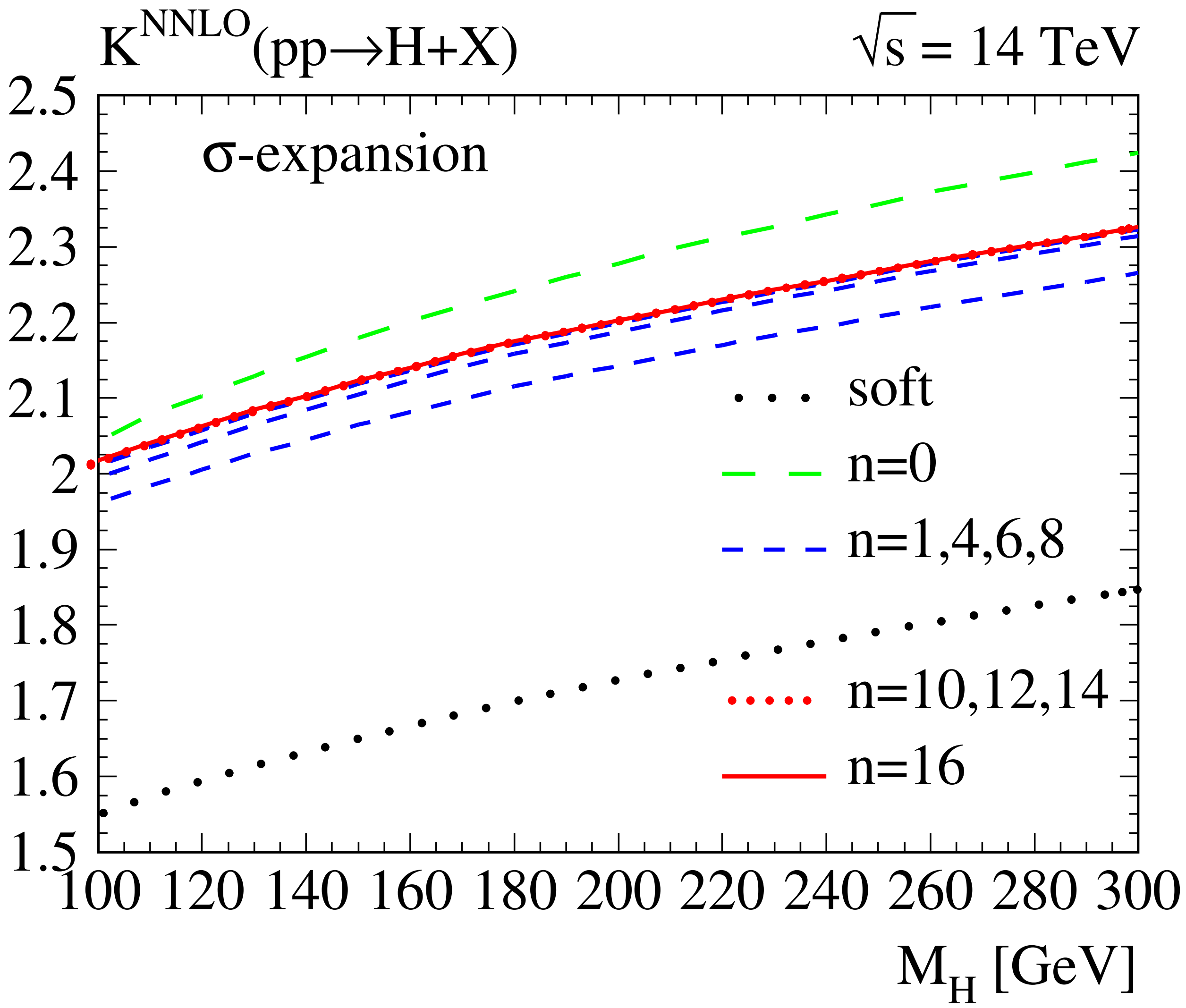


- independent check of [Hamberg, van Neerven, Matsuura, '91]
- found differences in NNLO result!

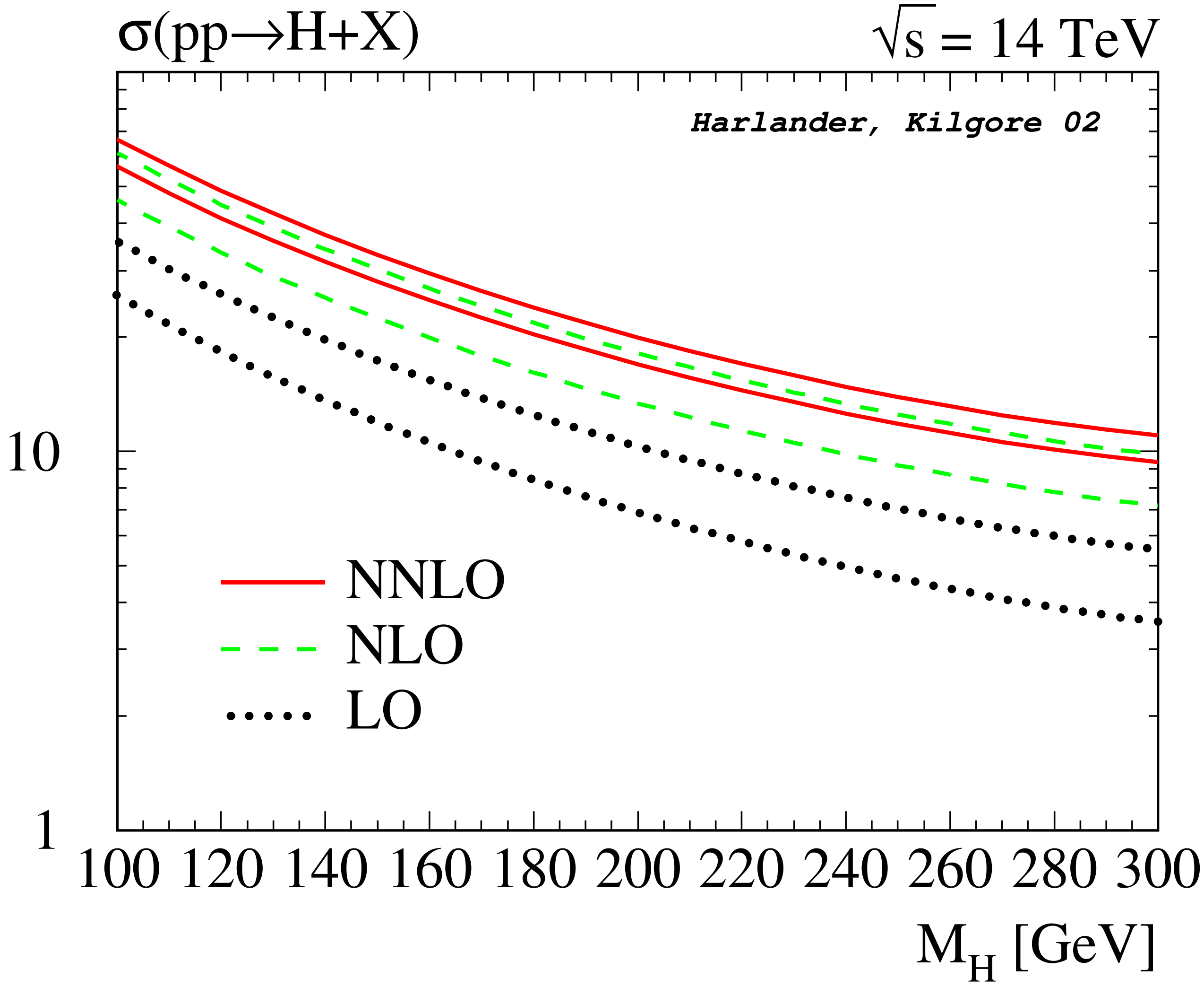
  $\longrightarrow$ confirmed by Ravindran, v.Neerven
- expansion method provides way to obtain

  systematic approximation *and* analytic result

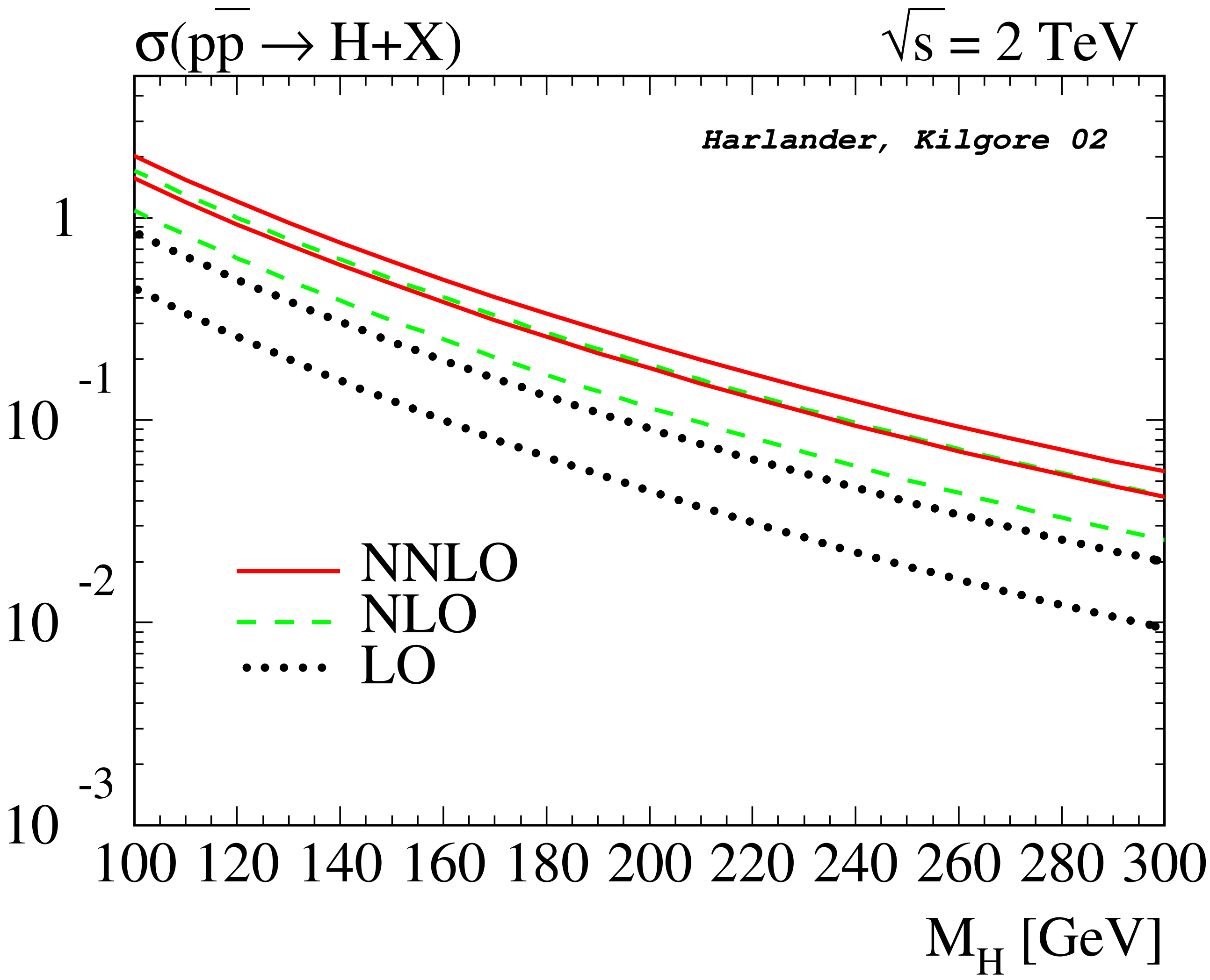
NNLO: convergence



NNLO result for LHC



NNLO result for Run II



Inverting the series

- full result: $\log$, Li_2, Li_3 of various arguments,
  multiplied by powers of x

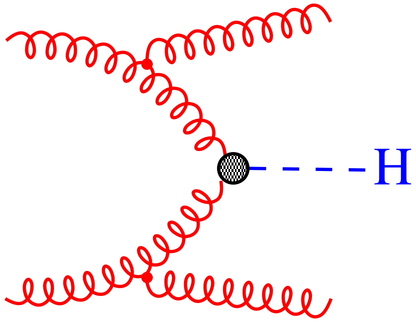
  $\Rightarrow$ 98 basis functions

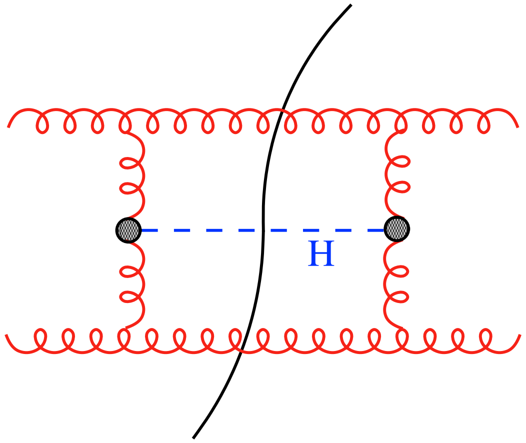
- evaluate expansion up to $(1-x)^{98}$

  $\rightarrow$ determine coefficients of basis functions [Kilgore '02]

Alternative approach: cutting method

[Anastasiou, Melnikov '02]

$\int \mathrm{dPS} \Big|$ 

$\Big|^2 =$ 

- apply multi-loop techiques to cut diagrams

  $\longrightarrow$ see talk by C. Anastasiou

- full agreement to expansion method

Final Remarks and Conclusions

- NNLO prediction for Higgs production complete:

$$\sigma \sim \sigma_0 \left[1 + 0.8 + 0.3\right], \qquad \frac{\delta\sigma}{\sigma} \approx \pm 15\%$$

- new methods:
  - phase space expansion [R.H., Kilgore '02]
  - cutting technique [Anastasiou, Melnikov '02]
- recent application: pseudo-scalar Higgs production

  [H+K, hep-ph/0208096], [A+M, hep-ph/0208115]