Jet Physics at Two-Loop Accurracy

RADCOR–Symposium 2002

- Techniques for Two-Loop Integrals
- Recent Results: Two-Loop Amplitudes

Loops and Legs–Workshop 2002

- Towards NNLO Parton Level Monte Carlo
- Real NNLO Corrections: the Open Issues

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RADCOR-Symposium 2002

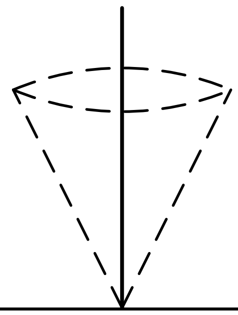
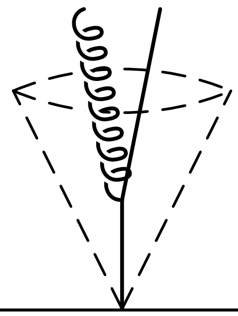
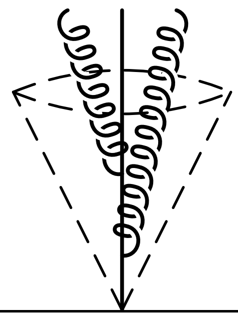
Introduction

Experimental precision for many $2 \to 2$ and $1 \to 3$ reactions has reached a level that demands theoretical predictions at next-to-next-to-leading order.

Examples:

- Bhabha-Scattering: $e^+e^- \to e^+e^-$
- Three Jet observables in e^+e^-
- DIS (2+1)-Jet production
- Hadron-Hadron 2-Jet and $V + 1$-Jet production

Jets in Perturbation Theory

LO	each parton forms one jet on its own	
NLO	up to two partons in one jet	
NNLO	up to three partons in one jet	

current state-of-the-art: NLO

Jet Observables: Results at NLO

- $e^+e^- \to 3$ Jets

  K. Ellis, D. Ross, A. Terrano

  G. Kramer et al.

  Z. Kunszt, P. Nason

  W. Giele, N. Glover

  S. Catani, M. Seymour

  D. Soper (purely numerical approach)

- $e^+e^- \to 4$ Jets

  L. Dixon, A. Signer

  Z. Nagy, Z. Trocsanyi

  J. Campbell, M. Cullen, N. Glover

  D. Kosower, S. Weinzierl

- $pp \to 2$ Jets

  S. Ellis, D. Soper

  W. Giele, N. Glover, D. Kosower

- $ep \to (2+1)$ Jets

  E. Mirkes, D. Zeppenfeld

  S. Catani, M. Seymour

  D. Graudenz

- $ep \to (3+1)$ Jets, $pp \to 3$ Jets

  Z. Nagy

Jet Observables

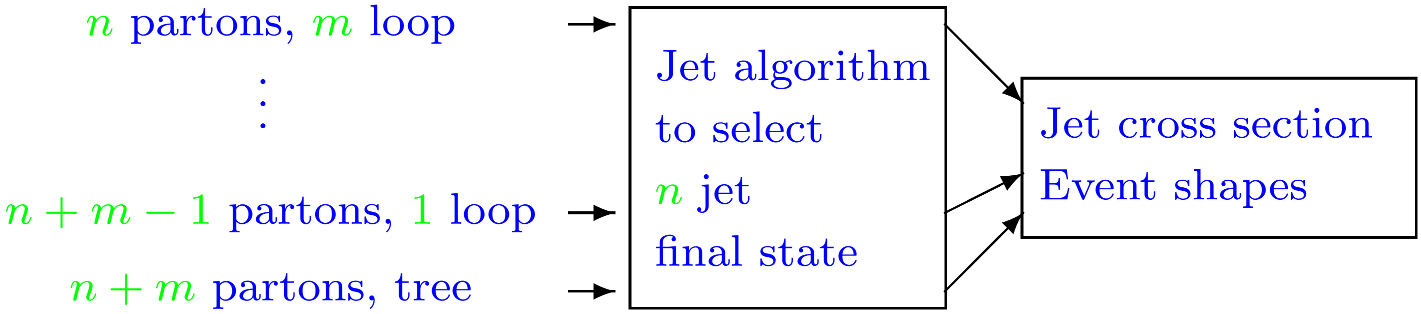
Expected improvements NLO $\longrightarrow$ NNLO

- reduced factorization scale dependence
- better matching of parton level jet algorithm with experimental hadron level jet algorithm
- better description of transverse momentum of final states at hadron colliders due to double radiation in the initial state
- reduced power corrections as higher perturbative powers $1/\ln(Q^2/\Lambda^2)$ can mimic genuine power corrections Q/Λ
- allow full NNLO global fits to parton distributions $\longrightarrow$ lower error on benchmark processes at LHC

Calculation of Jet Observables

General Structure:

n jets, m–th order in perturbation theory



- Jet algorithm acts differently on different partonic final states
  $\longrightarrow$ Optical theorem can not be applied
- Divergencies from soft and collinear real and virtual contributions must be extracted before application of jet algorithm

Virtual corrections at NNLO

Virtual corrections require the computation of 2-loop multi-leg amplitudes with massless internal propagators.

Process	Two-loop amplitudes
$e^+e^- \to e^+e^-$ $pp \to 2$ Jets	4-point, all legs on-shell
$e^+e^- \to 3$ Jets $ep \to (2+1)$ Jets $pp \to V+1$ Jet	4-point, one leg off-shell

Reduction of Scalar Two-Loop Four-Point Functions

Generic structure of scalar two-loop integrals:

$$
\begin{aligned}
& I_{t,r,s}(p_1,\dots,p_n) \\
& = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{1}{D_1^{m_1} \dots D_t^{m_t}} S_1^{n_1} \dots S_q^{n_q}
\end{aligned}
$$

D_i	:	massless scalar propagators
S_i	:	scalar products involving loop momenta
t	:	number of different propagators
$r = \sum_i m_i$	:	dimension of denominator
$s = \sum_i n_i$	:	dimension of numerator

Topology of Feynman graph defined by specifying the set of different propagators

$$\{D_1, \dots, D_t\}$$

Reduction of Scalar Two-Loop Four-Point Functions

Identities:

- Integration-by-parts (IBP)
  K. Chetyrkin, F. Tkachov

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{\partial}{\partial a^\mu} \left[b^\mu f(k, l, p_i) \right] = 0$$

with: $a^\mu = k^\mu, l^\mu$ and $b^\mu = k^\mu, l^\mu, p_i^\mu$

- Lorentz Invariance (LI)
  E. Remiddi, TG

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mathrm{d}^d l}{(2\pi)^d} \delta\varepsilon^\mu_\nu \left(\sum_i p_i^\nu \frac{\partial}{\partial p_i^\mu} \right) f(k, l, p_i) = 0$$

For each two-loop four-point integral, one has 10 IBP and 3 LI identities.

Reduction of Scalar Two-Loop Four-Point Functions

Using IBP and LI identities, any two-loop four-point integral can be expressed as linear combination of a small number of two-loop four-point master integrals:

- on-shell: 6 master integrals
- off-shell: 14 master integrals

plus simpler two- and three-point master integrals.

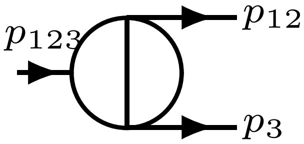
Calculation of Master Integrals

E. Remiddi

Multi-scale master integrals fulfil inhomogeneous differential equations in their external invariants.

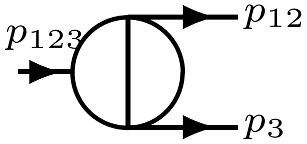
For example:

$$s_{123}\frac{\partial}{\partial s_{123}}$$

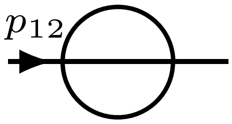


$$=$$

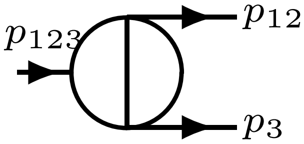
$$+\frac{d-4}{2}\,\frac{2s_{123}-s_{12}}{s_{123}-s_{12}}$$



$$-\frac{3d-8}{2}\,\frac{1}{s_{123}-s_{12}}$$

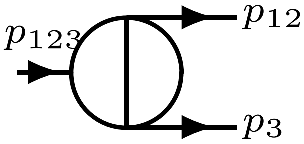


$$s_{12}\frac{\partial}{\partial s_{12}}$$

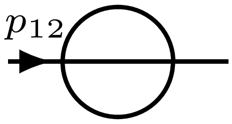


$$=$$

$$-\frac{d-4}{2}\,\frac{s_{12}}{s_{123}-s_{12}}$$



$$+\frac{3d-8}{2}\,\frac{1}{s_{123}-s_{12}}$$

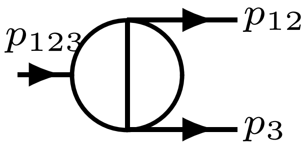


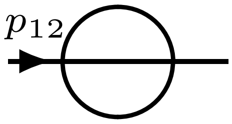
Calculation of Master Integrals

E. Remiddi, TG

Boundary conditions for differential equations correspond to simpler integrals with fewer scales.

For example:



$$(s_{123} = 0) = -\frac{3d-8}{d-4}\,\frac{1}{s_{12}}$$


Master integrals are computed by integration of differential equations and subsequent matching of boundary conditions.

Two-scale integrals (vertex functions and on-shell four-point functions) contain only polylogarithms: $\mathrm{Li}_n(s/t)$, $S_{n,p}(s/t)$.

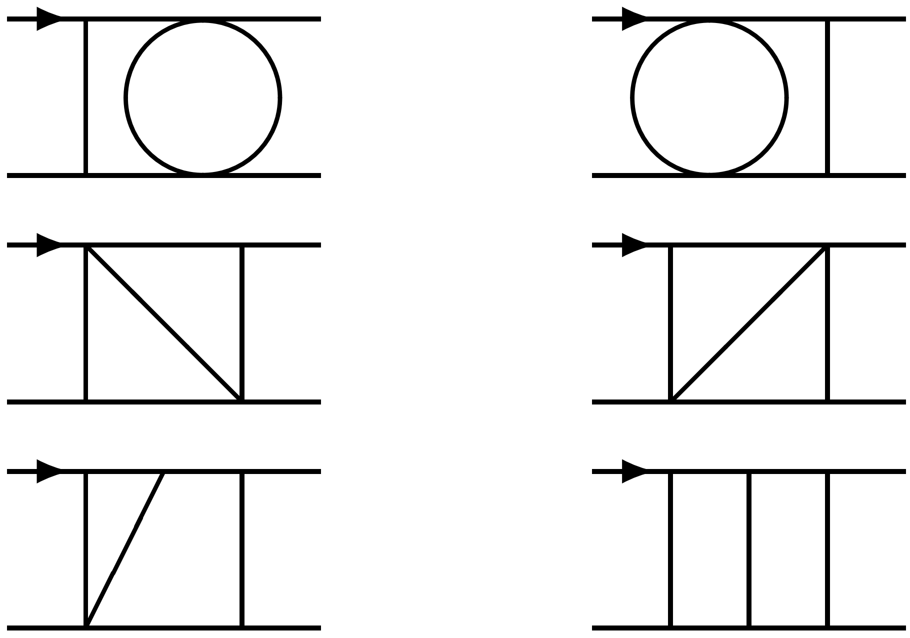
Three-scale integrals (four-point functions with one off-shell leg) require the introduction of new functions: HPL and 2dHPL.

Calculation of Master Integrals

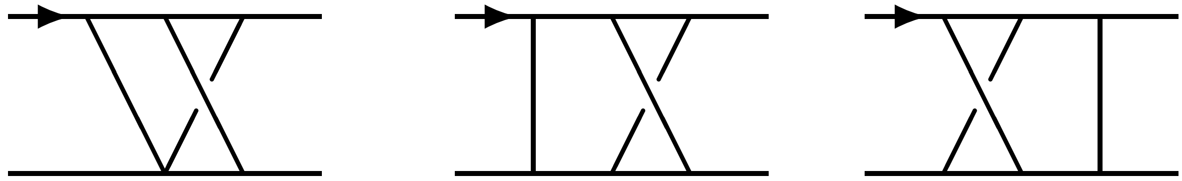
E. Remiddi, TG

All two-loop four-point master integrals with one leg off-shell have been computed from their differential equations:

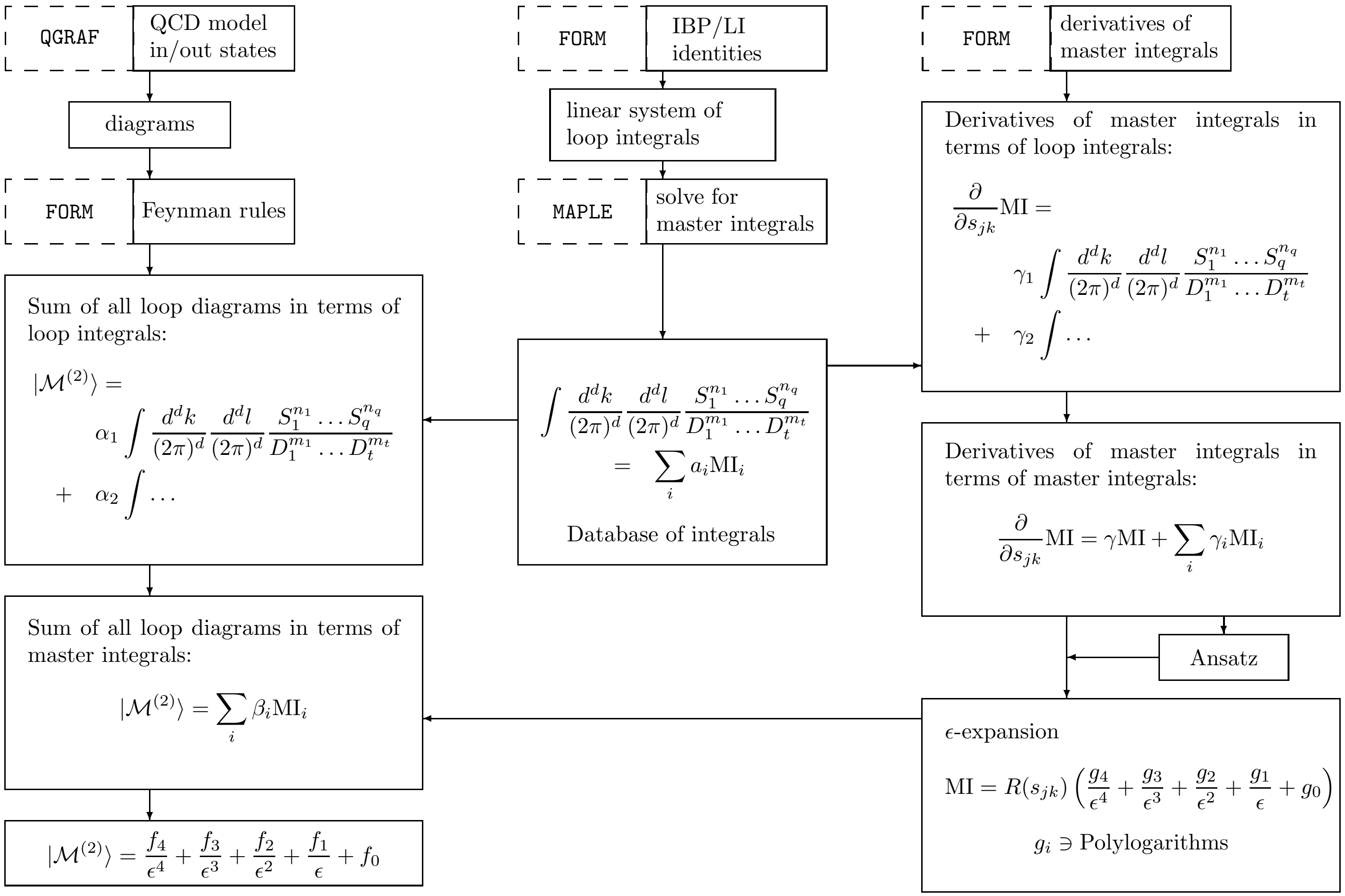
Planar topologies



Non-planar topologies



Computer Algebra for Two-Loop Amplitudes



Two-loop corrections to $\gamma^* \to q\bar{q}g$

L. Garland, N. Glover, A.Koukoutsakis, E. Remiddi, TG

renormalized martix element:

$$|\mathcal{M}\rangle = \sqrt{4\pi\alpha} e_q \sqrt{4\pi\alpha_s}$$
$$\left[|\mathcal{M}^{(0)}\rangle + \left(\frac{\alpha_s}{2\pi}\right) |\mathcal{M}^{(1)}\rangle + \left(\frac{\alpha_s}{2\pi}\right)^2 |\mathcal{M}^{(2)}\rangle + \mathcal{O}(\alpha_s^3) \right]$$

$|\mathcal{M}^{(0)}\rangle$: tree level, 2 diagrams

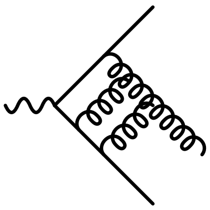
$|\mathcal{M}^{(1)}\rangle$: one-loop, 13 diagrams

$|\mathcal{M}^{(2)}\rangle$: two-loop, 229 diagrams

$\mathcal{O}(\alpha_s^3)$–contribution to the cross section:

$$\mathcal{T}^6(x,y,z) \quad = \quad \langle \mathcal{M}^{(1)} | \mathcal{M}^{(1)} \rangle$$
$$+ \langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \rangle + \langle \mathcal{M}^{(2)} | \mathcal{M}^{(0)} \rangle$$

e.g.



require reduction of integrals with

$t \leq 7; \quad r \leq 7; \quad s \leq 4$

Two-loop corrections to $\gamma^* \to q\bar{q}g$

L. Garland, N. Glover, A.Koukoutsakis, E. Remiddi, TG

structure of infrared poles of $\langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \rangle$:

predicted from infrared factorization formula

Catani

$$
\begin{aligned}
\mathcal{P} \quad = \quad & 2\mathrm{Re}\Big[-\frac{1}{2} \langle \mathcal{M}^{(0)} | \mathbf{I}^{(1)}(\epsilon) \mathbf{I}^{(1)}(\epsilon) | \mathcal{M}^{(0)} \rangle \\
& -\frac{\beta_0}{\epsilon} \langle \mathcal{M}^{(0)} | \mathbf{I}^{(1)}(\epsilon) | \mathcal{M}^{(0)} \rangle \\
& + \langle \mathcal{M}^{(0)} | \mathbf{I}^{(1)}(\epsilon) | \mathcal{M}^{(1)} \rangle \\
& + e^{-\epsilon\gamma} \frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} \left(\frac{\beta_0}{\epsilon} + K \right) \\
& \qquad \langle \mathcal{M}^{(0)} | \mathbf{I}^{(1)}(2\epsilon) | \mathcal{M}^{(0)} \rangle \\
& + \langle \mathcal{M}^{(0)} | \mathbf{H}^{(2)}(\epsilon) | \mathcal{M}^{(0)} \rangle \Big]
\end{aligned}
$$

$\mathbf{I}^{(1)}(\epsilon) \sim 1/\epsilon^2$: universal singularity operator

$K \sim 1$: from two-loop splitting function

$\mathbf{H}^{(2)}(\epsilon) \sim 1/\epsilon$: final-state dependent constant

full agreement with explicit calculation:

$$\epsilon^{-4}, \ldots, \epsilon^{-1}$$

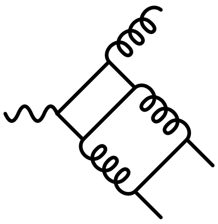
Two-loop corrections to $\gamma^* \to q\bar{q}g$

L. Garland, N. Glover, A.Koukoutsakis, E. Remiddi, TG

Finite part: seven colour structures

$$\mathcal{F}inite^{(2\times 0)} =$$
$$(N^2-1)\Bigg[N^2 A_{20} + B_{20} + \frac{1}{N^2}C_{20} + N N_F D_{20}$$
$$+\frac{N_F}{N}E_{20} + N_F^2 F_{20}(y,z) + N_{F,\gamma}\left(\frac{4}{N} - N\right) G_{20}\Bigg]$$

last term: diagrams with closed quark loop coupling to vector boson



$$N_{F,\gamma} = \frac{\left(\sum_q e_q\right)^2}{\sum_q e_q^2}$$

Coefficients are between 4k (D_{20}) and 35k (G_{20})

D_{20}, E_{20} recently confirmed independently

S. Moch, P. Uwer, S. Weinzierl

Two-loop corrections to $\gamma^* \to q\bar{q}g$

L. Garland, N. Glover, A.Koukoutsakis, E. Remiddi, TG

Helicity amplitudes

- to compute oriented event shapes
- to compute observables in polarized e^+e^- annihilation at TESLA
- to study infrared limits (splitting amplitudes)

All helicity amplitudes can be obtained from

$$\mathcal{S}_\mu(q+;g+;\overline{q}-) = R^V_{f_1 f_2}\sqrt{2}\,\sigma_\mu^{\dot{A}B}\mathcal{S}_{\dot{A}B}(q+;g+;\overline{q}-)$$

by charge and parity conjugation

Weyl–van der Waerden amplitude

$$\mathcal{S}_{\dot{A}B}(q+;g+;\overline{q}-) =$$

$$\alpha\ \frac{p_{1\dot{A}D}p_2^D p_{2B}}{\langle p_1 p_3\rangle\langle p_3 p_2\rangle} + \beta\ \frac{p_{3\dot{A}D}p_2^D p_{2B}}{\langle p_1 p_3\rangle\langle p_3 p_2\rangle} + \gamma\ \frac{p_{1\dot{C}B}p_3^{\dot{C}} p_{3\dot{A}}}{\langle p_1 p_3\rangle\langle p_3 p_2\rangle^*}$$

$$+\ \delta\ \frac{\langle p_1 p_3\rangle^*}{\langle p_1 p_3\rangle\langle p_1 p_2\rangle^*}\left(p_{1\dot{A}B} + p_{2\dot{A}B} + p_{3\dot{A}B}\right)\ .$$

Two-loop corrections to $\gamma^* \to q\bar{q}g$

L. Garland, N. Glover, A.Koukoutsakis, E. Remiddi, TG

Helicity coefficients $\alpha, \beta, \gamma, \delta$ can be extracted from two-loop matrix element by applying projectors.

Infrared pole structure of the coefficients as for the squared, helicity averaged matrix element

Color structure of the finite part as for the squared matrix element

D, E : S. Moch, P. Uwer, S. Weinzierl

Check: recover finite part

$$\mathcal{F}inite^{(2\times 0)} = 8\,(N^2-1)\,\mathcal{R}\Bigg[\frac{(s_{12}+s_{23})s_{12}}{s_{13}s_{23}}\alpha^{(2),\mathrm{finite}} + \frac{s_{12}+s_{23}}{s_{13}}\beta^{(2),\mathrm{finite}} - \gamma^{(2),\mathrm{finite}} + (s_{13}\leftrightarrow s_{23})\Bigg]$$

Crossing of $\gamma^* \to q\bar{q}g$

E. Remiddi, TG

Two-loop matrix element for $\gamma^* \to q\bar{q}g$ is relevant to $e^+e^- \to 3$ Jets at NNLO.

Kinematic region:

$$0 < y = \frac{s_{13}}{q^2} < 1 \, , \quad 0 < z = \frac{s_{23}}{q^2} < 1 - y$$

Processes related by crossing and analytic continuation are:

- DIS (2+1)-Jet production
- Hadron-Hadron $V + 1$-Jet production

Requires analytic continuation of helicity amplitudes outside the triangle

$$0 < y < 1 \, , \quad 0 < z < 1 - y$$

Virtual corrections at NNLO

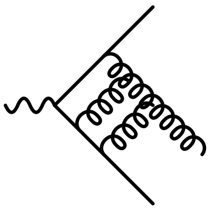
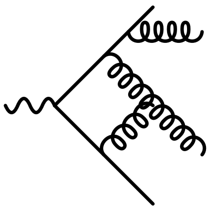
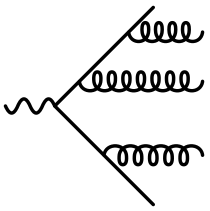
Using similar techniques, virtual two-loop matrix elements have recently been computed for:

- Bhabha-Scattering: $e^+e^- \to e^+e^-$
  Z. Bern, L. Dixon, A. Ghinculov
- Hadron-Hadron 2-Jet production:
  $qq' \to qq'$, $q\bar{q} \to q\bar{q}$, $q\bar{q} \to gg$, $gg \to gg$
  C. Anastasiou, N. Glover, C. Oleari, M. Yeomans-Tejeda
  Z. Bern, A. De Freitas, L. Dixon [SUSY-YM]
- Photon pair production at LHC:
  $gg \to \gamma\gamma$
  Z. Bern, A. De Freitas, L. Dixon
  $q\bar{q} \to \gamma\gamma$
  C. Anastasiou, N. Glover, M. Yeomans-Tejeda
- Light-by-Light scattering (QED, QCD):
  $\gamma\gamma \to \gamma\gamma$
  Z. Bern, A. De Freitas, L. Dixon, A. Ghinculov, H. Wong

Loops and Legs-Workshop 2002

Jet Physics at NNLO

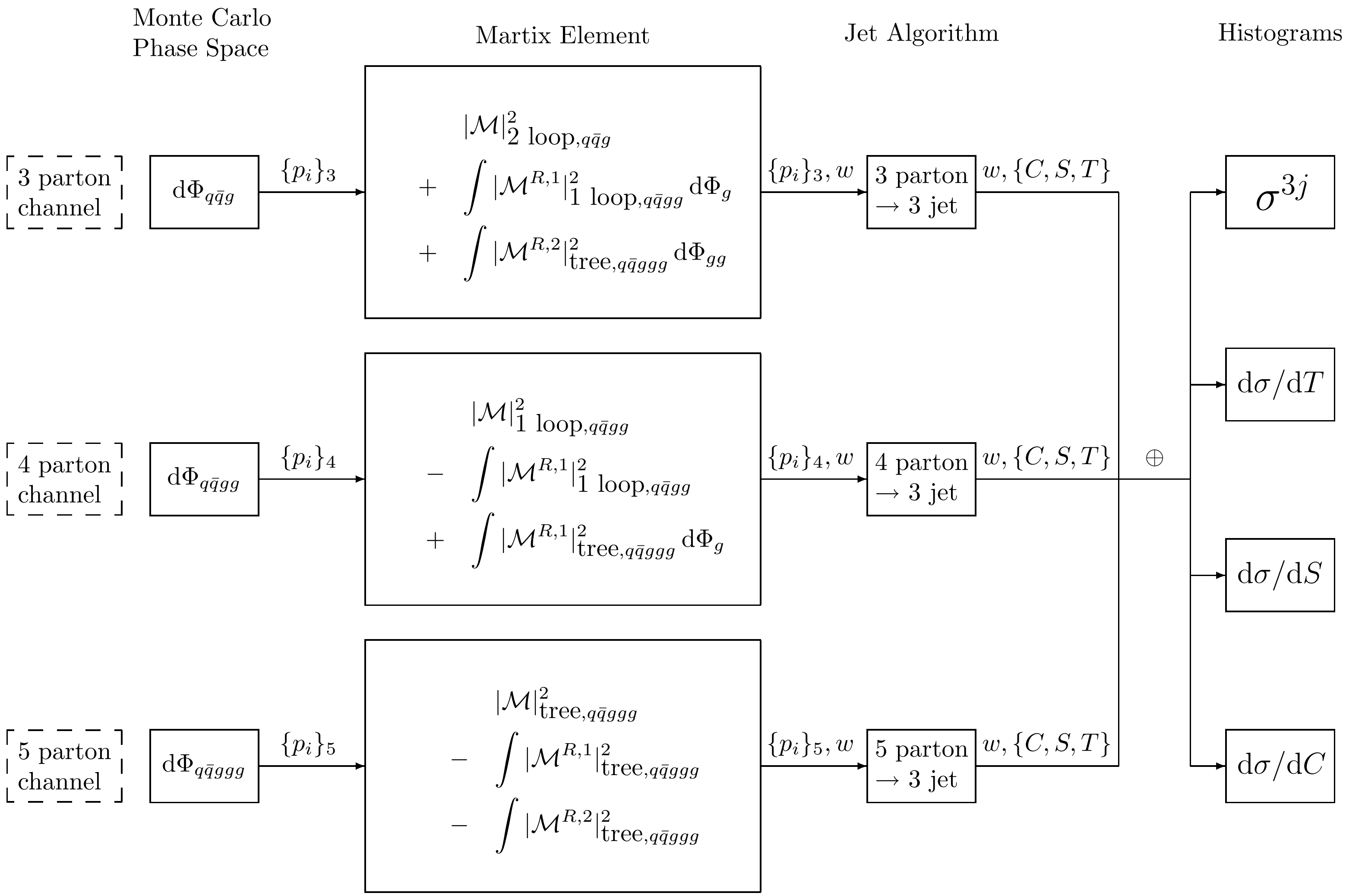
Structure of $e^+e^- \to$ 3 jets at NNLO:

Subprocess	partonic final state	partons in jets
$\gamma^* \to$ 3 partons, 2 loop e.g. 	3 partons	(1) (1) (1)
$\gamma^* \to$ 4 partons, 1 loop e.g. 	4 partons (3+1) partons	(2) (1) (1) (1) (1) (1)
$\gamma^* \to$ 5 partons, tree e.g. 	5 partons (4+1) partons (3+2) partons	(3) (1) (1) (2) (2) (1) (2) (1) (1) (1) (1) (1)

Partons indicated in red are soft or collinear.

They are theoretically unresolved.

Structure of 3 Jet Program



Jet Physics at NNLO

Ingredients to 3 jet calculation:

- Two-loop matrix elements

  $|\mathcal{M}|^2_{2\ \mathrm{loop}, q\bar{q}g}$

  L.W. Garland, N. Glover, A. Koukoutsakis, E. Remiddi, TG

- One-loop matrix elements

  $|\mathcal{M}|^2_{1\ \mathrm{loop}, q\bar{q}gg}$, $|\mathcal{M}|^2_{1\ \mathrm{loop}, q\bar{q}q\bar{q}}$

  Z. Bern, L.J. Dixon, D. Kosower, S. Weinzierl

  J. Campbell, N. Glover, D.J. Miller

- One-loop one-particle subtraction terms

  $\int |\mathcal{M}^{R,1}|^2_{1\ \mathrm{loop}, q\bar{q}gg} \mathrm{d}\Phi_g$

  D. Kosower, P. Uwer

  Z. Bern, V. Del Duca, W.B. Kilgore, C.R. Schmidt

Jet Physics at NNLO

Ingredients to 3 jet calculation (cont.):

- Tree level matrix elements

  $|\mathcal{M}|^2_{\mathrm{tree},q\bar{q}ggg}$, $|\mathcal{M}|^2_{\mathrm{tree},q\bar{q}q\bar{q}g}$

  K. Hagiwara, D. Zeppenfeld

  N. Falck, D. Graudenz, G. Kramer

- Tree-level one-particle subtraction terms

  $\int |\mathcal{M}^{R,1}|^2_{\mathrm{tree},q\bar{q}ggg} \mathrm{d}\Phi_g$, $\int |\mathcal{M}^{R,1}|^2_{\mathrm{tree},q\bar{q}q\bar{q}g} \mathrm{d}\Phi_g$

  W. Giele, N. Glover

  S. Catani, M. Seymour

- Tree-level two-particle subtraction terms

  $\int |\mathcal{M}^{R,2}|^2_{\mathrm{tree},q\bar{q}ggg} \mathrm{d}\Phi_{gg}$, $\int |\mathcal{M}^{R,2}|^2_{\mathrm{tree},q\bar{q}q\bar{q}g} \mathrm{d}\Phi_{q\bar{q}}$

  remain to be calculated

Calculation of Jet Observables

Infrared subtraction terms

Combining real and virtual contributions:
general task is to evaluate

$$I = \lim_{\epsilon \to 0} \left\{ \int_0^1 \frac{\mathrm{d}x}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right\}$$

Subtraction:

K. Ellis, D. Ross, A. Terrano; S. Catani, M. Seymour

- integrate over full phase space
- must not overcount singularities

Slicing:

G. Kramer et al.; W. Giele, N. Glover

- integrate only over unresolved phase space
- finding subtraction term easy
  J. Campbell, N. Glover; S. Catani, M. Grazzini
- unresolved phase space complicated
  A. Gehrmann-De Ridder, N. Glover, TG

Summary

Calculations of jet observables at NNLO appear to become realistic

- enormous progress on two-loop vitrual corrections: two-loop helicity amplitudes for all important processes known now
- systematic procedures for treatment of one-loop single-unresolved contributions available

... but still some obstacles to be overcome:

- need procedure to treat tree-level double-unresolved contributions
- and efficient numerical implementation into parton level Monte Carlo