Fermionic decays of sfermions in the MSSM: a full one-loop calculation

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Phys. Lett. **B510** (2001) 211, `hep-ph/0101086`; `hep-ph/0207364`

- Introduction and Motivation
- Renormalization
- Computation
- Results
- Conclusions and outlook

Fermionic decays of sfermions

- Check of SUSY: equality of particles and sparticles couplings
  - Can not be checked on S-channel pair-production
- In this work we concentrate on fermionic decays of sfermions

$$\boxed{\tilde{f} \rightarrow f^{(\prime)}\chi^{(0,+,-)}}$$

- Some of these decays channels are *always* available: $\tilde{b} \rightarrow b\chi^0_1$.

  χ^0_1 is usually the Lightest SUSY Particle (LSP).
- Whenever open, some of these channels have a non-negligible Branching Ratio.
- Previous computations:
  - QCD corrections

S. Kraml *et al.*, *Phys. Lett.* **B386** (1996) 175, `hep-ph/9605412`.
A. Djouadi, W. Hollik, C. Jünger, *Phys. Rev.* **D55** (1 997) 6975, `hep-ph/9609419`.
J. Guasch, Ph.D. Thesis, UAB 1999.

  - Yukawa corrections (higgsino approximation).

J. Guasch, W. Hollik, J. Solà, *Phys. Lett.* **B437** (1998) 88, `hep-ph/9802329`.

Large corrections were found. Are they "real"?

- We present the Full Electroweak one-loop quantum corrections to $\Gamma(\tilde{f} \rightarrow f^{(\prime)}\chi^{(0,+,-)})$.
- In the Minimal Supersymmetric Standard Model (MSSM).

$\boxed{\text{Renormalization}}$

- It is **NOT** possible to separate between SM-like and non-SM-like corrections

  $\Rightarrow$ Almost the Full MSSM must be renormalized.

- We use an On-Shell renormalization scheme:
  - Usual On-Shell scheme for SM particles.
  - $\tan\beta \Rightarrow \frac{\delta v_2}{v_2} = \frac{\delta v_1}{v_1}$

Sfermion sector

$$\mathcal{M}^2_{\tilde{q}} = \begin{pmatrix} M^2_{\tilde{q}_L} + m_q^2 + c_{2\beta}(T_3^{q_L} - Q_q s_W^2) M_Z^2 & m_q M^q_{LR} \\ m_q M^q_{LR} & M^2_{\tilde{q}_R} + m_q^2 + c_{2\beta} Q_q s_W^2 M_Z^2 \end{pmatrix},$$

$$M^t_{LR} = A_t - \mu \cot\beta \;,\; M^b_{LR} = A_b - \mu \tan\beta \,.$$

$$\tilde{f}^{weak} = R \tilde{f}^{mass} \;,\; R = \begin{pmatrix} C_\theta & -S_\theta \\ S_\theta & C_\theta \end{pmatrix}$$

- 5 (3) input parameters for $\tilde{q}$ ($\tilde{l}$) doublet

$$M_{\tilde{q}_L} \,,\, M_{\tilde{b}_R} \,,\, M_{\tilde{t}_R} \,,\, A_b \,,\, A_t$$

$\Rightarrow$ $m_{\tilde{b}_1} \,,\, m_{\tilde{b}_2} \,,\, m_{\tilde{t}_2} \,,\, \theta_{\tilde{b}} \,,\, \theta_{\tilde{t}} \quad (m_1 > m_2)$

- $$\left.\begin{aligned} A_b &= \mu \tan\beta + \frac{m^2_{\tilde{b}_2} - m^2_{\tilde{b}_1}}{2 m_b} \sin 2\theta_{\tilde{b}} \\ A_t &= \mu \cot\beta + \frac{m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1}}{2 m_t} \sin 2\theta_{\tilde{t}} \end{aligned}\right\} A_f \lesssim 3 m_{\tilde{q}} \text{ (Colour breaking)}$$
- $\delta\theta_{\tilde{q}} = \frac{1}{2} \frac{\Sigma^{12}(m_{\tilde{q}_2}) + \Sigma^{21}(m_{\tilde{q}_1})}{m^2_{\tilde{q}_2} - m^2_{\tilde{q}_1}}$
- $\Delta m^2_{\tilde{t}_1} = \delta m^2_{\tilde{t}_1} + \Sigma_{\tilde{t}_1}(m^2_{\tilde{t}_1}) \neq 0$

Chargino-Neutralino sector

$$\mathcal{M} = \begin{pmatrix} M & \sqrt{2}M_W s_\beta \\ \sqrt{2}M_W c_\beta & \mu \end{pmatrix} ,$$

$$\mathcal{M}^0 = \begin{pmatrix} M' & 0 & M_Z c_\beta s_W & -M_z s_\beta s_W \\ 0 & M & -M_Z c_\beta c_W & M_Z s_\beta c_W \\ M_Z c_\beta s_W & -M_Z c_\beta c_W & 0 & -\mu \\ -M_Z s_\beta s_W & M_Z s_\beta c_W & -\mu & 0 \end{pmatrix} .$$

- 3 parameters (M, M', μ) and 6 particles.

  $\Rightarrow$ It is not possible to build on-shell conditions for all of them.

  1. Choose 3 "input" particles $(\chi^{\pm}_{1,2}, \chi^0_1)$
  2. Ask for on-shell conditions for them
  3. Invert the relations, and find counterterms for the original parameters (M, M', μ).
  4. Compute the mass-counterterms for the other particles *and* the mixing self-energies.

  – it is non-trivial to check the finiteness of the computed masses.

- Ex: Chargino sector

  M_1, M_2: Chargino Masses

$$\boxed{\begin{aligned} M_1\,\delta M_1 + M_2\,\delta M_2 &= M\,\delta M + \mu\,\delta\mu + \delta M_W^2 \\ M_1 M_2\,(M_1\,\delta M_2 + M_2\,\delta M_1) &= \left(M\mu - M_W^2\sin(2\beta)\right)\left[M\,\delta\mu + \mu\,\delta M \right. \\ &\quad \left. - M_W^2\,\delta\sin(2\beta) - \sin(2\beta)\,\delta M_W^2\right] \end{aligned}}$$

On Shell condition:

$$\delta M_i = -\frac{1}{2}\left(\Sigma_L^i(M_i^2) + \Sigma_R^i(M_i^2)\right) - \Sigma_S^i(M_i^2) ,$$

Other Approaches:

H. Eberl, M. Kincel, W. Majerotto, Y. Yamada , hep-ph/0104109, Phys.Rev.D64:115013,2001

W. Hollik, T. Fritzsche, hep-ph/0203159

Computation

- Three kind of terms contribute:
  - Counterterms
  - one-particle-irreducible three-point-functions
  - Mixing self-energies $\chi_i - \chi_j$
- Infrared divergent:
  - Addition of the soft- and hard- photon bremsstrahlung:

$$\Gamma(\tilde{q} \to q^{(\prime)} \chi^{(0,+,-)} \gamma)$$

- Loops computation: Heavy use of Computer Algebra Systems:
  - FeynArts (Creation of Amplitudes)
  - FormCalc (One-Loop integration and simplification of expressions)
  - LoopTools (Numerical Evaluation of one-loop integrals)

  T. Hahn, `http://www.feynarts.de`
- Checks:
  - The finiteness of the result is a non-trivial check.
  - Reproduce the one-loop chargino-neutralino masses ($\overline{\text{MS}}$-bar scheme)

  D. Pierce, A. Papadopoulos, *Phys. Rev.* **D50** (1994) 565, `hep-ph/9312248`; *Nucl. Phys.* **B430** (1994) 278, `hep-ph/9403240`.
  - Recover the results of the Yukawa approximation.
  - With small modifications, we can reproduce the QCD corrections.

Non-decoupling effects

- Non-decoupling of *gauginos* is known

A. Djouadi, W. Hollik, C. Jünger, *Phys. Rev.* **D55** (1997) 6975, hep-ph/9609419

Sfermions not in the external states do not decouple

$$\left.\begin{array}{l} \sim \Delta_\varepsilon - \log\left(\frac{m_l}{\mu_\varepsilon}\right) \\ \\ \sim \Delta_\varepsilon - \log\left(\frac{M_{\tilde{l}}}{\mu_\varepsilon}\right) \end{array}\right\} \Rightarrow \delta g \ , \ \delta M_V^2$$

$$\sim \Delta_\varepsilon - \log\left(\frac{M_{\tilde{l}}}{\mu_\varepsilon}\right) \Rightarrow \delta Z_i \ , \ \delta\mu, \ \delta M \ \ldots \ \Sigma_{ij}$$

- The Δ_ε must cancel between the VV and $\chi\chi$ diagrams
  $\Rightarrow$ A term $\log\left(\frac{M_{\tilde{l}}}{m_l}\right)$ appears!!

Effective coupling matrices

$$\tilde{U} = U + \Delta U \quad , \quad \tilde{V} = V + \Delta V \quad , \quad \tilde{N} = N + \Delta N \quad ,$$

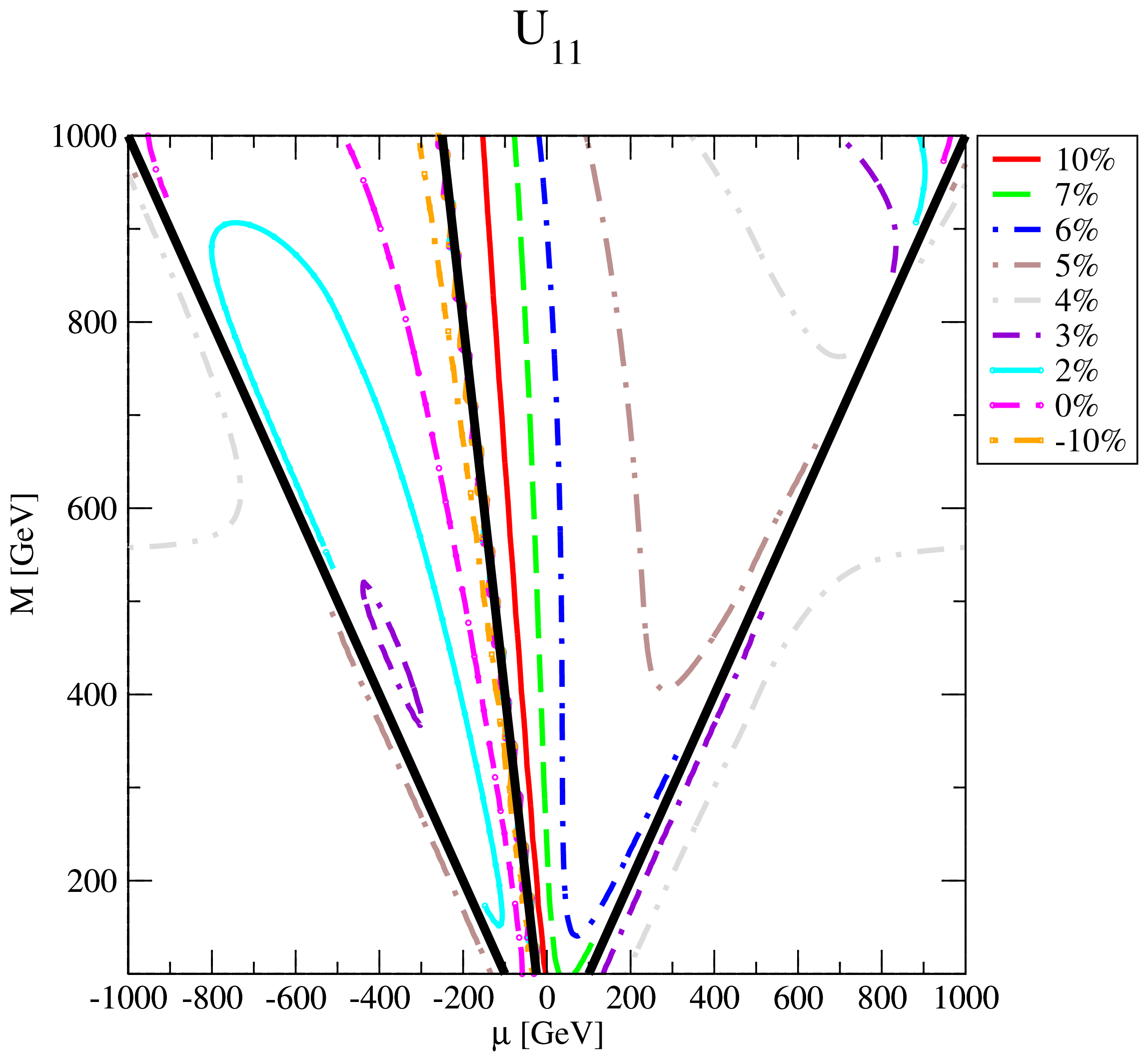
$$\begin{aligned}
\Delta U_{i1} &\equiv U_{i1}\left(\frac{\delta g}{g} + \frac{\delta Z_L^{+\,i}}{2}\right) + U_{j1} Z_R^{-\,ji}\,, \\
\Delta U_{i2} &\equiv U_{i2}\left(\frac{\delta g}{g} + \frac{\delta Z_L^{+\,i}}{2} - \frac{1}{2}\frac{\delta M_W^2}{M_W^2} - \frac{\delta \cos\beta}{\cos\beta}\right) + U_{j2} Z_L^{+\,ji}\,,
\end{aligned}$$

- Only the fermion-sfermion contributions are finite
- Universal Corrections (*Super-oblique*)

  Hikasa, Nakamura '96; Katz, Randall, Su '98
- limit $m_e \ll M_Z \ll M_{\tilde{e}}$, heavy masses log terms:

$$\begin{aligned}
\Delta U_{i1}^{(f)} &= \frac{\alpha}{4\pi s_W^2} \log\left(\frac{M_{\tilde{L}}^2}{M_W^2}\right) [\frac{U_{i1}^3}{6} - \\
& U_{i2} \frac{\sqrt{2} M_W \left(M c_\beta + \mu s_\beta\right)}{3\left(M^2 - \mu^2\right)\left(M_1^2 - M_2^2\right)^2} \left(M^4 - M^2 \mu^2 + 3 M^2 M_W^2 + \right. \\
+ & \left. \mu^2 M_W^2 + M_W^4 + M_W^4 c_{4\beta} + (\mu^2 - M^2) M_i^2 + 4 M \mu M_W^2 s_{2\beta}\right)] \\
\Delta U_{i2}^{(f)} &= \frac{\alpha}{4\pi s_W^2} \log\left(\frac{M_{\tilde{L}}^2}{M_W^2}\right) U_{i1} \frac{M_W \left(M c_\beta + \mu s_\beta\right)}{3\sqrt{2}\left(M^2 - \mu^2\right)\left(M_1^2 - M_2^2\right)^2} \times \\
\times & \left((M^2 - \mu^2)^2 + 4 M^2 M_W^2 + 4 \mu^2 M_W^2 + 2 M_W^4 + 2 M_W^4 c_{4\beta} \right. \\
& \left. + 8 M \mu M_W^2 s_{2\beta}\right)\,,
\end{aligned}$$

- fermion-sfermion contributions to $\Delta U_{11}/U_{11}$, $M_{\tilde{f}} = 1\,TeV$



- Mismatch between
  - U-matrix that diagonalizes the chargino
  - U'-matrix that couples fermion-sfermion-chargino
- Finite shift between
  - Charge in the compton limit ($e^+e^-\gamma$)
  - Yukawa coupling of fermion-sfermion-fotino ($e^+\tilde{e}^-\tilde{\gamma}$)

Renormalization Group: SUSY-QED

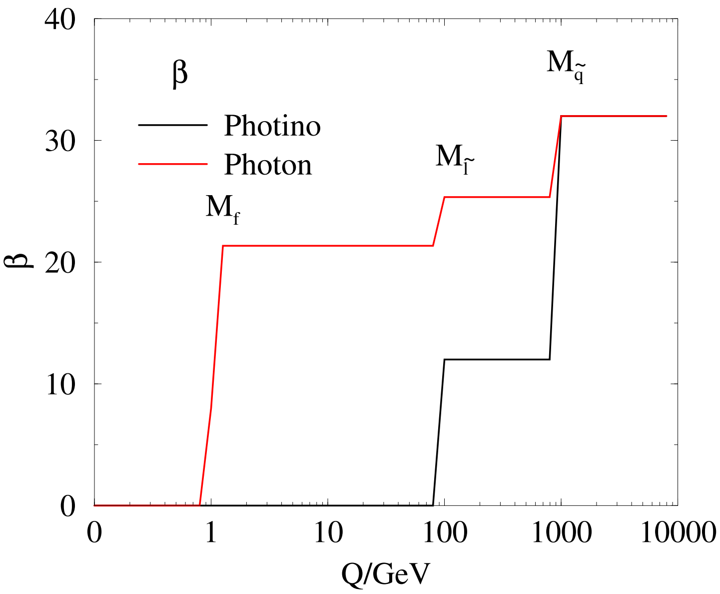
$$\boxed{e^+e^-\gamma \equiv \alpha \;\;, \;\; e^+\tilde{e}^-\tilde{\gamma} \equiv \tilde{\alpha}}$$

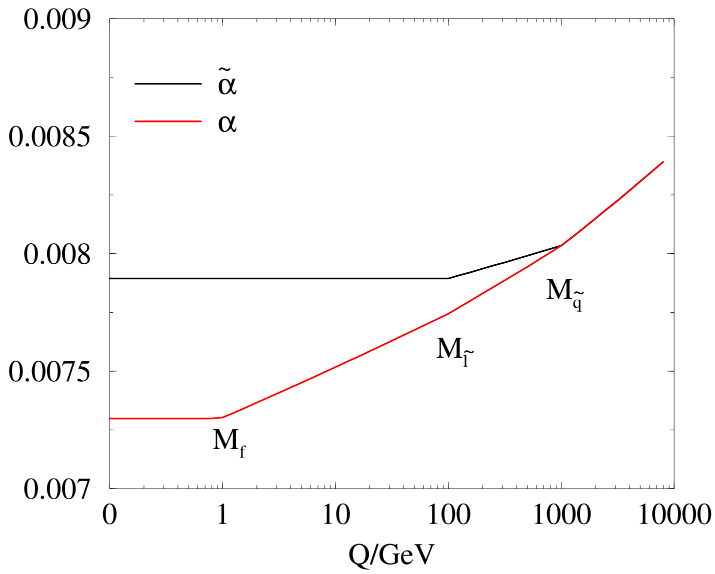
- Assume: $M_{\tilde{l}} < M_{\tilde{q}}$, take only q-$\tilde{q}$ loops into account
  - $Q > M_{\tilde{q}} \Longrightarrow \tilde{\alpha}(Q) = \alpha(Q)$
  - $Q = M_{\tilde{q}}$:
    - $\tilde{q}$ decouple from α
    - $\tilde{q}$ and q decouple from $\tilde{\alpha}$

    $\Rightarrow$ $\tilde{\alpha}$ is frozen at $M_{\tilde{q}}$
  - $Q' < M_{\tilde{q}} \Longrightarrow \tilde{\alpha}(Q') = \alpha(M_{\tilde{q}})$

    $\Rightarrow \dfrac{\tilde{\alpha}(Q')}{\alpha(Q')} - 1 = \dfrac{\alpha(M_{\tilde{q}})}{\alpha(Q')} - 1 \propto \log\left(\dfrac{M_{\tilde{q}}}{Q'}\right)$
- On-Shell renormalization:

$$\frac{\tilde{\alpha}(Q')}{\alpha(M_{Weak})} - 1 \propto \log\left(\frac{M_{\tilde{q}}}{M_{Weak}}\right)$$





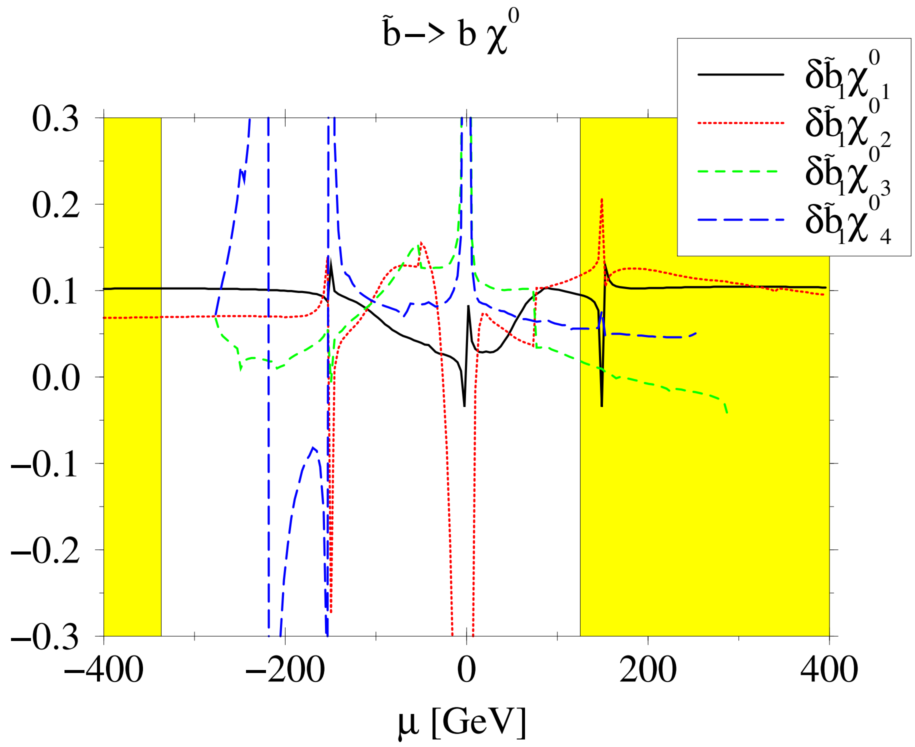
Results

- Universal Corrections: fermion-sfermion contributions to gauge-bosons, Higgs-bosons and chargino-neutralino self-energies.
- Non-Universal Corrections: Rest of one-loop diagrams contributions (3-point 1PI vertices, self-energies with gauge/Higgs/chargino/neutralino)
- Phase space: Change in the phase space factor due to the corrections to the sfermion and neutralino masses.

$$\delta \tilde{f}_i \chi^0 = \frac{\sum_\alpha \left[\Gamma^{1-loop}(\tilde{f}_i \to f \chi^0_\alpha) - \Gamma^{tree}(\tilde{f}_i \to f \chi^0_\alpha) \right]}{\sum_\alpha \Gamma^{tree}(\tilde{f}_i \to f \chi^0_\alpha)}$$

Rich structure

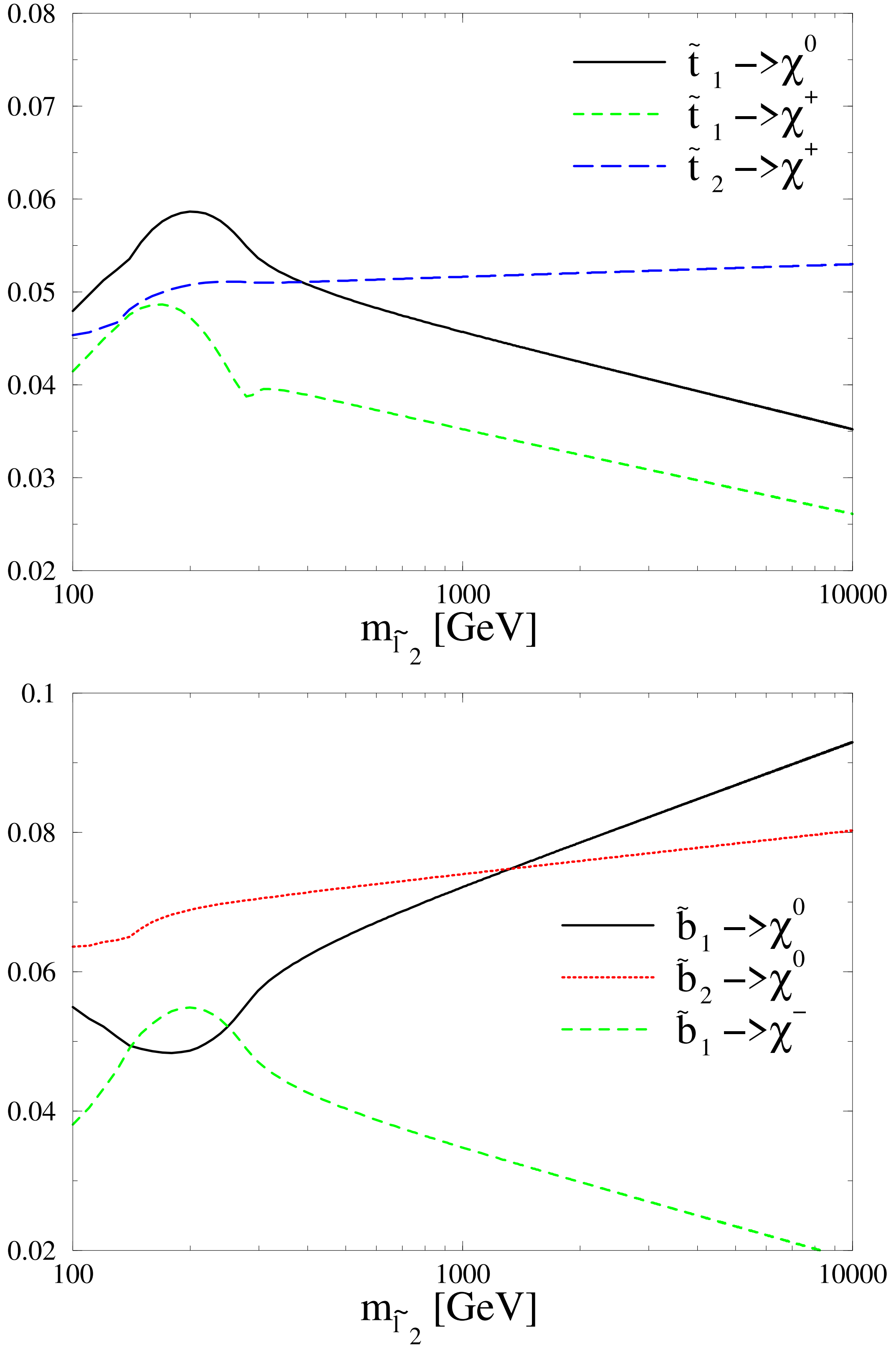
- μ determines:
  - $\chi^{\pm}$ and χ^0 mass.
  - $\tilde{t}_2$ mass.
  - the soft-SUSY-breaking trilinear couplings A_t,A_b.
  - directly the $\tilde{q} - \tilde{q}' - H$ coupling.
  - Higgs masses (Rad. Corr.) ...



$\tan\beta = 4\ ,\ M = 150\,\mathrm{GeV}, m_{\tilde{b}} = (300, 310)\,\mathrm{GeV}$

$\Rightarrow$ The value of the corrections is extremelly dependent on the correlation between the different parameters of the model!!

Universal Corrections

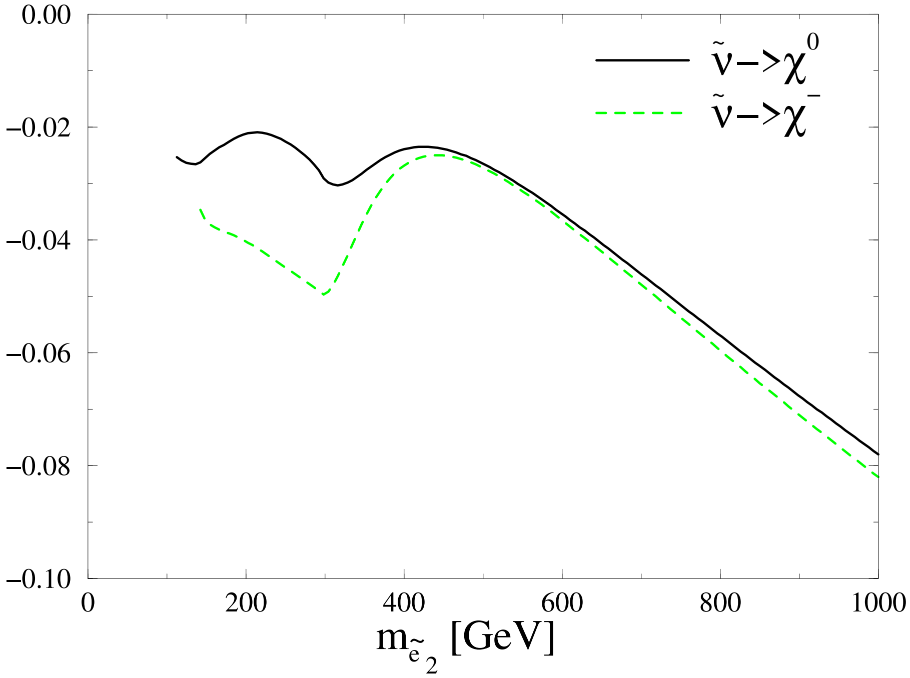


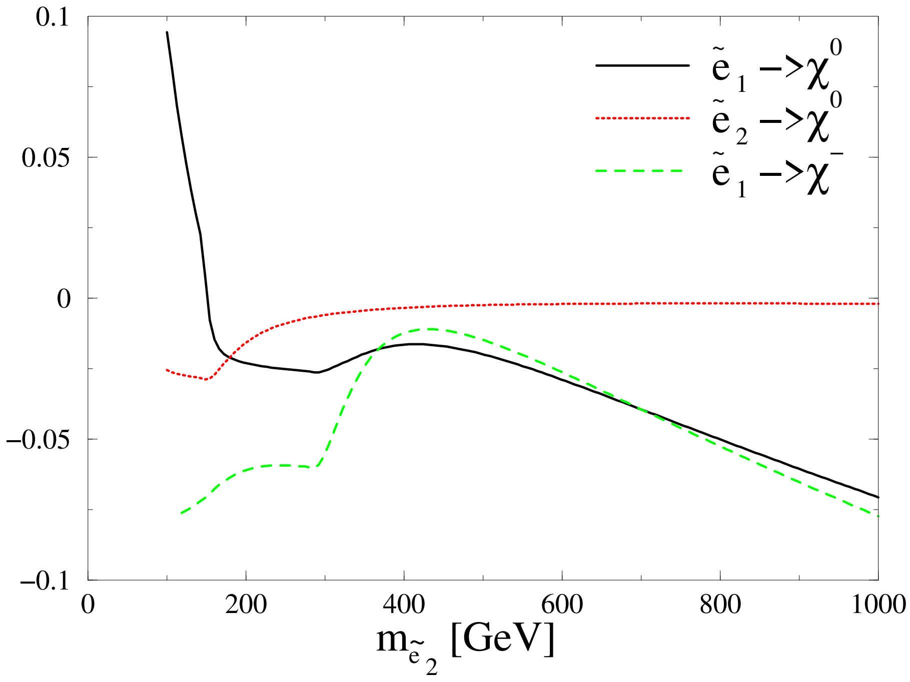
Non-Universal Electroweak Corrections

- Sudakov-type double logs:

$$\delta \sim A + B \log^2\left(\frac{S}{M_W^2}\right)$$

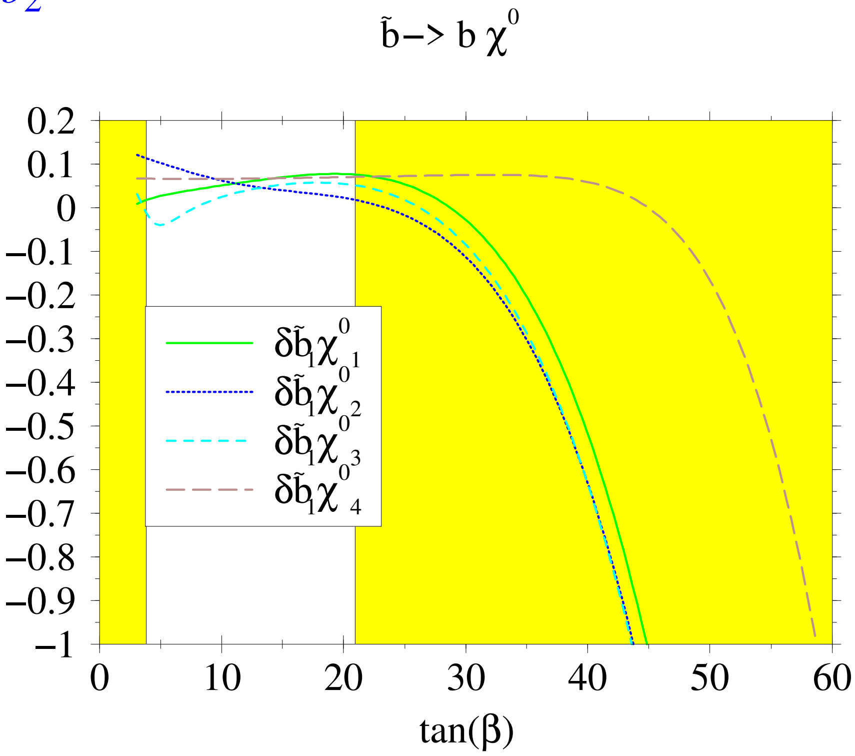
P. Ciafaloni, D. Comelli, *Phys. Lett.* **B 446** (1999) 278, `hep-ph/9809321`
V. S. Fadin, et al. *Phys. Rev.* **D61** (2000) 094002, `hep-ph/9910338`;
M. Beccaria, et al., `hep-ph/0112273`.





Problems with perturbativity ?

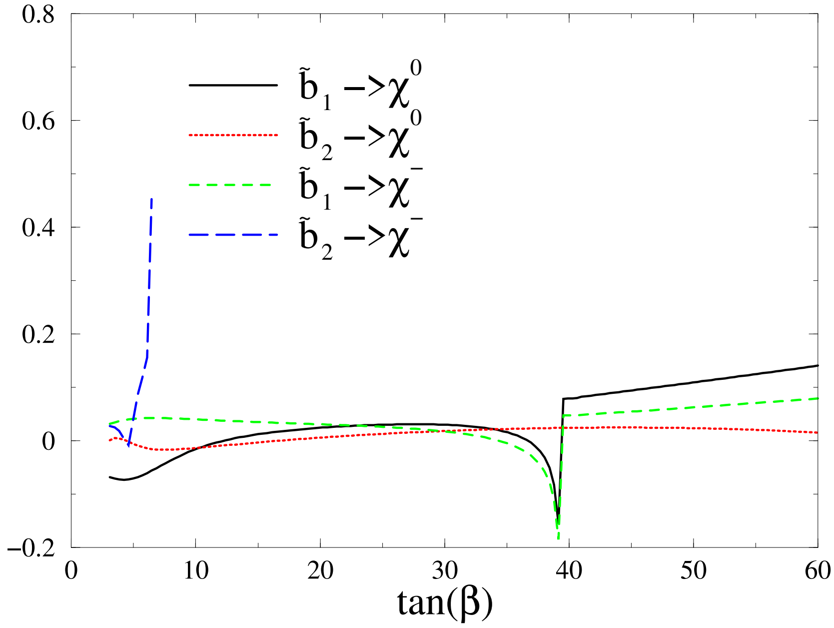
$m_{\tilde{b}_1} - m_{\tilde{b}_2} = 5\,\mathrm{GeV}$:



- No problems if A_q is maintained fixed

  $\Rightarrow$ Use Soft-SUSY-breaking parameters as input!

A_b fixed:



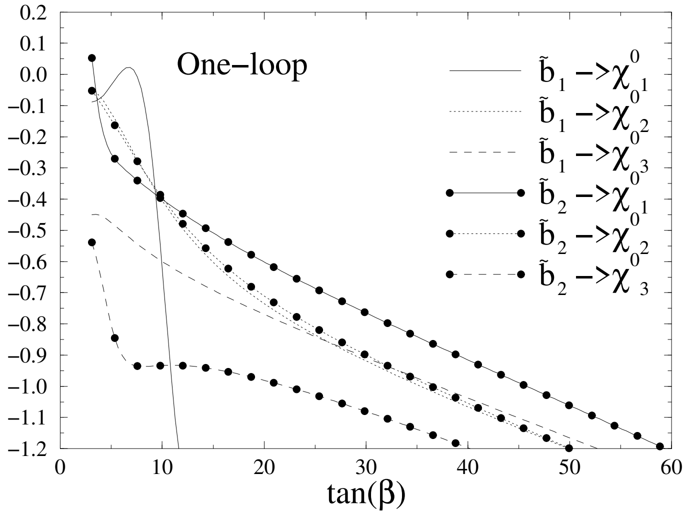
QCD and threshold corrections

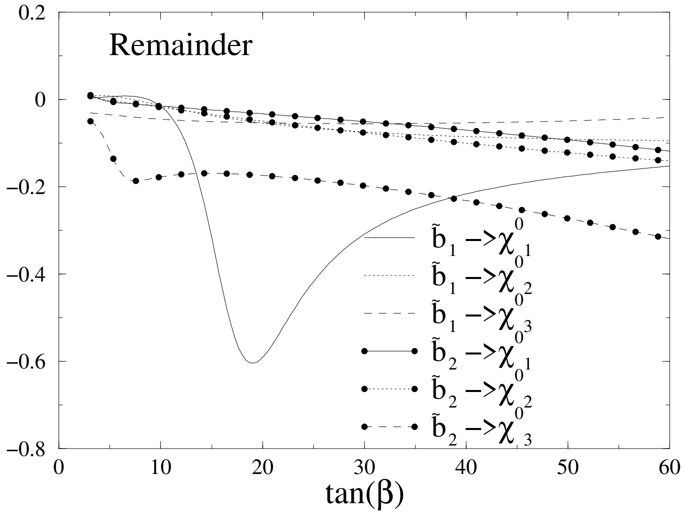
$$m_b^{eff} = \frac{m_b(Q)}{1+\Delta_b} \quad , \quad \Delta_b = \frac{2\alpha_S}{3\pi}\mu \tan\beta \, m_{\tilde{g}} \, I(m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_{\tilde{g}})$$

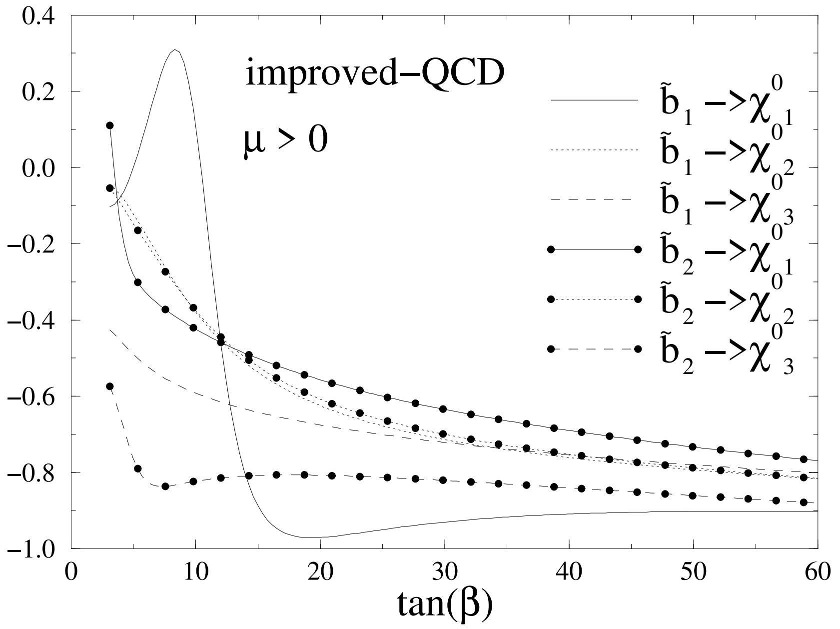
Hall et al.; Carena et al.; Pierce et al.; Babu, Kolda; J.G. et al.; Coarasa et al.; ...

- Similar kind of corrections from $\tilde{b}_L - \tilde{b}_R$ self-energies.
- Relevant for: 3rd generation-higgsino interaccion

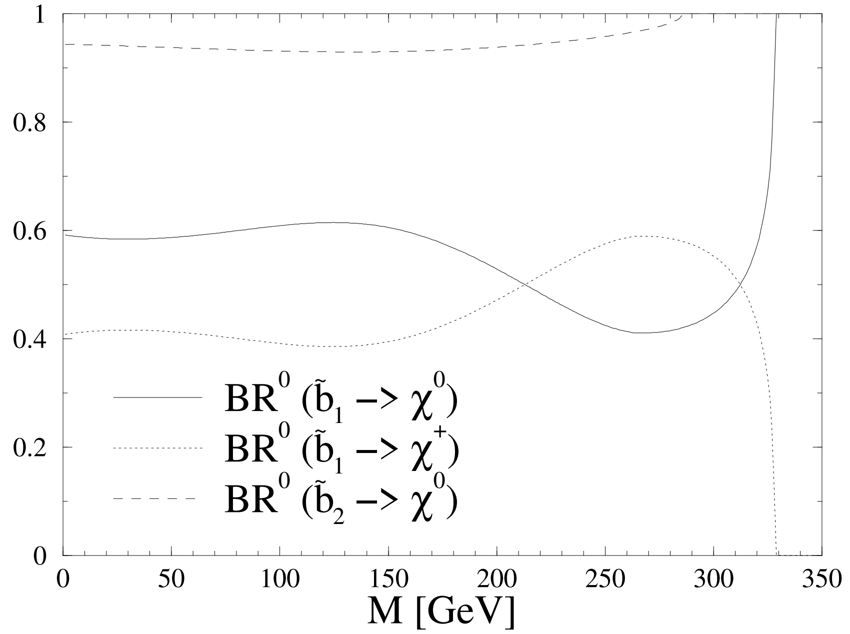
$(\mu > 0)$

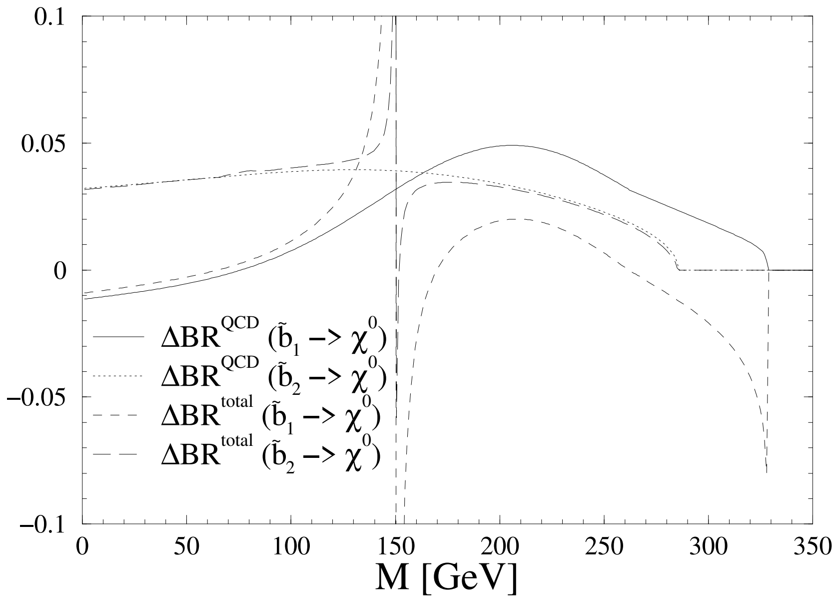






Branching Ratios: QCD + EW





Conclusions and Outlook

- We have performed an On-Shell renormalization of the MSSM
  - Chargino-neutralino
  - Sfermions
- Full Electroweak one-loop corrections to sfermions decays.
- The results are valid for all kind of sfermions.
- Use of Computer Algebra System + several checks $\Rightarrow$ High confidence on the results.

Results

- Show a rich structure.
  - Universal corrections $\Longrightarrow$ RG explanation
    - use of *effective coupling matrices*
  - Non-universal corrections
    - *Threshold*-like corrections (Δm_b) for 3rd generation sfermions–higgsino-like particles
    - Sudakov-type double logs
    - *effective coupling angles*?
- $\sim 10\%$ EW corrections to processes with large Branching Ratio are a common feature

  $\Rightarrow$ The corrections are needed for precision determination of the parameters.
- new non-decoupling effects have been identified
  - Process where SUSY is manifest

    $\Rightarrow$ *Nothing decouples*
  - Simple expressions are available for the non-decoupling of sfermions
- Perturbatibity is OK when using appropriate parameters (A_q)
- A full combination of all the corrections has been analyzed.
  - Corrections to the sfermion and the neutralino masses

    $\Rightarrow$ only important in small regions of the parameter space
  - Combination of EW and QCD corrections

    $\Rightarrow$ Analysis of the Branching ratios
- Programs are available:

`http://www-itp.physik.uni-karlsruhe.de/~guasch/progs/`