Full $O(\alpha)$ corrections to $e^+e^- \to \nu\bar{\nu}H$ by GRACE system

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for the collaboration with

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Contents

1. Introduction

$e^+e^- \to \nu\bar{\nu}H$ is one of the most important processes for the production of Higgs bosons in e^+e^- collisions, here, ν stands for all 3 types of neutrinos. There are two mechanisms: Higgs-strahlung and WW fusion.

- tree level analysis by W. Kilian, M. Krämer and P.M. Zerwas, Pys. Lett. B373(1996)135.
- complete $O(\alpha)$ corrections to $e^+e^- \to ZH$ by A. Denner, J. Küblbeck, R. Mertig and M. Böhm, Z. Phys. C56(1992) 261.
- Higher order corrections to Higgs decay into W-pair by B. A. Kniehl, Phys. rep. 240(1994) 211.
- 1-loop corrections from fermion and sfermion loops to $e^+e^- \to \nu\bar{\nu}$H by E. Eberl, W. Majerotto and V.C. Spanos, Phys. Lett. B538(2002) 353.

2. GRACE/1-LOOP

- Based on the on-mass shell renormalization scheme proposed by Kyoto group, all the counterterms are automatically generated.
- Two libraries for loop integration are available for comparison: Numerical integration method and `FF` package & Brown-Feynman reduction of numerators.
- Ultra-violet divergence:

Dimensional regularization is used.

$$C_{\rm UV} = 1/\varepsilon - \gamma_E + \log 4\pi, \qquad n = 4 - 2\varepsilon.$$

- Infrared divergence:

Fictitious photon mass λ,

$$C_{\rm IR} \;=\; \ln \lambda.$$

Symbolic expression for each Feynman diagrams.
A sequence of procedures is written in `REDUCE/FORM` and then `FORTRAN` codes are generated.

1. Write down the product

$$T^{\rm loop}\, T^{\rm tree\dagger}$$

   in a symbolic code.

2. Indices inside loop are contracted in n dimension.

3. If there are fermion loops, trace is taken. The cyclicity of γ matrix in a trace is *not* used to avoid the anomaly from γ_5. The ordering of γ's is fixed common to a set of graphs with the same topology.

4. Shift momentum is determined by inspecting the denominator structure after combining propagators by Feynman parameters. Then loop momentum l is shifted.

5. Terms in odd power of l are dropped.

6. The following replacement of even l is done:

$$l^\mu l^\nu \to \frac{l^2}{n} g^{\mu\nu}, \tag{1}$$

$$l^\mu l^\nu l^\rho l^\sigma \to \frac{(l^2)^2}{n(n+2)} \left(g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\rho\nu}\right) \tag{2}$$

7. Numerators are polynomials in Feynman parameters. The set of coefficients of polynomial are extracted.

Following is an example of a vertex integral:

$$\int \mathrm{d}x\mathrm{d}y\Gamma(3)\int \frac{\mathrm{d}^n l}{(2\pi)^4 i}\frac{f(x,y)+\left(g(x,y)+\varepsilon g_1(x,y)\right)l^2}{(D_3(l)-i\varepsilon)^3} \tag{3}$$

8. Here $f(x,y), g(x,y), ...$ are product of coupling constants and invariants of external particles.

9. Loop library is called to calculate loop integrals with polynomials in the numerator.

10. All the above procedures are done automatically by the system.

3. 1-loop diagrams of $e^+e^- \to \nu\bar{\nu}H$

1350 diagrams for 1-loop and **12** ones for tree, when the electron-scalar couplings are kept.

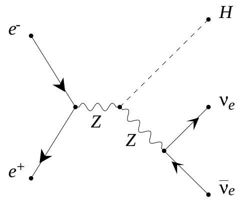
$\Longrightarrow$ The approximation with the neglect of m_e^2 has no problems.

$\Longrightarrow$ **249** 1-loops(**15** 5-point functions) and **2** trees.

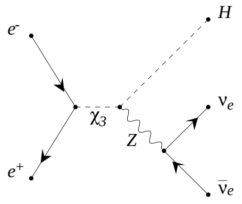
There exists 5-point functions: **98** graphs.
$\Longrightarrow$ 5-point functions are reduced to linear combinations of 4-point functions with Gram determinant.

$\Longrightarrow$ Figures

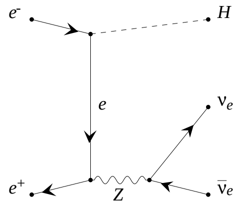
Graph 1



Graph 2



Graph 3



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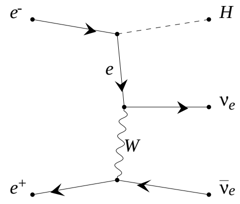

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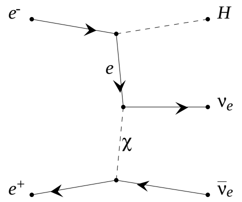

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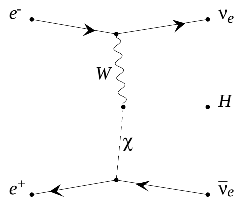

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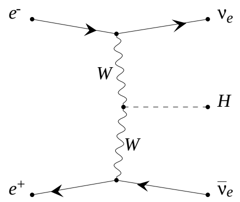
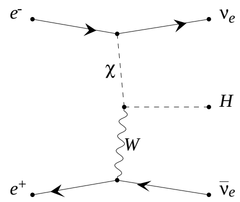


Graph 8



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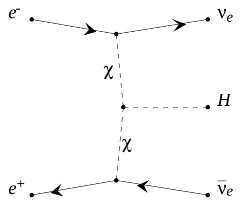
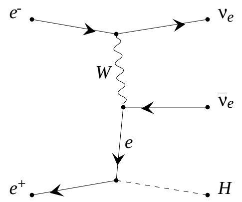
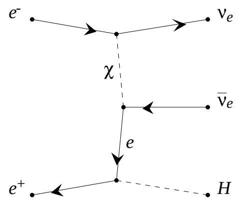


Graph 10



Graph 11



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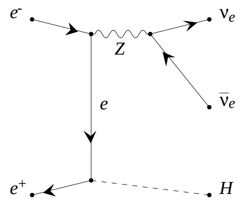
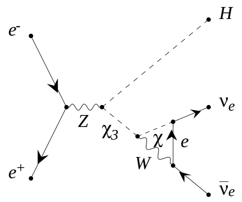


Graph 1



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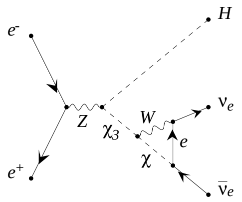

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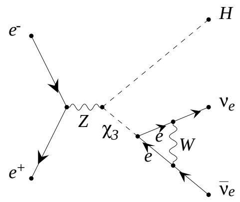

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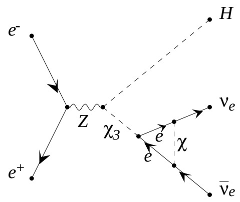

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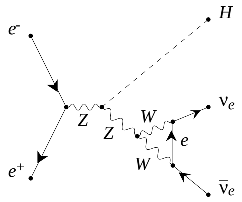

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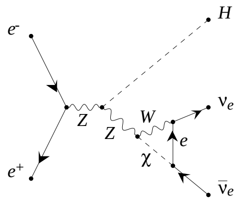

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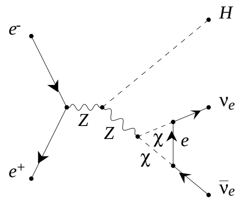
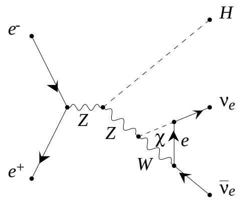


Graph 8



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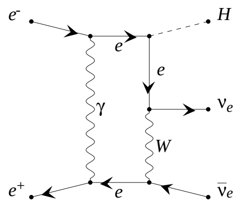
Graph 809



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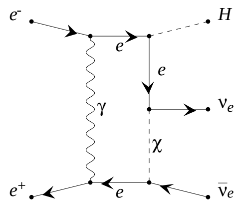

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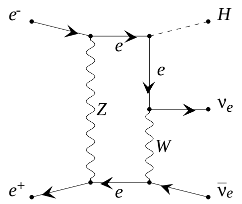

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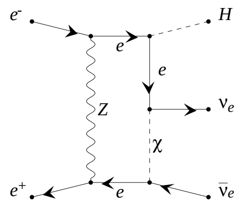

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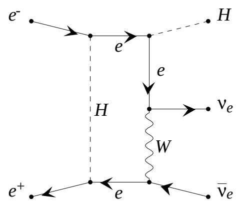

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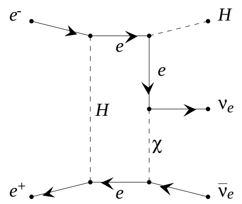

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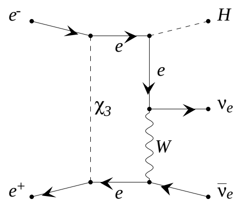

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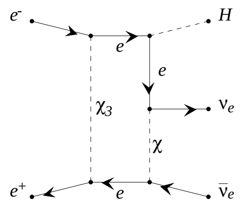

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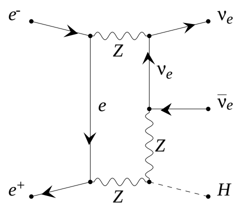

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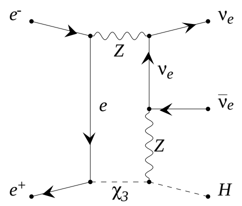

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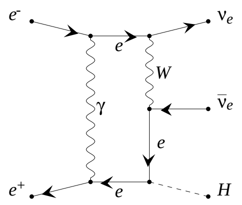

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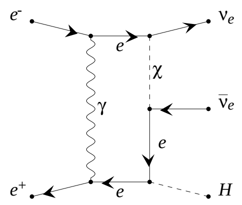

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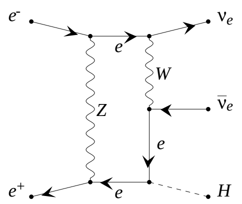

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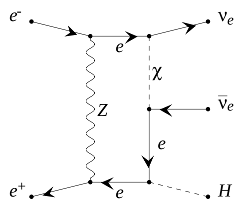

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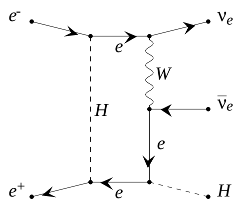

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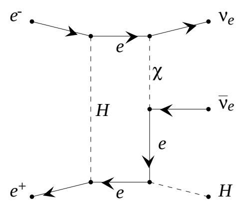

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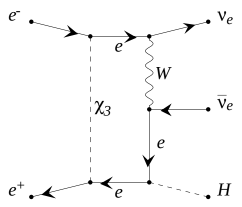

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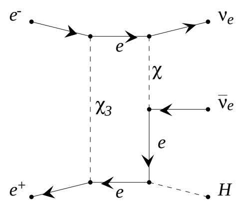

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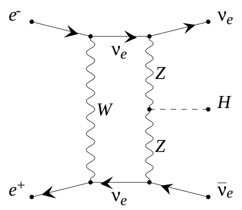

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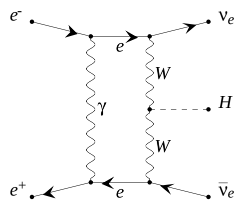

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4. Checks of calculations

- Quadruple precision is taken.
- Single phase space point.
- Renormalization:
  Keep $C_{UV} = 1/\varepsilon - \gamma_E + \log 4\pi$ in the code and check independence of it.
  $\Longrightarrow$ stable more than **30** digits.
- Infrared divergence.
  Check independence of λ.
  $\Longrightarrow$ stable more than **23** digits.
- Gauge invariance in the 1-loop level
  $\Longrightarrow$ non-linear gauge (NLG) fixing is applied in collaboration with LAPTH
  (F. Boudjema and E. Chopin, Z.Phys. C73(1996) 85.)

$\xi_W = \xi_Z = \xi_A = 1 \Longrightarrow$ Numerator $= g^{\mu\nu}$.

$$
\begin{aligned}
\mathcal{L}_{\mathrm{GF}} \;=\; & -\frac{1}{\xi_W}\Big|(\partial_\mu - ie\tilde{\alpha}A_\mu - ig\cos\theta_W\tilde{\beta}Z_\mu)W^{+\mu} \\
& \qquad\qquad + \xi_W\frac{g}{2}(v + \tilde{\delta}H + i\tilde{\kappa}\chi_3)\chi^+\Big|^2 \\
& -\frac{1}{2\xi_Z}\left(\partial_\mu Z^\mu + \xi_Z\frac{g}{2\cos\theta_W}(v + \tilde{\varepsilon}H)\chi_3\right)^2 - \frac{1}{2\xi_A}(\partial_\mu A^\mu)^2
\end{aligned}
$$

- In total the amplitude contains 5 NLG gauge parameters:

$$\tilde{\zeta} = \tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\varepsilon}, \tilde{\kappa} \tag{4}$$

NLG invariance check:

The sum of graphs is written as

$$\begin{aligned} \sigma \;=\; & \sum_{g\in\mathrm{Graph}} \Big[\mathrm{coef}(g,4)\tilde{\zeta}^4 + \mathrm{coef}(g,3)\tilde{\zeta}^3 + \mathrm{coef}(g,2)\tilde{\zeta}^2 \\ & \qquad + \mathrm{coef}(g,1)\tilde{\zeta}^1 + \mathrm{coef}(g,0)\tilde{\zeta}^0\Big] \end{aligned}$$

After summing up all diagrams, one should have

$$\mathrm{coef}(n) = 0 \qquad (n = 1 \sim 4), \tag{5}$$

if NLG invariance is satisfied.

$$\mathrm{coef}(n) = \sum_{g\in\mathrm{Graph}} \mathrm{coef}(g,n) \tag{6}$$

For each parameter, five σs are calculated to get coef(n)

$$\mathrm{s/m}(n) \equiv \frac{\mathrm{coef}(n)}{\mathrm{Max}_g(|\mathrm{coef}(g,n)|)} \tag{7}$$

	# of graphs	s/m(4)	s/m(3)	s/m(2)	s/m(1)
$\tilde{\alpha}$	149	0	0	10^{-27}	10^{-29}
$\tilde{\beta}$	314	0	0	10^{-31}	10^{-22}
$\tilde{\delta}$	1132	0	10^{-19}	10^{-19}	10^{-25}
$\tilde{\kappa}$	112	0	0	10^{-20}	10^{-30}
$\tilde{\varepsilon}$	132	0	0	10^{-22}	10^{-22}

5. Numerical results

- Input parameters:

$$\begin{aligned}
M_Z &= 91.1876\text{GeV}, \Gamma_Z = 2.4952\text{GeV}, \\
M_t &= 174\text{GeV}, \\
M_u &= M_d = 58\text{MeV}, \\
M_s &= 92\text{MeV}, \\
M_c &= 1.5\text{GeV}, \\
M_b &= 4.7\text{GeV}, \\
\alpha &= 1/137.0359895
\end{aligned}$$

- M_W is corrected in $O(\alpha)$ and thus is a function of M_H.

$$M_W = M_W(M_H)$$

- **Total cross section**
  - Phase space integration is carried out by `BASES` package.
  - Hard photon emission is also included.
  - Results of integration are consistent between double precision and quadruple precision.

At $\sqrt{s} = 500$ GeV[1]:

M_H (GeV)	σ_{tree} (fb)	$\sigma_{O(\alpha)}$ (fb)	δ
150	61.12	60.99 $\pm$0.07	-0.2%
200	37.33	37.16 $\pm$0.04	-0.4%
250	21.17	20.63 $\pm$0.02	-2.5%
300	10.76	10.30 $\pm$0.01	-4.2%
350	4.603	4.184 $\pm$ 0.004	-9.1%

where $\delta = \sigma_{O(\alpha)}/\sigma_{tree} - 1$.

- Distributions
  - Energy of Higgs boson
  - Polar angle of Higgs Boson

[1] in the talk the factor 2 for the hard 's-channel' was missing.

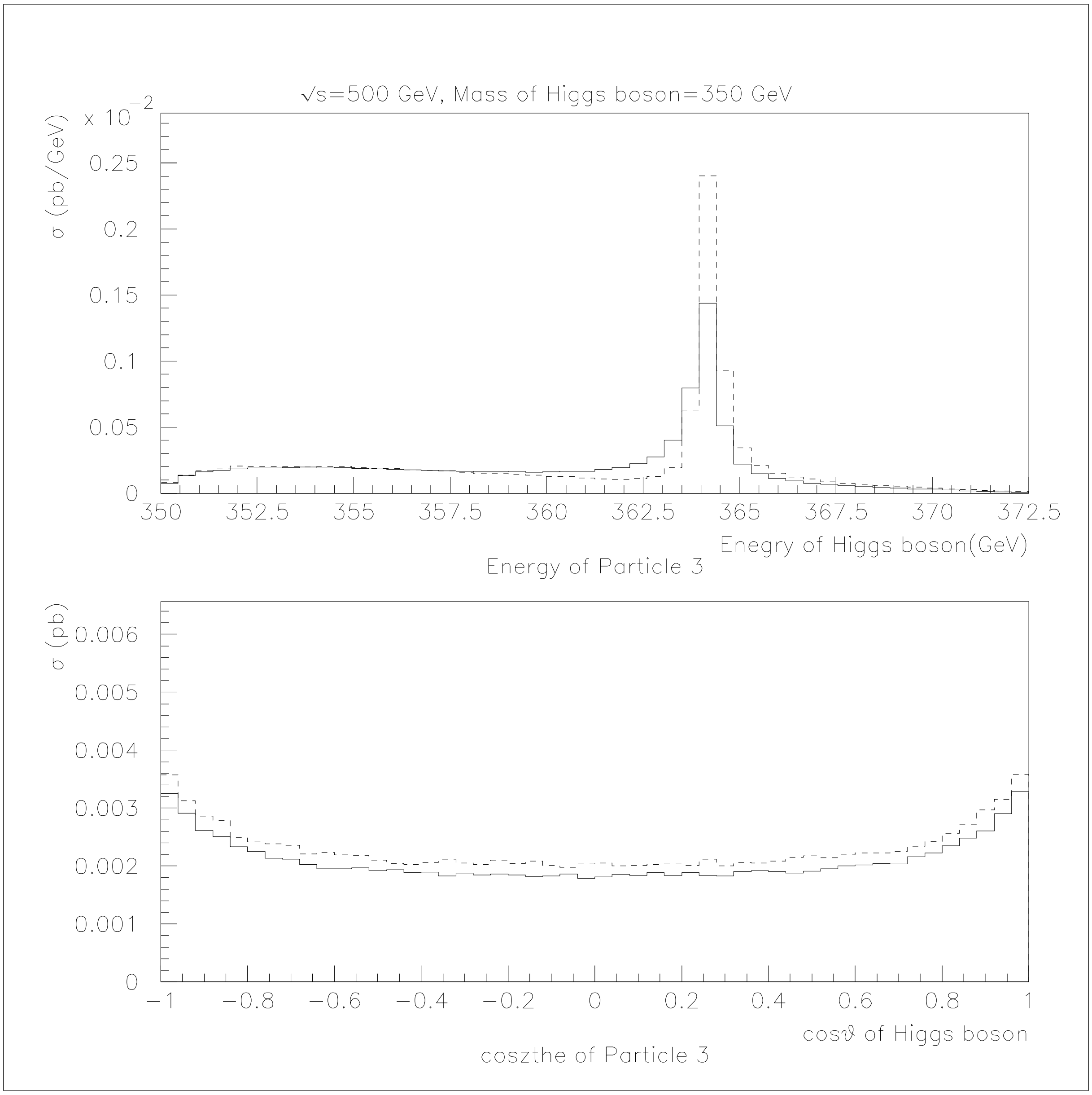

Figure 1: Energy and $\cos\theta$ distributions of Higgs boson at $\sqrt{s} = 500$GeV and $M_{Higgs} = 350$ GeV. The solid and dashed lines show $O(\alpha)$ corrected and tree level distributions, respectively.

6. Summary

- One-loop amplitudes of **full** 1350 diagrams for $e^+e^- \to \nu\bar{\nu}H$ were generated by `GRACE/1-LOOP` system.
- The Non-Linear Gauge invariance has shown that it is a very powerful tool to check the consistency of the full set of amplitudes and the system itself.
- At $\sqrt{s} = 500$ GeV the correction factor reaches to $\sim -10\%$ when the mass of Higgs boson is 350 GeV.
- At higher energies than 500 GeV, the contribution from the fusion type amplitude will become more dominant.
- Also to get more realistic predictions one has somehow to include multiple photon emissions.