Decoupling and the Radiatively-corrected MSSM Higgs Sector

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This talk is based on work that appears in:

1. M. Carena, H.E. Haber, H.E. Logan and S. Mrenna, hep-ph/0106116, *Phys. Rev.* **D65** (2002) 055005 [E: **D65** (2002) 099902].

2. J.F. Gunion and H.E. Haber, "The CP-conserving two-Higgs-doublet model: the approach to the decoupling limit," hep-ph/0207010.

3. M. Carena and H.E. Haber, "Higgs boson theory and phenomenology," hep-ph/0208209.

Outline

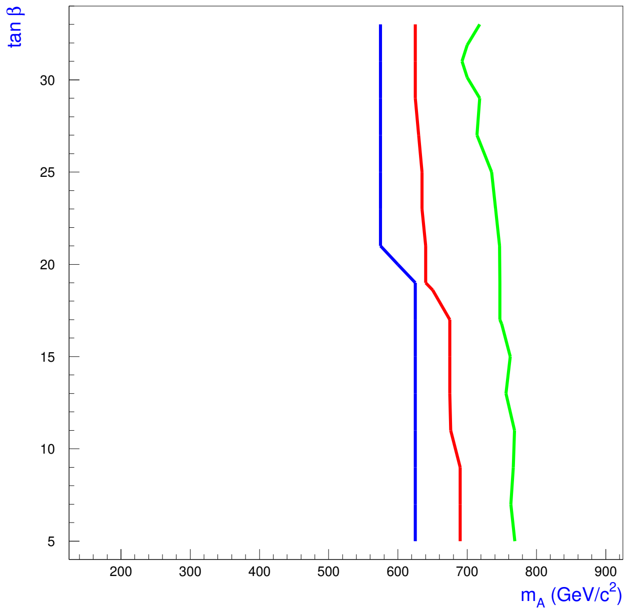
- A Challenge for the Higgs Search at the LC
- The Decoupling Limit of the 2HDM
- MSSM Higgs Sector at Tree Level
- MSSM Higgs Sector at One-Loop (and beyond)
- Precision Higgs physics at the LC
- Conclusions

Challenge for the Higgs Search at the LC

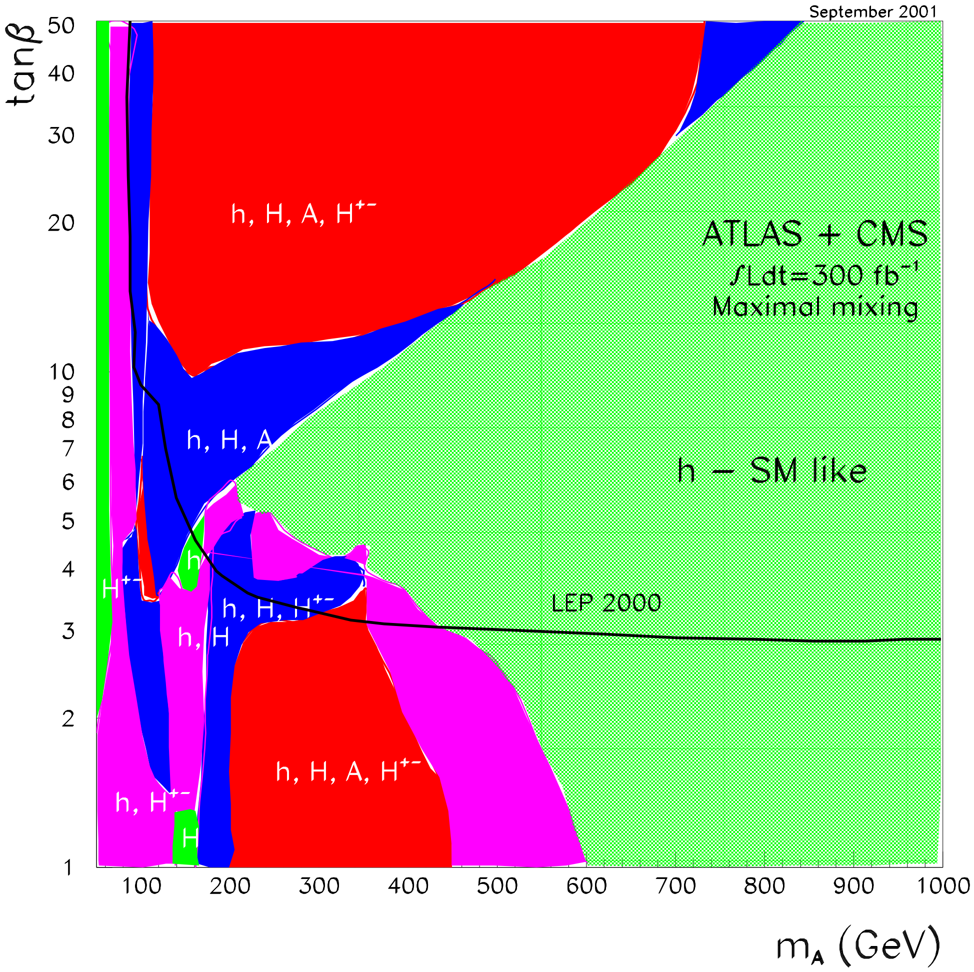
Suppose a Higgs boson is discovered at the LHC, and its properties are observed to coincide (to within experimental error) to those of the Standard Model (SM) Higgs boson.

Moreover, suppose evidence for SUSY is found suggesting that the Higgs boson is the lightest state of a two-Higgs-doublet model (2HDM). But no evidence for the heavier Higgs states is found.

With precision measurements at the e^+e^- Linear Collider (LC), can one determine the mass scale of the heavier Higgs states?



From right to left, the upper bound on the regions in which 68%, 90% and 95% of the MSSM parameter space, respectively, yields h branching ratio predictions that differ from those of the SM at the 95% CL. Based on a simulation of 1000 fb^{-1} of data at the LC with $\sqrt{s} = 500$ GeV [Battaglia and Desch].



At LHC, more than one Higgs state will often be observed if $\tan\beta \gg 1$ (due to enhanced couplings to down-type fermions). But, there is also a substantial region in which only h is observed.

In particular, this illustrates the challenge of the decoupling limit, in which $h \simeq h_{\rm SM}$. If one cannot detect the non-minimal Higgs states, one must look for a deviation of the h couplings from SM expectations.

The Decoupling Limit of the 2HDM

- Given a non-minimal Higgs sector, the decoupling limit corresponds to the parameter regime in which all but one CP-even Higgs scalar is significantly heavier than the Z. The properties of the lightest CP-even Higgs boson are nearly indistinguishable from those of the Standard Model (SM) Higgs boson.
- The decoupling limit is very general. Many models with non-minimal Higgs sectors possess a decoupling limit. The MSSM Higgs sector is one such example.

A case study: the general two-Higgs-doublet model (2HDM)

$$
\begin{aligned}
\mathcal{V} = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \tfrac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 \\
& + \tfrac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\
& + \left\{ \tfrac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2) \right] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\}
\end{aligned}
$$

Assume CP conservation (for simplicity). The vevs are $\langle \Phi_i^0 \rangle = v_i/\sqrt{2}$, with $\tan\beta \equiv v_2/v_1$ and $v^2 \equiv v_1^2 + v_2^2 = (246~\text{GeV})^2$.

Diagonalizing the CP-even Higgs squared-mass matrix yields two CP-even eigenstates:

$$H = (\sqrt{2}\mathrm{Re}\,\Phi_1^0 - v_1)\cos\alpha + (\sqrt{2}\mathrm{Re}\,\Phi_2^0 - v_2)\sin\alpha\,,$$

$$h = -(\sqrt{2}\mathrm{Re}\,\Phi_1^0 - v_1)\sin\alpha + (\sqrt{2}\mathrm{Re}\,\Phi_2^0 - v_2)\cos\alpha\,.$$

The other physical Higgs scalars: are a CP-odd state, A and a charged scalar pair, $H^\pm$. The decoupling limit is defined as the limit of $m_A \gg m_Z$, assuming $\lambda_i \lesssim \mathcal{O}(1)$. One can show that this limit corresponds to taking $\beta - \alpha \to \pi/2$; *i.e.*, $\cos(\beta - \alpha) \to 0$. In the approach to the decoupling limit, one finds:

$$\begin{aligned}
m_A^2 &\simeq v^2\left[\frac{\widehat{\lambda}}{c_{\beta-\alpha}} + \lambda_A - \tfrac{3}{2}\widehat{\lambda}\, c_{\beta-\alpha}\right]\,, \\
m_h^2 &\simeq v^2(\lambda - \widehat{\lambda}\, c_{\beta-\alpha})\,, \\
m_H^2 &\simeq m_A^2 + (\lambda - \lambda_A + \widehat{\lambda}\, c_{\beta-\alpha})v^2\,, \\
m_{H^\pm}^2 &= m_A^2 + \tfrac{1}{2}(\lambda_5 - \lambda_4)v^2\,.
\end{aligned}$$

where $c_{\beta-\alpha} \equiv \cos(\beta - \alpha)$ and

$$
\begin{aligned}
\lambda \equiv &\lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 + 2\lambda_6 c_\beta^2 s_{2\beta} + 2\lambda_7 s_\beta^2 s_{2\beta}\,, \\
\widehat{\lambda} \equiv &\tfrac{1}{2} s_{2\beta}\left[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta}\right] - \lambda_6 c_\beta c_{3\beta} - \lambda_7 s_\beta s_{3\beta}\,, \\
\lambda_A \equiv &c_{2\beta}(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2) + \lambda_{345} s_{2\beta}^2 - \lambda_5 + 2\lambda_6 c_\beta s_{3\beta} - 2\lambda_7 s_\beta c_{3\beta}\,, \\
\lambda_{345} \equiv &\lambda_3 + \lambda_4 + \lambda_5\,.
\end{aligned}
$$

In particular,

$$
\boxed{\cos(\beta-\alpha) \simeq \frac{\widehat{\lambda} v^2}{m_A^2 - \lambda_A v^2} \simeq \frac{\widehat{\lambda} v^2}{m_H^2 - m_h^2}}
$$

Implications

$$m_h \sim \mathcal{O}(m_Z)\,,$$

$$m_H \simeq m_A \simeq m_{H^\pm}\,, \quad [\text{up to corrections of } \mathcal{O}(m_Z^2/m_A)]\,,$$

$$\cos(\beta-\alpha) \sim \mathcal{O}(\frac{m_Z^2}{m_A^2})\,.$$

2HDM Tree-Level Higgs Couplings

Higgs couplings to gauge bosons: suppression factors

$\cos(\beta-\alpha)$	$\sin(\beta-\alpha)$	no angle factor
HW^+W^-	hW^+W^-	
HZZ	hZZ	
ZAh	ZAH	$ZH^+H^-, \gamma H^+H^-$
$W^\pm H^\mp h$	$W^\pm H^\mp H$	$W^\pm H^\mp A$

Model-II Tree-level Higgs-fermion Yukawa couplings

The MSSM employs the so-called Type-II couplings in which Φ_1^0 [Φ_2^0] couples exclusively to down-type [up-type] fermions.

$$h b\bar{b}: \quad -\frac{\sin\alpha}{\cos\beta} = 1\sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha)\,,$$

$$h t\bar{t}: \quad \frac{\cos\alpha}{\sin\beta} = 1\sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha)\,,$$

$$H b\bar{b}: \quad \frac{\cos\alpha}{\cos\beta} = \cos(\beta-\alpha) + \tan\beta\,\sin(\beta-\alpha)\,,$$

$$H t\bar{t}: \quad \frac{\sin\alpha}{\sin\beta} = \cos(\beta-\alpha) - \cot\beta\,\sin(\beta-\alpha)\,.$$

SM-like Higgs boson without decoupling

Recall that

$$c_{\beta-\alpha} \equiv \cos(\beta-\alpha) \simeq \frac{\widehat{\lambda} v^2}{m_A^2 - \lambda_A v^2} \simeq \frac{\widehat{\lambda} v^2}{m_H^2 - m_h^2}$$

where

$$\widehat{\lambda} \equiv \tfrac{1}{2} s_{2\beta} \left[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta} \right] - \lambda_6 c_\beta c_{3\beta} - \lambda_7 s_\beta s_{3\beta}$$

Assume that $\tan\beta \geq 1$. Two cases arise in which $|c_{\beta-\alpha}| \ll 1$:

1. decoupling limit: $m_A^2 \gg \lambda_i v^2 \tan\beta$, with $\lambda_i \lesssim \mathcal{O}(1)$
2. $|\widehat{\lambda}| \ll 1$, m_A arbitrary

In Case 1, even if $\tan\beta \gg 1$, we have $|\tan\beta\, c_{\beta-\alpha}| \ll 1$, which implies that the $hb\bar{b}$ coupling is SM-like. That is, $h \simeq h_{\rm SM}$.

Case 2 does not correspond to decoupling if $m_A^2 \sim \mathcal{O}(\lambda_i v^2)$. If $\tan\beta \gg 1$, it is possible to have $|\tan\beta\, c_{\beta-\alpha}| \sim \mathcal{O}(1)$ even if $|c_{\beta-\alpha}| \ll 1$, in which case the $hb\bar{b}$ coupling deviates from the $h_{\rm SM}b\bar{b}$ coupling. Nevertheless, for other couplings, h is SM-like.

If $\tan\beta \leq 1$, replace $\tan\beta$ with $\cot\beta$ and $b\bar{b}$ with $t\bar{t}$ above.

MSSM Higgs sector at Tree-Level

Five physical Higgs scalars:

- $H^{\pm}$: a charged Higgs pair
- h, H: two CP-even Higgs scalars ($m_h \leq m_H$)
- A: a CP-odd Higgs scalar

All Higgs masses and couplings can be expressed in terms of two parameters usually chosen to be m_A and $\tan\beta \equiv v_u/v_d$. The CP-even Higgs mixing angle, α, is given by

$$\cos^2(\beta - \alpha) = \frac{m_h^2(m_Z^2 - m_h^2)}{m_A^2(m_H^2 - m_h^2)} \,.$$

As expected, in the decoupling limit where $m_h \sim \mathcal{O}(m_Z)$ and $m_A \gg m_Z$, we see that $\cos^2(\beta - \alpha) \to 0$.

Decoupling Limit of the MSSM Higgs Sector

In the limit $m_A \gg m_Z$, tree-level Higgs masses are given by:

$$m_h^2 \simeq m_Z^2 \cos^2 2\beta \,,$$

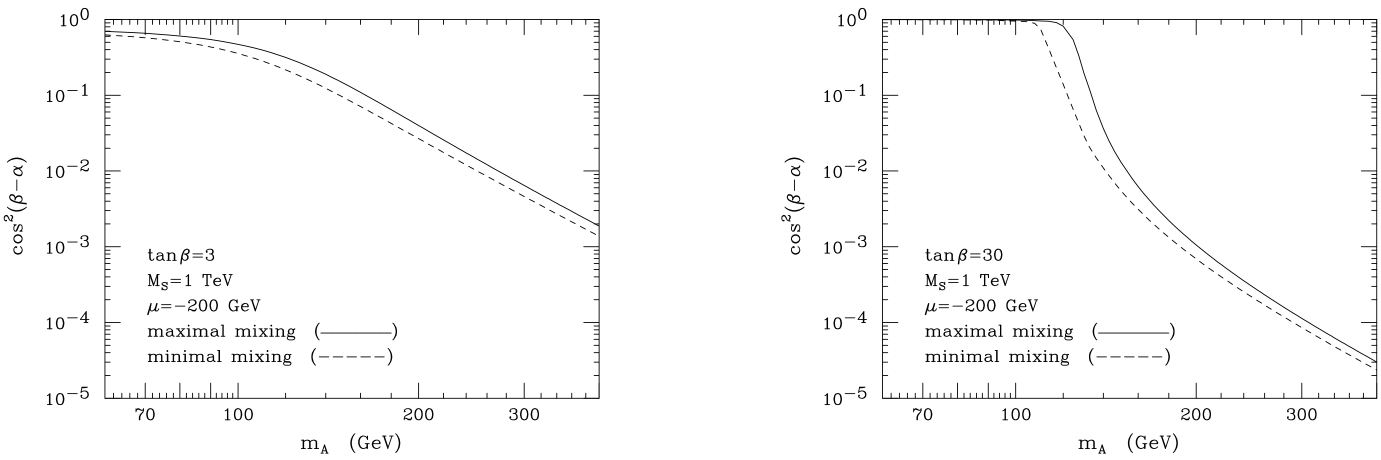
$$m_H^2 \simeq m_A^2 + m_Z^2 \sin^2 2\beta \,,$$

$$m_{H^\pm}^2 = m_A^2 + m_W^2 \,,$$

$$\cos^2(\beta - \alpha) \simeq \frac{m_Z^4 \sin^2 4\beta}{4m_A^4} \,,$$

$$\cot\alpha + \tan\beta = -\frac{2m_Z^2}{m_A^2} \tan\beta \cos 2\beta + \mathcal{O}(m_Z^4/m_A^4) \,.$$

Thus, $m_A \simeq m_H \simeq m_{H^\pm}$, up to corrections of $\mathcal{O}(m_Z^2/m_A)$, and $\cos(\beta - \alpha) = 0$ up to corrections of $\mathcal{O}(m_Z^2/m_A^2)$.



The effective low-energy theory below the scale m_A is a theory with an effective Higgs sector consisting of a SM-like CP-even Higgs boson, h.

Radiatively-corrected MSSM Higgs sector

Due to supersymmetric relations among couplings, one finds that $m_h \leq m_Z$ (a result already ruled out by LEP data). But, this inequality receives quantum corrections. The Higgs mass can be shifted due to loops of particles and their superpartners (an incomplete cancelation, which would have been exact if supersymmetry were unbroken):

The CP-even Higgs mass matrix takes the form:

$$\mathcal{M}^2 \equiv \begin{pmatrix} \mathcal{M}_{11}^2 & \mathcal{M}_{12}^2 \\ \mathcal{M}_{12}^2 & \mathcal{M}_{22}^2 \end{pmatrix} = \mathcal{M}_0^2 + \delta\mathcal{M}^2 \,,$$

where $\mathcal{M}_0^2$ is the tree-level contribution and $\delta\mathcal{M}^2$ is the contribution from the radiative corrections. The dominant corrections to $\mathcal{M}^2$, coming from the one-loop top and bottom quark and top and bottom squark contributions plus the two-loop leading logarithmic contributions, are given to $\mathcal{O}(h_t^4, h_b^4)$ by

$$\delta\mathcal{M}^2_{11} \simeq -\bar{\mu}^2 x_t^2 \frac{h_t^4 v^2}{32\pi^2} s_\beta^2 \left[1 + c_{11} \ln\left(\frac{M_S^2}{m_t^2}\right)\right]$$
$$-\bar{\mu}^2 a_b^2 \frac{h_b^4 v^2}{32\pi^2} s_\beta^2 \left[1 + c_{12} \ln\left(\frac{M_S^2}{m_t^2}\right)\right],$$

$$\delta\mathcal{M}^2_{12} \simeq -\bar{\mu} x_t \frac{h_t^4 v^2}{32\pi^2} (6 - x_t a_t) s_\beta^2 \left[1 + c_{31} \ln\left(\frac{M_S^2}{m_t^2}\right)\right]$$
$$+\bar{\mu}^3 a_b \frac{h_b^4 v^2}{32\pi^2} s_\beta^2 \left[1 + c_{32} \ln\left(\frac{M_S^2}{m_t^2}\right)\right],$$

$$\delta\mathcal{M}^2_{22} \simeq \frac{3h_t^4 v^2}{8\pi^2} s_\beta^2 \ln\left(\frac{M_S^2}{m_t^2}\right) \left[1 + \tfrac{1}{2} c_{21} \ln\left(\frac{M_S^2}{m_t^2}\right)\right]$$
$$+\frac{h_t^4 v^2}{32\pi^2} s_\beta^2 x_t a_t (12 - x_t a_t) \left[1 + c_{21} \ln\left(\frac{M_S^2}{m_t^2}\right)\right]$$
$$-\bar{\mu}^4 \frac{h_b^4 v^2}{32\pi^2} s_\beta^2 \left[1 + c_{22} \ln\left(\frac{M_S^2}{m_t^2}\right)\right],$$

where $s_\beta \equiv \sin\beta$, $c_\beta \equiv \cos\beta$, and the coefficients c_{ij} are:

$$c_{ij} \equiv \frac{t_{ij} h_t^2 + b_{ij} h_b^2 - 32 g_3^2}{32\pi^2},$$

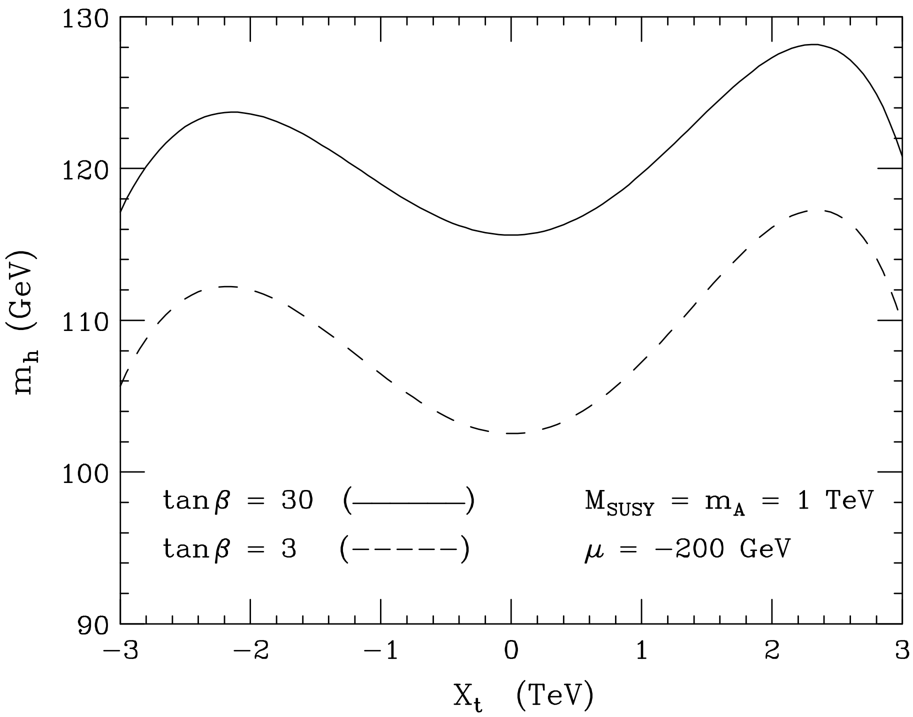
and $(t_{11}, t_{12}, t_{21}, t_{22}, t_{31}, t_{32}) = (12, -4, 6, -10, 9, -7)$, $(b_{11}, b_{12}, b_{21}, b_{22}, b_{31}, b_{32}) = (-4, 12, 2, 18, -1, 15)$.

In addition, $\bar{\mu} \equiv \mu/M_S$, $a_t \equiv A_t/M_S$, $a_b \equiv A_b/M_S$ and $x_t \equiv X_t/M_S$, where $X_t \equiv A_t - \mu \cot\beta$, govern the third generation squark mixing, and M_S^2 is the average squared-top squark mass. Diagonalizing the CP-even Higgs squared-mass matrix yields radiatively-corrected values for m_h^2, m_H^2 and the mixing angle α.

In particular, the upper bound on the lightest CP-even Higgs mass is approximately given by:

$$m_h^2 \lesssim m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right] .$$

End result: $m_h \lesssim 130$ GeV [assuming that $m_t = 175$ GeV and the top-squark mass is no heavier than about 2 TeV].



Leading radiative corrections to h couplings

For Higgs couplings to vector bosons, the dominant corrections arise from corrections to $\cos(\beta-\alpha)$.

For Higgs couplings to fermions, in addition to corrections to $\cos(\beta-\alpha)$, one must also consider Yukawa vertex corrections, which modify the effective Lagrangian that describes the coupling of the Higgs bosons to the third generation quarks:

$$
\begin{aligned}
-\mathcal{L}_{\rm eff} = \epsilon_{ij}\Big[(h_b+\delta h_b)\bar{b}_R H_d^i Q_L^j + (h_t+\delta h_t)\bar{t}_R Q_L^i H_u^j\Big] \\
+\Delta h_b \bar{b}_R Q_L^k H_u^{k*} + \Delta h_t \bar{t}_R Q_L^k H_d^{k*} + {\rm h.c.}\,,
\end{aligned}
$$

resulting in a modification of the tree-level relation between h_t $[h_b]$ and m_t $[m_b]$ as follows:

$$
\begin{aligned}
m_b &= \frac{h_b v}{\sqrt{2}}\cos\beta\left(1+\frac{\delta h_b}{h_b}+\frac{\Delta h_b\tan\beta}{h_b}\right) \equiv \frac{h_b v}{\sqrt{2}}\cos\beta(1+\Delta_b)\,, \\
m_t &= \frac{h_t v}{\sqrt{2}}\sin\beta\left(1+\frac{\delta h_t}{h_t}+\frac{\Delta h_t\cot\beta}{h_t}\right) \equiv \frac{h_t v}{\sqrt{2}}\sin\beta(1+\Delta_t)\,.
\end{aligned}
$$

The dominant contributions to Δ_b are $\tan\beta$-enhanced. In particular, for $\tan\beta\gg 1$, $\Delta_b\simeq(\Delta h_b/h_b)\tan\beta$; whereas $\delta h_b/h_b$ provides a small correction to Δ_b. In the same limit, $\Delta_t\simeq\delta h_t/h_t$, with the additional contribution of $(\Delta h_t/h_t)\cot\beta$ providing a small correction.

Caveats

1. CP-violating effects (that can enter through radiative corrections due to complex SUSY parameters) have been neglected.

2. By keeping only the dimension-4 terms in $\mathcal{L}_{\rm eff}$ and neglecting higher-dimensional terms, we are implicitly assuming that $M_{\rm SUSY} \gg m_Z$ (where $M_{\rm SUSY}$ is a typical SUSY mass that appears in the loops).

3. Due to weak isospin breaking, one should allow for different radiatively induced couplings to charged and neutral Higgs bosons. For example, one should write $\Delta h_b \bar{b}_R b_L H_u^{0*} + \Delta\overline{h}_b \bar{b}_R t_L H_u^-$ in place of $\Delta h_b \bar{b}_R Q_L^k H_u^{k*}$, *etc.* To the extent that weak isospin breaking effects are small in the loop diagrams that generate Δh_b and $\Delta\overline{h}_b$, it follows that $\Delta h_b \approx \Delta\overline{h}_b$ (and similarly for the other radiatively generated coefficients), and we may use our result for $\mathcal{L}_{\rm eff}$ as written.

In these approximations, we obtain:

$$
\begin{aligned}
g_{H^- t\bar{b}} \simeq \frac{\sqrt{2}}{v} \Bigg\{ & m_t \cot\beta \left[1 - \frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}(\cot\beta + \tan\beta)\right] P_R \\
& + m_b \tan\beta \left[1 + \frac{1}{(1+\Delta_b)\sin^2\beta}\left(\frac{\delta h_b}{h_b} - \Delta_b\right)\right] P_L \Bigg\}
\end{aligned}
$$

Explicit expressions

For $\tan\beta \gg 1$,

$$\Delta_b \simeq \left[\frac{2\alpha_s}{3\pi}\mu M_{\tilde{g}}\, I(M^2_{\tilde{b}_1}, M^2_{\tilde{b}_2}, M^2_{\tilde{g}}) + \frac{h_t^2}{16\pi^2}\mu A_t\, I(M^2_{\tilde{t}_1}, M^2_{\tilde{t}_2}, \mu^2)\right] \tan\beta$$

$$\Delta_t \simeq -\frac{2\alpha_s}{3\pi}A_t M_{\tilde{g}} I(M^2_{\tilde{t}_1}, M^2_{\tilde{t}_2}, M^2_{\tilde{g}}) - \frac{h_b^2}{16\pi^2}\mu^2 I(M^2_{\tilde{b}_1}, M^2_{\tilde{b}_2}, \mu^2)\,,$$

and the function I is defined by:

$$I(a,b,c) = \frac{ab\ln(a/b) + bc\ln(b/c) + ca\ln(c/a)}{(a-b)(b-c)(a-c)},$$

Note that I is manifestly positive and $I(a,a,a) = 1/(2a)$.

The τ couplings are obtained by replacing m_b, Δ_b and δh_b with m_τ, Δ_τ and δh_τ, respectively. At large $\tan\beta$,

$$\Delta_\tau \simeq \left[\frac{\alpha_1}{4\pi}M_1 \mu I(M^2_{\tilde{\tau}_1}, M^2_{\tilde{\tau}_2}, M^2_1) - \frac{\alpha_2}{4\pi}M_2\mu\, I(M^2_{\tilde{\nu}_\tau}, M^2_2, \mu^2)\right] \tan\beta\,,$$

where $\alpha_2 \equiv g^2/4\pi$ and $\alpha_1 \equiv g'^2/4\pi$ are the electroweak gauge couplings. One expects that $|\Delta_\tau| \ll |\Delta_b|$.

Leading corrections to Higgs-fermion couplings

$$h b\bar{b}: \qquad -\frac{m_b}{v}\frac{\sin\alpha}{\cos\beta}\left[1+\frac{1}{1+\Delta_b}\left(\frac{\delta h_b}{h_b}-\Delta_b\right)(1+\cot\alpha\cot\beta)\right]$$

$$H b\bar{b}: \qquad \frac{m_b}{v}\frac{\cos\alpha}{\cos\beta}\left[1+\frac{1}{1+\Delta_b}\left(\frac{\delta h_b}{h_b}-\Delta_b\right)(1-\tan\alpha\cot\beta)\right]$$

$$A b\bar{b}: \qquad \frac{m_t}{v}\frac{\cos\alpha}{\sin\beta}\left[1-\frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}(\cot\beta+\tan\alpha)\right]\gamma_5$$

$$h t\bar{t}: \qquad \frac{m_t}{v}\frac{\cos\alpha}{\sin\beta}\left[1-\frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}(\cot\beta+\tan\alpha)\right]$$

$$H t\bar{t}: \qquad \frac{m_t}{v}\frac{\sin\alpha}{\sin\beta}\left[1-\frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}(\cot\beta-\cot\alpha)\right]$$

$$H t\bar{t}: \qquad \frac{m_t}{v}\cot\beta\left[1-\frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}(\cot\beta+\tan\beta)\right]\gamma_5$$

where $\Delta_b \sim \alpha_s \tan\beta\, f(M_S)$, and $f(M_S)$ is a dimensionless function of supersymmetric masses.

Radiative corrections to $\cos(\beta-\alpha)$ and implications for decoupling

Writing the CP-even Higgs mass matrix:

$$\mathcal{M}^2 = \begin{pmatrix} m_A^2 s_\beta^2 + m_Z^2 c_\beta^2 & -(m_A^2+m_Z^2)s_\beta c_\beta \\ -(m_A^2+m_Z^2)s_\beta c_\beta & m_A^2 c_\beta^2 + m_Z^2 s_\beta^2 \end{pmatrix} + \delta\mathcal{M}^2 \,,$$

and noting that $\delta\mathcal{M}^2_{ij} \sim \mathcal{O}(m_Z^2)$, and $m_H^2 - m_h^2 = m_A^2 + \mathcal{O}(m_Z^2)$, one obtains for $m_A \gg m_Z$

$$\cos(\beta-\alpha) = c\left[\frac{m_Z^2 \sin 4\beta}{2m_A^2} + \mathcal{O}\left(\frac{m_Z^4}{m_A^4}\right)\right] \,,$$

where

$$c \equiv 1 + \frac{\delta\mathcal{M}^2_{11} - \delta\mathcal{M}^2_{22}}{2m_Z^2\cos 2\beta} - \frac{\delta\mathcal{M}^2_{12}}{m_Z^2 \sin 2\beta} \,.$$

These formulae exhibit the expected decoupling behavior for $m_A \gg m_Z$. But, they also allow for $c_{\beta-\alpha} = 0$, independently of the value of m_A, when $c=0$. Assuming large $\tan\beta$, a solution to $c=0$ arises for:

$$\tan\beta \simeq \frac{2m_Z^2 - \delta\mathcal{M}^2_{11} + \delta\mathcal{M}^2_{22}}{\delta\mathcal{M}^2_{12}} \,.$$

The Decoupling Limit Revisited

Working to first order in $\cos(\beta-\alpha)$, and using

$$\tan\alpha\tan\beta = -1+(\tan\beta+\cot\beta)\cos(\beta-\alpha)+\mathcal{O}\left(\cos^2(\beta-\alpha)\right)\,,$$

it follows that

$$g_{hbb} \simeq g_{h_{\rm SM}bb}\Bigg[1+(\tan\beta+\cot\beta)\cos(\beta-\alpha) \times\left(\cos^2\beta-\frac{1+\delta h_b/h_b}{1+\Delta_b}\right)\Bigg]\,.$$

Note that $(\tan\beta+\cot\beta)\cos(\beta-\alpha)\simeq\mathcal{O}(m_Z^2/m_A^2)$, even if $\tan\beta$ is very large (or small). Thus, the deviation from decoupling limit vanishes as m_Z^2/m_A^2 for all values of $\tan\beta$.

Similarly,

$$g_{htt} \simeq g_{h_{\rm SM}tt}\left[1+\cos(\beta-\alpha)\left(\cot\beta-\frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}\frac{1}{\sin^2\beta}\right)\right]\,.$$

The deviation from the decoupling limit is suppressed at large $\tan\beta$.

A SM-Like H

If $\sin(\beta-\alpha)=0$, then $g_{HVV}=g_{h_{\rm SM}VV}$ [$V=W$ or Z]. Expanding about $\sin(\beta-\alpha)=0$ yields:

$$g_{Hbb} \simeq g_{h_{\rm SM}bb}\Bigg[1-(\tan\beta+\cot\beta)\sin(\beta-\alpha) \\ \times\left(\cos^2\beta-\frac{1+\delta h_b/h_b}{1+\Delta_b}\right)\Bigg].$$

In this case, $g_{Hbb}=g_{h_{\rm SM}bb}$ only if $|(\tan\beta+\cot\beta)\sin(\beta-\alpha)|\ll 1$, which need not be satisfied if $\tan\beta$ is very large (or small).

Similarly,

$$g_{Htt}\simeq g_{h_{\rm SM}tt}\left[1-\sin(\beta-\alpha)\left(\cot\beta-\frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}\frac{1}{\sin^2\beta}\right)\right].$$

The parameter region where $\sin(\beta-\alpha)\simeq 0$ corresponds to large $\tan\beta$ and $m_A<(m_h)_{\rm max}$. In this region, H has SM-like couplings to vector bosons and up-type fermions. However, $\tan\beta\sin(\beta-\alpha)$ need not be small in this regime, and so the Hbb couplings deviate from the corresponding SM value.

Summary: the Approach to Decoupling

If we only keep the leading $\tan\beta$-enhanced radiative corrections, then

$$\frac{g^2_{hVV}}{g^2_{h_{\rm SM}VV}} \simeq 1 - \frac{c^2 m_Z^4 \sin^2 4\beta}{4m_A^4}\,,$$

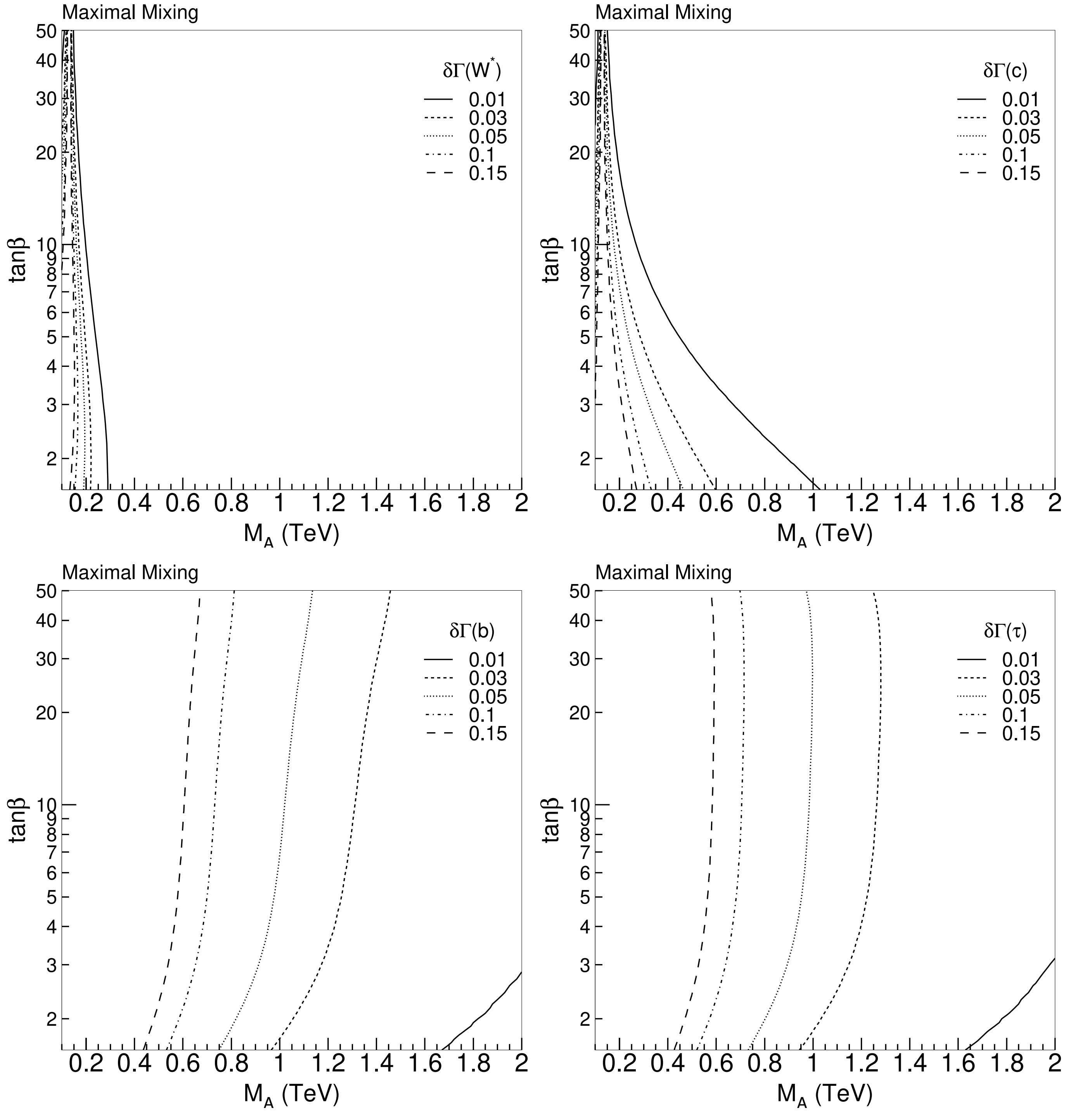
$$\frac{g^2_{htt}}{g^2_{h_{\rm SM}tt}} \simeq 1 - \frac{c m_Z^2 \sin 4\beta \cot\beta}{m_A^2}\,,$$

$$\frac{g^2_{hbb}}{g^2_{h_{\rm SM}bb}} \simeq 1 + \frac{4c m_Z^2 \cos 2\beta}{m_A^2}\left[\cos^2\beta - \frac{1}{1+\Delta_b}\right]\,.$$

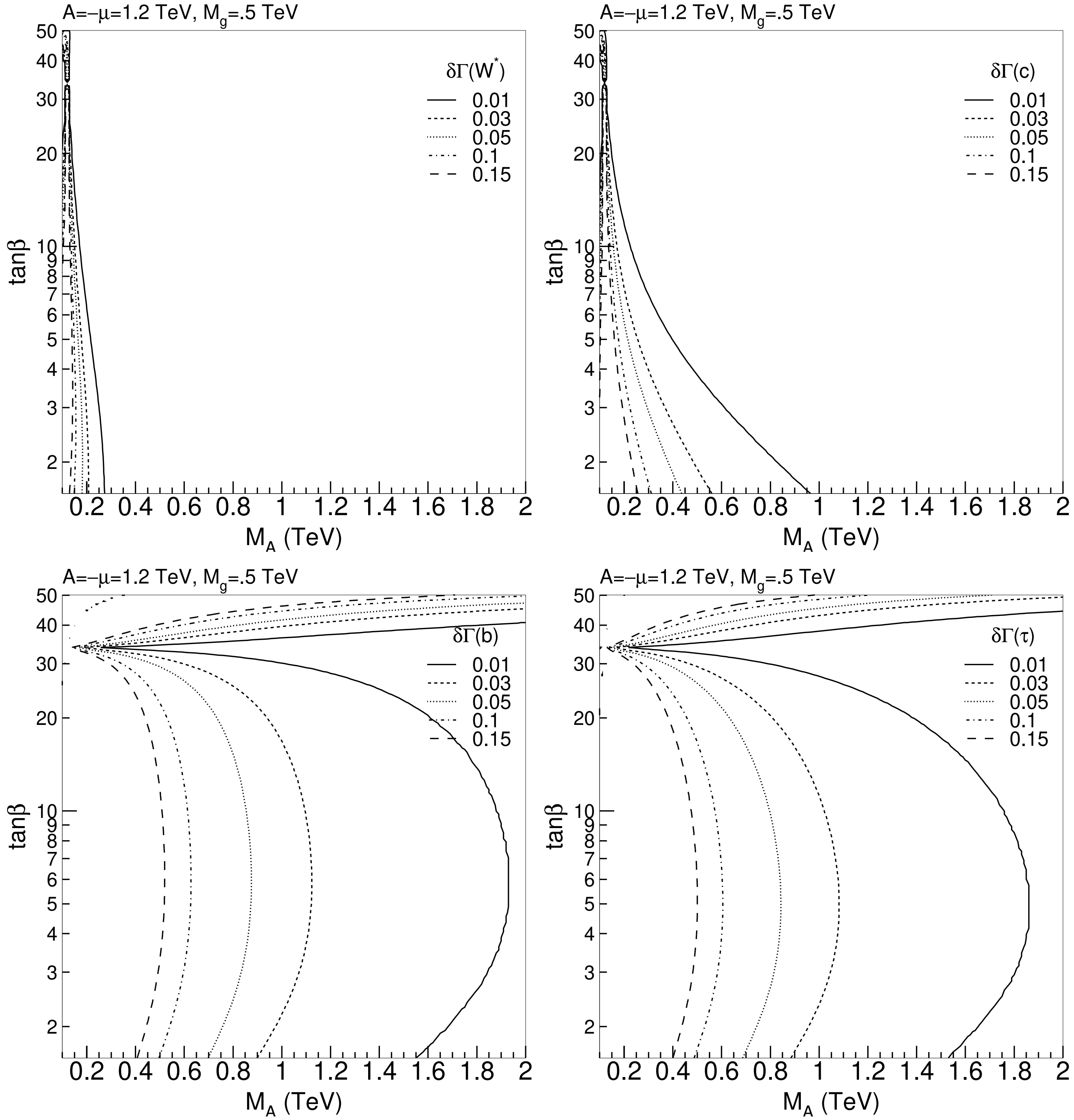
The approach to decoupling is fastest for the h couplings to vector bosons and slowest for the couplings to down-type quarks.

If $c \simeq 0$, which can occur at large $\tan\beta$, then we have a SM-like h, independent of the value of m_A! Sensitivity to m_A through precision measurements of h BRs is lost.

For loop-induced Higgs decays, such as $h \to gg$ or $\gamma\gamma$ there are two decoupling limits of relevance: $m_A \gg m_Z$ and $M_{\rm SUSY} \gg m_Z$. If only the former holds, then squark-loops can generate a small deviation in the loop-induced partial widths from the corresponding SM values.



Deviations of Higgs partial widths from their SM values in the maximal-mixing scenario.

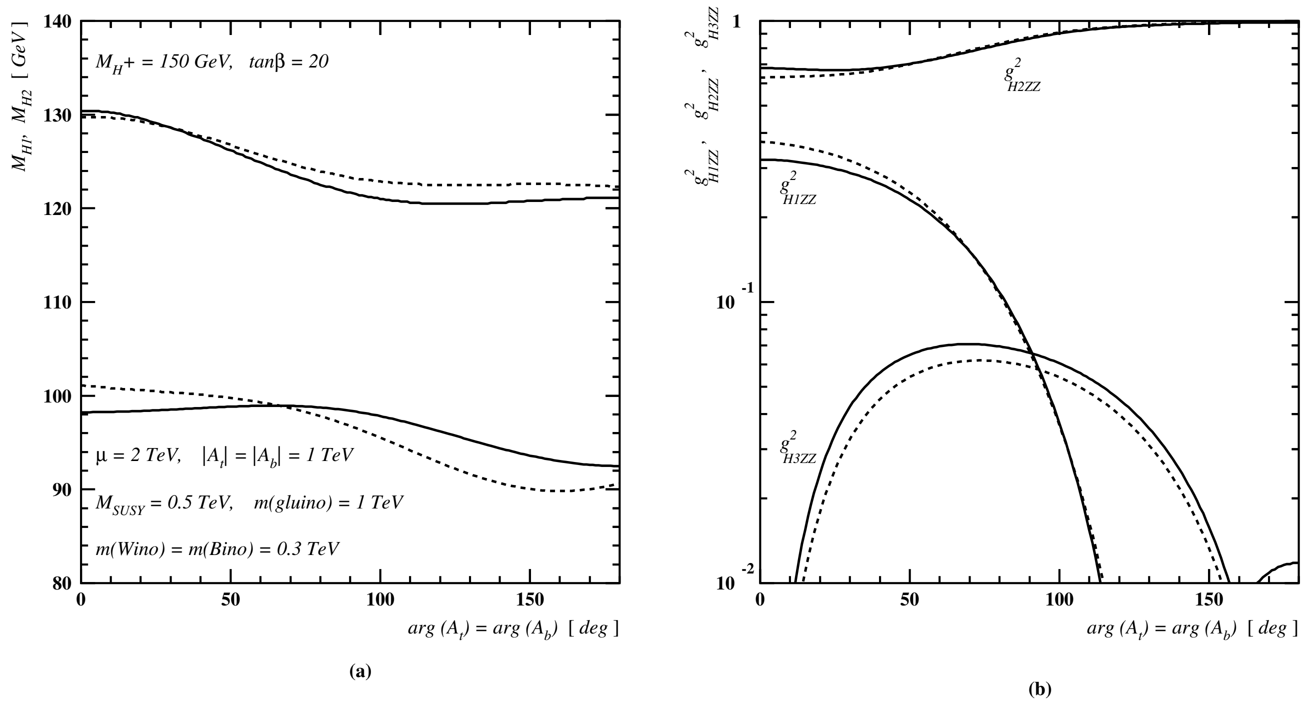


Deviations of Higgs partial widths from their SM values in the large μ and A_t scenario, with $A_t = -\mu = 1.2$ TeV.

CP-violation induced by radiative corrections

CP-violation can enter through the MSSM parameters that control the loop corrections to the Higgs mass matrix and couplings. As a result δh_b, Δh_b, δh_t and δh_t can be complex. Three immediate consequences:

- Mixing of h, H, and A
- Generation of CP-violating Higgs fermion couplings
- Higgs-vector-vector and Higgs-Higgs-vector couplings exist for all combinations of neutral Higgs bosons



(Carena, Ellis, Pilaftsis, Wagner)

When CP-violation effects are included, we denote the three neutral Higgs bosons (of indefinite CP) by H_1, H_2 and H_3, where H_1 is the lightest neutral Higgs boson. Explicitly,

$$
\begin{aligned}
H_i = (\sqrt{2}\,\mathrm{Re}\,\Phi_d^0 - v_d)O_{1i} + (\sqrt{2}\,\mathrm{Re}\,\Phi_u^0 - v_u)O_{2i} + \\
\sqrt{2}\left(\mathrm{Im}\,\Phi_d^0 \sin\beta + \mathrm{Im}\,\Phi_u^0 \cos\beta\right) O_{3i}\,,
\end{aligned}
$$

where O is a 3×3 real orthogonal matrix. In this more general case, $\sin\alpha$ and $\cos\alpha$ of the CP-conserving theory are replaced by the matrix elements of O. All the results of this talk can be generalized to the CP-violating case.

For example, the decoupling limit is governed by the condition $m_{H^\pm} \gg m_W$. In this limit, one can show that

- The properties of H_1 are identical to those of the SM Higgs boson. That is, H_1 is a pure CP-even state, up to corrections of $\mathcal{O}(m_W^2/m_{H^\pm}^2)$.
- There can still be large mixing between H and A (*i.e.*, H_2 and H_3 can have large admixtures of CP-even and odd components). However, H_2 and H_3 will be nearly mass-degenerate.

Explicit results

Higgs coupings to gauge bosons:

$$\begin{aligned} g_{H_iVV} &= O_{1i}\cos\beta + O_{2i}\sin\beta\,, \\ g_{H_iH^-W^+} &= \tfrac{1}{2}g\,[O_{1i}\sin\beta - O_{2i}\cos\beta - iO_{3i}]\ , \\ g_{H_iH_jZ} &= \frac{g}{2\cos\theta_W}\,[(O_{3i}O_{1j} - O_{1i}O_{1j})\sin\beta \\ &\qquad -(O_{3i}O_{2j} - O_{2i}O_{3j})\cos\beta]\ . \end{aligned}$$

From the orthogonality of O (and $\det\,O = 1$), it follows that

$$g_{H_iH_jZ} = \frac{m_Z^2}{2m_V^2}\,\epsilon_{ijk}\,g_{H_kVV}\ ,$$

where $V = W$ or Z.

Higgs couplings to fermion pairs:

$$\mathcal{L}_{H\bar{f}f} = -\sum_{i=1}^{3} H_i\left[\frac{m_b}{v}\,\bar{b}\left(g^S_{H_ibb} + ig^P_{H_ibb}\gamma_5\right)b + \frac{m_t}{v}\,\bar{t}\left(g^S_{H_itt} + ig^P_{H_itt}\gamma_5\right)t\right],$$

with

$$g^S_{H_ibb} = \frac{1}{h_b + \delta h_b + \Delta h_b\tan\beta}\left\{\mathrm{Re}(h_b+\delta h_b)\,\frac{O_{1i}}{\cos\beta} + \mathrm{Re}(\Delta h_b)\,\frac{O_{2i}}{\cos\beta}\right.$$
$$\left. - \Big[\mathrm{Im}(h_b+\delta h_b)\tan\beta - \mathrm{Im}(\Delta h_b)\Big]\,O_{3i}\right\},$$

$$g^{P}_{H_i bb} = \frac{1}{h_b\,+\,\delta h_b\,+\,\Delta h_b \tan\beta}\,\Bigg\{\,\Big[\mathrm{Re}\,(\Delta h_b)\;-\;\mathrm{Re}(h_b+\delta h_b)\tan\beta\Big]\,O_{3i}$$

$$-\,\mathrm{Im}(h_b+\delta h_b)\,\frac{O_{1i}}{\cos\beta}\;-\;\mathrm{Im}(\Delta h_b)\,\frac{O_{2i}}{\cos\beta}\,\Bigg\}\,,$$

$$g^{S}_{H_i tt} = \frac{1}{h_t\,+\,\delta h_t\,+\,\Delta h_t \cot\beta}\,\Bigg\{\mathrm{Re}(h_t+\delta h_t)\,\frac{O_{2i}}{\sin\beta}+\;\mathrm{Re}(\Delta h_t)\,\frac{O_{1i}}{\sin\beta}$$

$$-\Big[\mathrm{Im}(h_t+\delta h_t)\cot\beta\;-\;\mathrm{Im}(\Delta h_t)\Big]\,O_{3i}\Bigg\}\,,$$

$$g^{P}_{H_i tt} = \frac{1}{h_t\,+\,\delta h_t\,+\,\Delta h_t \cot\beta}\,\Bigg\{\,\Big[\mathrm{Re}(\Delta h_t)\;-\;\mathrm{Re}(h_t+\delta h_t)\cot\beta\Big]\,O_{3i}$$

$$-\,\mathrm{Im}(h_t+\delta h_t)\,\frac{O_{2i}}{\sin\beta}\;-\;\mathrm{Im}(\Delta h_t)\,\frac{O_{1i}}{\sin\beta}\,\Bigg\}\,.$$

Higgs scalar couplings are normalized with respect to the corresponding SM values. The phases of the fields have been chosen so that the quantities $h_b + \delta h_b + \Delta h_b \tan\beta$ and $h_t + \delta h_t + \Delta h_t \cot\beta$ are both real and positive.

With the third caveat mentioned previously, the couplings of the charged Higgs bosons to fermions are of the form $\mathcal{L}_{\rm int} = \bar{b}\, g_{H^- t\bar{b}}\, tH^- + \mathrm{h.c.}$, with

$$g_{H^- t\bar{b}} \simeq \left(\frac{\sqrt{2}\,m_b}{v}\tan\beta - \frac{\Delta h_b}{\cos\beta}\right)P_L + \left(\frac{\sqrt{2}\,m_t}{v}\cot\beta - \frac{(\Delta h_t)^*}{\sin\beta}\right)P_R\,.$$

Precision Higgs Physics at the LC

A robust program of precision Higgs physics requires a lepton collider.

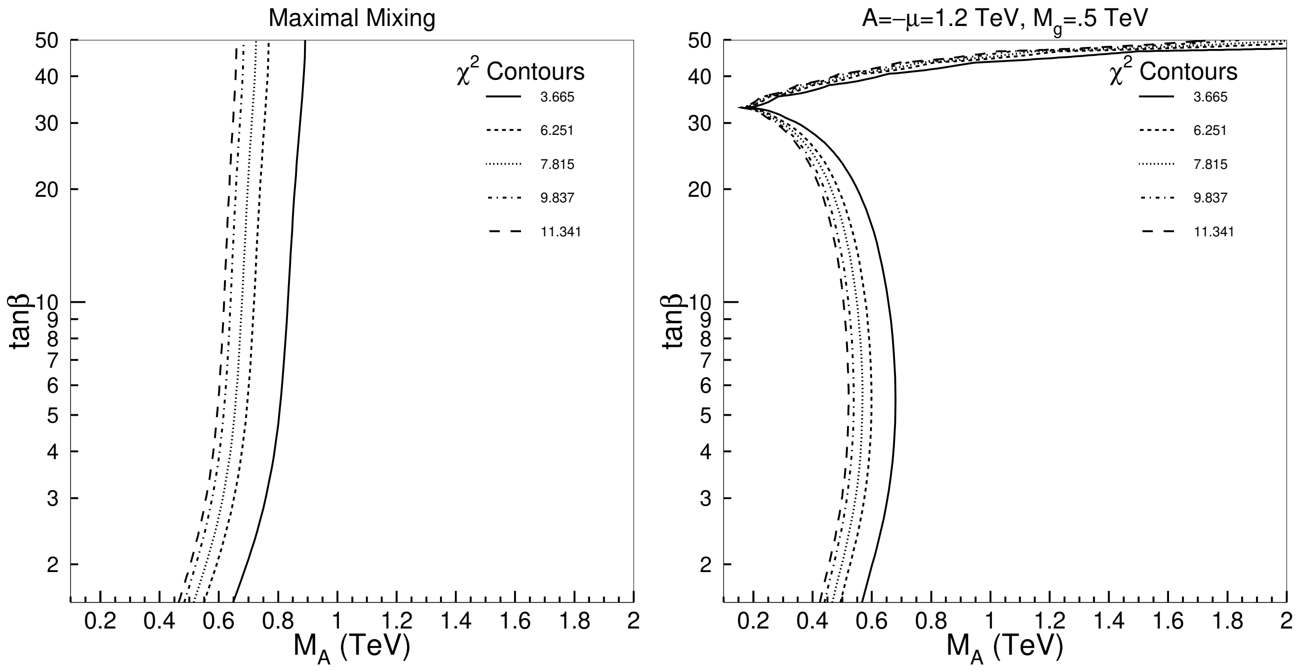
Higgs couplings for $m_{h_{\rm SM}} = 120$ GeV at $\sqrt{s} = 500$ GeV [800 GeV for $h_{\rm SM}t\bar{t}$] can be extracted (Battaglia and Desch)

Higgs coupling	δBR/BR	$\delta g/g$	$\mathcal{L}$ (fb^{-1})
$h_{\rm SM}WW$	5.1%	1.2%	500
$h_{\rm SM}ZZ$	—	1.2%	500
$h_{\rm SM}t\bar{t}$	—	2.2%	1000
$h_{\rm SM}b\bar{b}$	2.4%	2.1%	500
$h_{\rm SM}c\bar{c}$	8.3%	3.1%	500
$h_{\rm SM}\tau\tau$	5.0%	3.2%	500
$h_{\rm SM}gg$	5.5%		500
$h_{\rm SM}\gamma\gamma$	16%		1000

Beware: theoretical uncertainty in $h_{\rm SM}c\bar{c}$ [$h_{\rm SM}b\bar{b}$] coupling due to uncertainty in m_c [m_b] and α_s is about 12% [1.8%].

Implications of Precision Higgs Measurements at the LC

In the decoupling regime, one can measure the h branching ratios and couplings, and test for deviations of the h couplings from SM predictions. The size of the deviation measures the departure from decoupling. If no deviation is seen, then one can deduce a bound on m_A, which depends on the choice of MSSM parameters.



Contours of χ^2 for Higgs boson decay observables. The contours correspond to 68, 90, 95, 98 and 99% CLs (right to left) for the three observables g^2_{hbb}, $g^2_{h\tau\tau}$, and g^2_{hgg}. Note the region of m_A-independent "decoupling" in the right panel. (Carena, Haber, Logan and Mrenna)

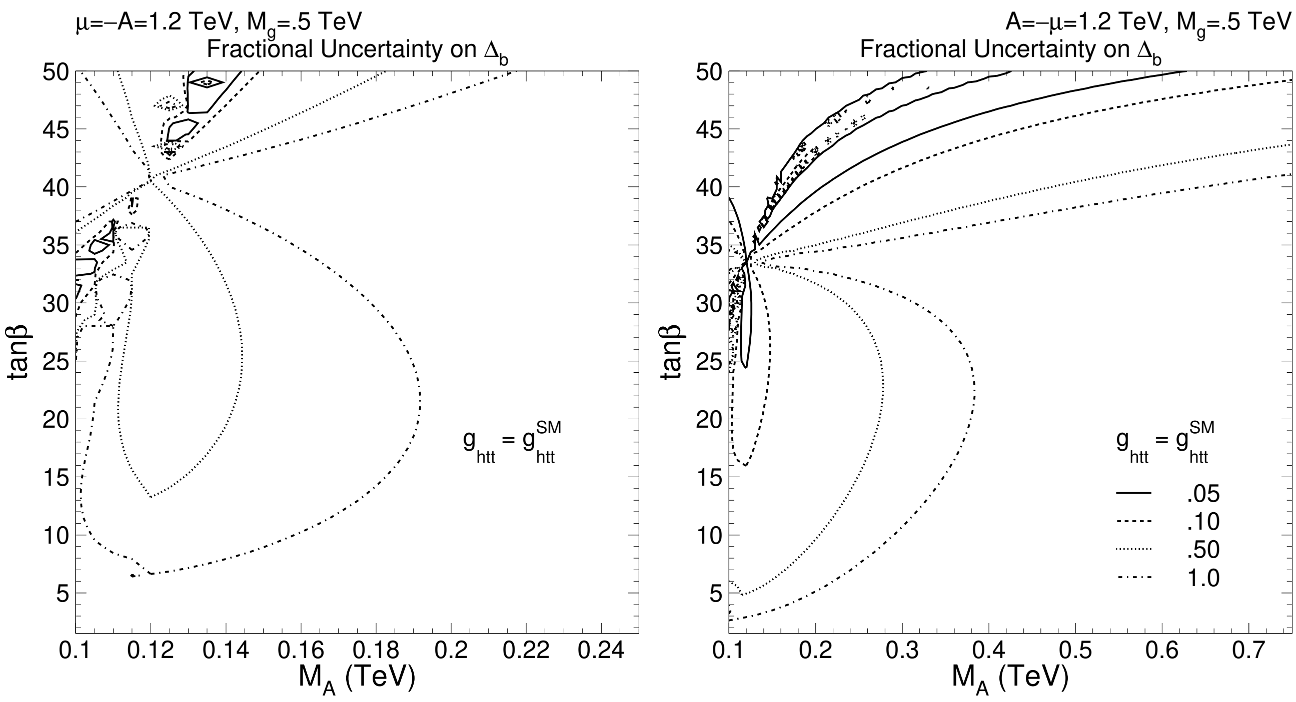
Extracting the SUSY parameter Δ_b

Consider the ratio of couplings

$$\frac{\hat{g}_{hbb}-\hat{g}_{h\tau\tau}}{\hat{g}_{htt}-\hat{g}_{hbb}} = \frac{\hat{g}_{Hbb}-\hat{g}_{H\tau\tau}}{\hat{g}_{Htt}-\hat{g}_{hbb}} \simeq \frac{\dfrac{\Delta_b-\Delta_\tau}{1+\Delta_\tau} - \dfrac{\delta h_b}{h_b} + \left(\dfrac{1+\Delta_b}{1+\Delta_\tau}\right)\dfrac{\delta h_\tau}{h_\tau}}{1-\left(\dfrac{1+\Delta_b}{1+\Delta_t}\right)\dfrac{\Delta h_t}{h_t}\cot\beta + \dfrac{\delta h_b}{h_b}}\,,$$

where $\hat{g}_{\phi ff} \equiv g_{\phi ff}/g_{h_{SM}ff}$ $[\phi \equiv h,\, H]$. If desired, $t\bar{t}$ couplings can be replaced by $c\bar{c}$ couplings. Keeping only the leading $\tan\beta$-enhanced one-loop terms, and assuming $|\Delta_\tau| \ll 1$,

$$\frac{\hat{g}_{hbb}-\hat{g}_{h\tau\tau}}{\hat{g}_{htt}-\hat{g}_{hbb}} = \frac{\hat{g}_{Hbb}-\hat{g}_{H\tau\tau}}{\hat{g}_{Htt}-\hat{g}_{hbb}} \simeq \frac{\Delta_b-\Delta_\tau}{1+\Delta_\tau} \simeq \Delta_b\,,$$



Conclusions

• The decoupling limit presents a severe challenge for future Higgs studies. It is therefore essential to find evidence for departures from Standard Model Higgs predictions. Such departures will reveal crucial information about the nature of the electroweak symmetry breaking dynamics.

• In the MSSM, deviations from the decoupling limit provide useful information about the non-minimal Higgs sector and can yield indirect information about the MSSM parameters. At large $\tan\beta$, there can be additional sensitivity to MSSM parameters via enhanced radiative corrections.

• There are some regions of MSSM parameter space at large $\tan\beta$ where sensitivity to m_A is particularly weak, even if m_A is not too large. This is an example of a case in which $h \simeq h_{\rm SM}$ *outside* the decoupling limit.

• Precision Higgs measurements can provide critical tests of the supersymmetric interpretation of new physics beyond the Standard Model.