DIANA and selected applications

(1) DIANA

(a) Graphics

(b) Distribution of integration momenta

(c) Parallelization

(2) $t\bar{t}$ in one-loop including hard Bremsstrahlung

(3) Factorizing one-loop contributions in two-loop Bhabha scattering.

DIANA (DIagram ANAlyzer)
written in C
M. Tentyukov and J. F.,
Computer Physics Communications 132 (2000) 124-141

– QGRAF Nogueira
– TM text manipulating language:
creation of text files Tentyukov

By preparing a 'create' (style) file, the user specifies the model and the process and DIANA will generate all other necessary files. The user types:

diana - c create.ext
(prepared by the user),
then calls: diana -c config.Bhabha
config.Bhabha contains the TM program for the evaluation of Feynman diagrams, produced by DIANA.
Enter the output filename: $\underbrace{Bhabha.frm}_{\text{contains FORM input}}$
from DIANA user

Recent development:

various graphic representations (ps) of F.d.'s.
encapsulated postscript files for publications.
choice of integration momenta by klicking on the line.
parallelization of DIANA.
see also : web-site
http://www.physik.uni-bielefeld.de/~tentukov/diana.html

```
SET _processname = Bhabha
SET _TOPOLOGYEDITORNAME = tedi
only interpret
\openlanguage(create.tml)
\Begin(program)
   \Begin(model,QED.model)
   \End(model)
   \Begin(process)
      ingoing e(;p1),E(;p4);
      outgoing e(;-p2),E(;-p3);
      loops = 1;
   \End(process)
   \Begin(qgrafoptions)
      options=notadp,onshell;
      ...
   \End(qgrafoptions)
   \Begin(tmlprogram,folders.tml)
   ...
   \End(tmlprogram)
   \indices(mu,mu1,mu2,mu3,... )
   \vectors(p1,p2,...,q1,q2,...,k,...,zero)
   \zeromomentum(zero)
   \loopmomenta(k)
\End(program)
```

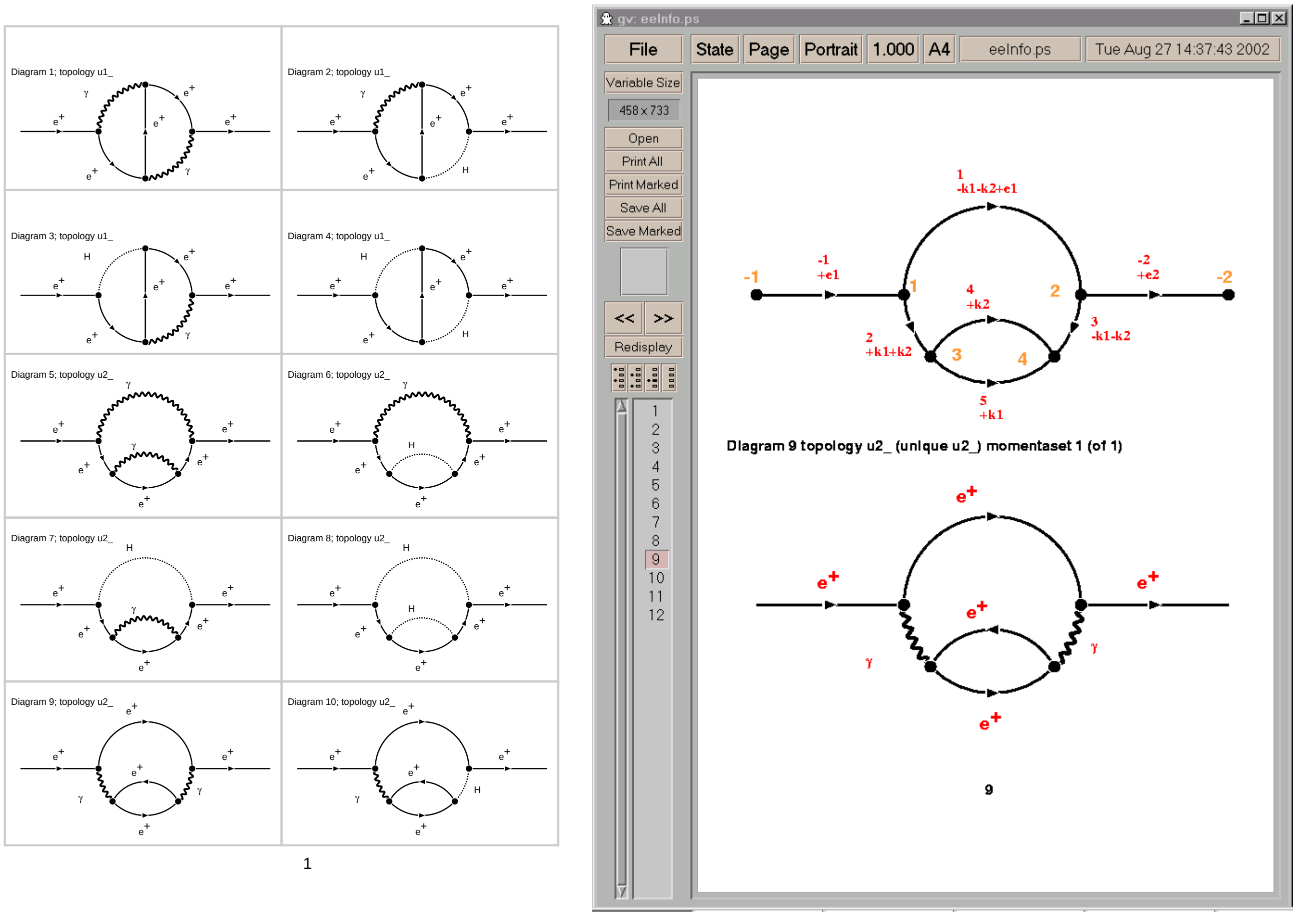
Diagram 1; topology u1_
Diagram 2; topology u1_
Diagram 3; topology u1_
Diagram 4; topology u1_
Diagram 5; topology u2_
Diagram 6; topology u2_
Diagram 7; topology u2_
Diagram 8; topology u2_
Diagram 9; topology u2_
Diagram 10; topology u2_
1
gv: eeInfo.ps
File
State
Page
Portrait
1.000
A4
eeInfo.ps
Tue Aug 27 14:37:43 2002
Variable Size
458 x 733
Open
Print All
Print Marked
Save All
Save Marked
<<
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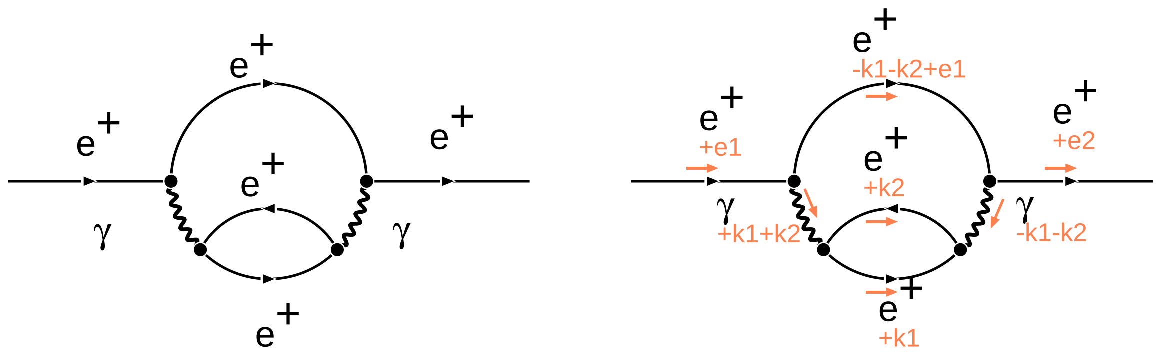
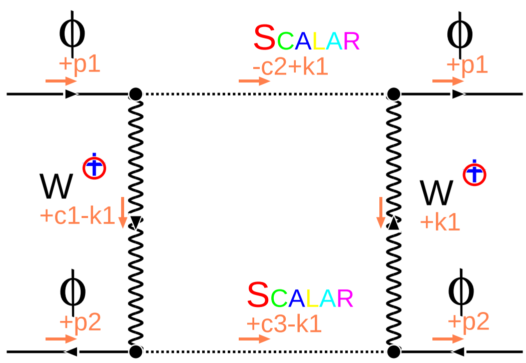
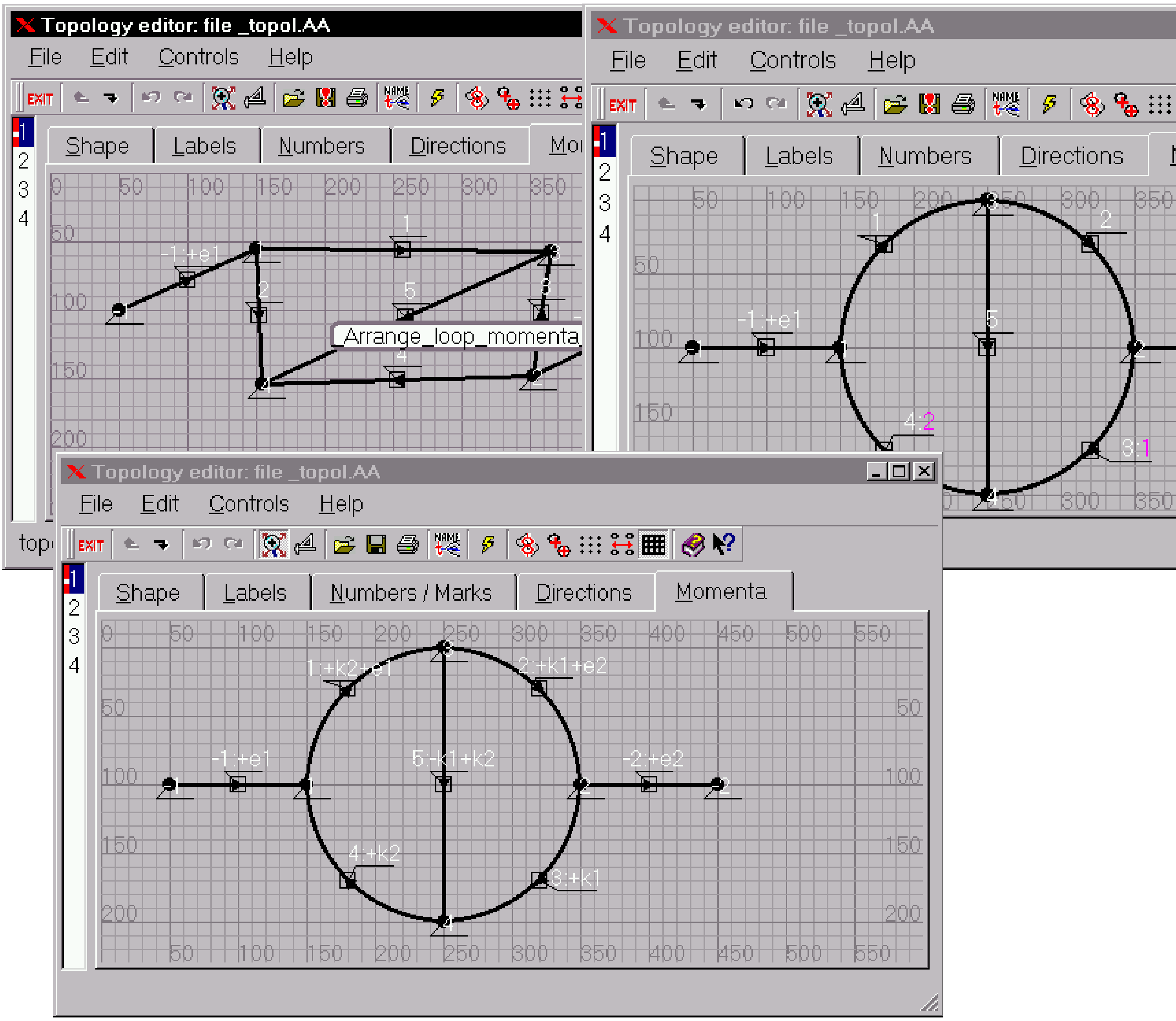
Redisplay
1
2
3
4
5
6
7
8
9
10
11
12
-k1-k2+e1
+e1
+e2
+k2
+k1+k2
-k1-k2
+k1
Diagram 9 topology u2_ (unique u2_) momentaset 1 (of 1)
9


e+
γ
e+
-k1-k2+e1
+e1
+e2
+k2
+k1+k2
-k1-k2
+k1


ϕ
+p1
SCALAR
-c2+k1
W
+c1-k1
+k1
+p2
+c3-k1

```
SET _processname = ee
SET _TOPOLOGYEDITORNAME = tedi
SET _MARK_LOOP = YES
```




```
\zeromomentum(zero)
\loopmomenta(k1,k2)

\End(program)
```

File “runfrm” (*executable*):

```
#!diana -smp 3 -c runpar.tml
\STARTSERVERS(phya25,phya26,phya27,phya28)
\system(echo > log.all)
\REPEAT(N)
\exec(form -d i=\get(N) do_tt.frm > /tmp/log.\get(N))
\stick(cat /tmp/log.\get(N)>>log.all;rm /tmp/log.\get(N))
\ENDREPEAT()
\waitall(2000)
```

Example: The user enters:

`runfrm 186 200`

The system executes:

`diana -smp 3 -c runpar.tml runfrm 186 200`

The option `-smp 3`: *Symmetric MultiProcessing,* three-processor host (default=1).

`\STARTSERVERS(..., phya26, ...):`
`ssh phya26 cd /home/user/jobs ; diana -d 1 -q`

The option `-d 1`: DIANA starts a daemon (server running in the background) accepting one connection.

`\REPEAT(N) ... \ENDREPEAT():`
The variable N runs from 186 to 200. E.g., for N=190:
`form -d i=190 do_tt.frm > /tmp/log.190`
`cat /tmp/log.190>>log.all;rm /tmp/log.190`
The `\stick()` operator: the `cat` will wait until all previous jobs are completed and acts on the same node as the form job.

`\waitall(2000)` - reports every 2000 milliseconds how many jobs are not yet finished.

$t\bar{t}$ in one-loop

(1) COMPLETE ELECTROWEAK ONE LOOP RADIATIVE CORRECTIONS TO TOP PAIR PRODUCTION AT TESLA: A COMPARISON.

J. Fleischer, T. Hahn , W. Hollik, T. Riemann, C. Schappacher, A. Werthenbach,

Feb. 2002, hep-ph/0202109 .

(2) ONE LOOP CORRECTIONS TO THE PROCESS $e^+e^- \to t\bar{t}$ INCLUDING HARD BREMSSTRAHLUNG

J. Fleischer, J. Fujimoto, T. Ishikawa, A. Leike, T. Riemann, Y. Shimizu, A. Werthenbach,

hep-ph/0203220

Diagram 280; topology top4_

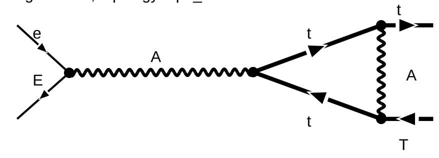


Diagram 281; topology top4_

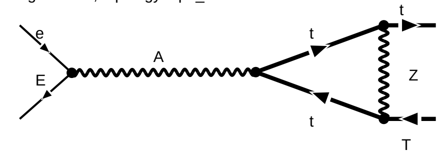


Diagram 282; topology top4_

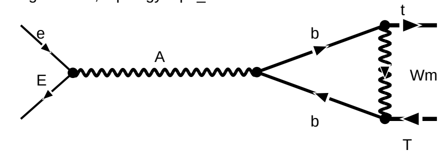


Diagram 286; topology top4_

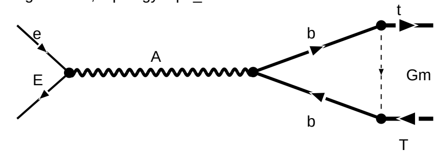


Diagram 287; topology top4_

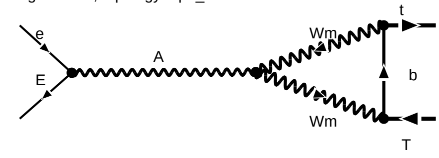


Diagram 288; topology top4_

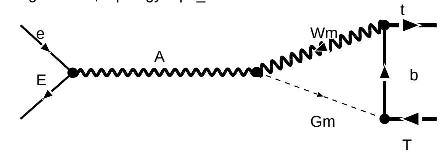


Diagram 289; topology top4_

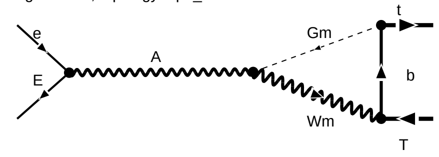


Diagram 290; topology top4_

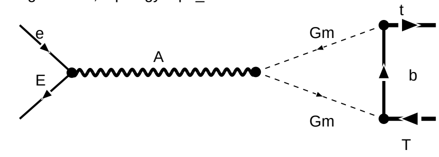


Diagram 291; topology top4_

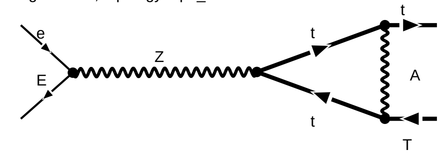


Diagram 292; topology top4_

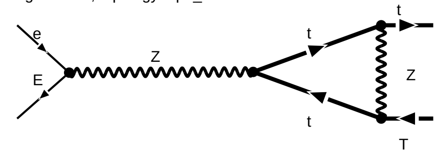


Diagram 293; topology top4_

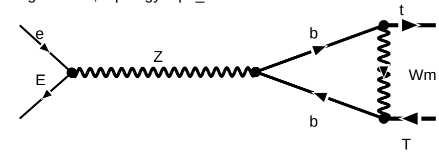


Diagram 294; topology top4_

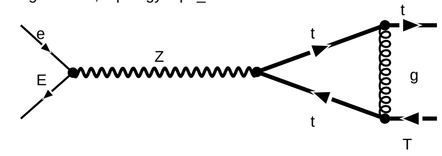


Diagram 295; topology top4_

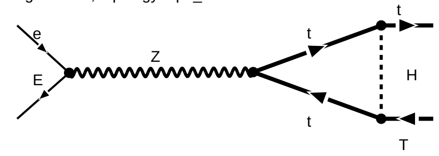


Diagram 296; topology top4_

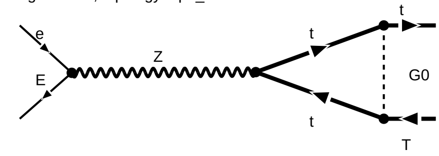


Diagram 297; topology top4_

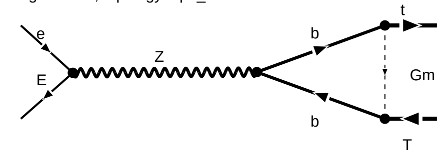


Diagram 298; topology top4_

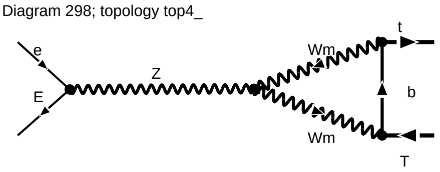


Diagram 299; topology top4_

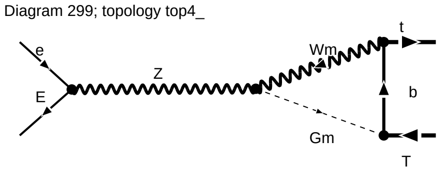


Diagram 300; topology top4_

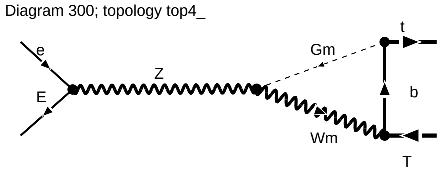


Diagram 301; topology top4_

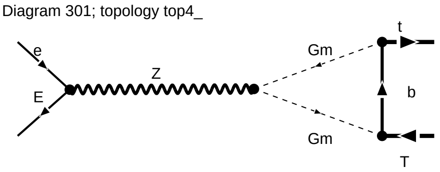


Diagram 302; topology top4_

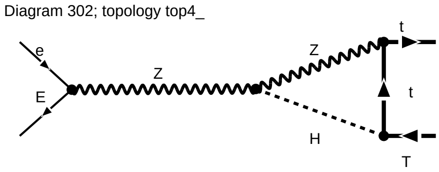


Diagram 303; topology top4_

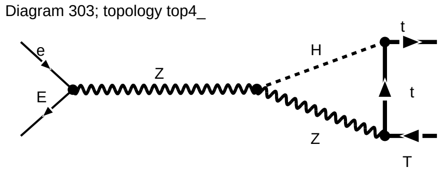


Diagram 304; topology top4_

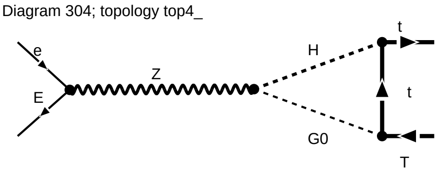


Diagram 305; topology top4_

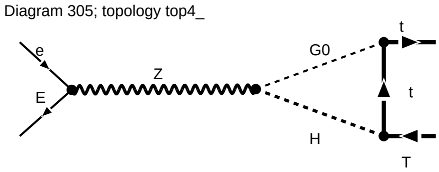


Diagram 335; topology top5_

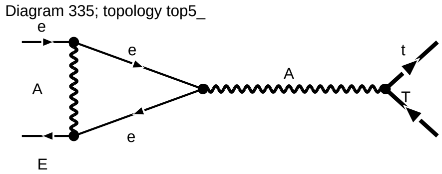


Diagram 336; topology top5_

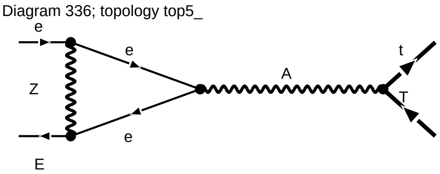


Diagram 339; topology top5_

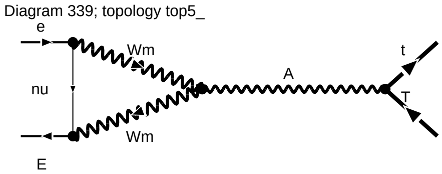


Diagram 343; topology top5_

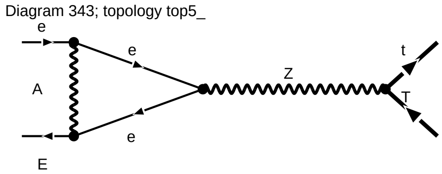


Diagram 344; topology top5_

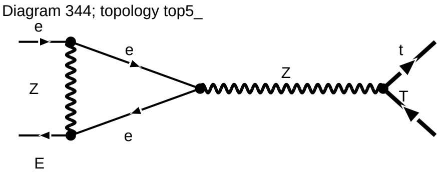


Diagram 345; topology top5_

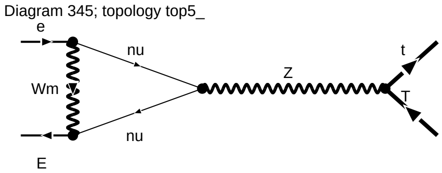


Diagram 349; topology top5_

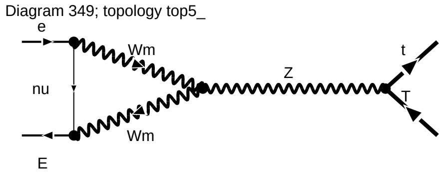


Diagram 1012; topology top14_

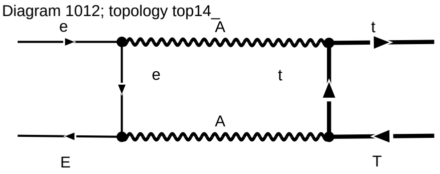


Diagram 1013; topology top14_

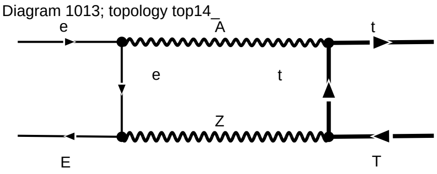


Diagram 1016; topology top14_

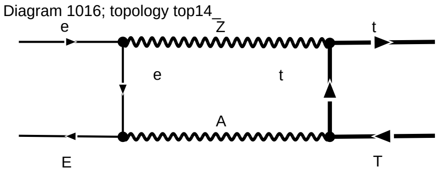


Diagram 1017; topology top14_

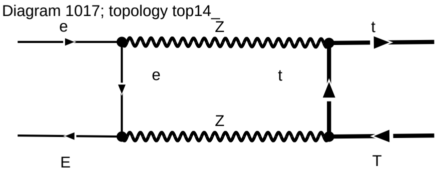


Diagram 1028; topology top15_

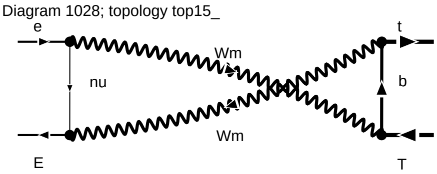


Diagram 1032; topology top15_

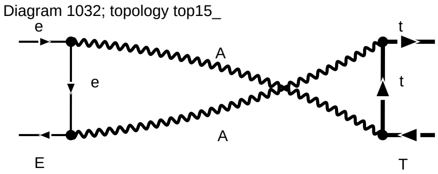


Diagram 1033; topology top15_

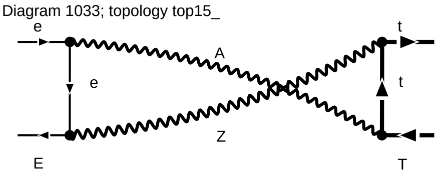


Diagram 1036; topology top15_

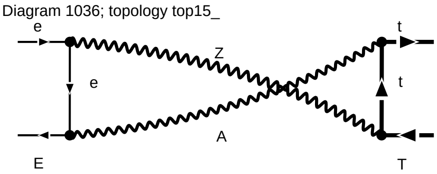
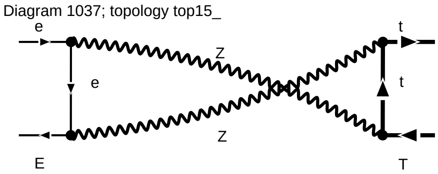


Diagram 1037; topology top15_



Born-like amplitudes: (for $m_e = 0$ total of 6 amplitudes)

Vertex (γ-pole):

$$V_\gamma = \gamma_\mu^{(1)} \times \gamma^{\mu(2)} F_{111}^\gamma + \gamma_\mu^{(1)} \times \gamma^{\mu(2)} \gamma_5^{(2)} F_{115}^\gamma + \gamma_\mu^{(1)} \gamma_5^{(1)} \times \gamma^{\mu(2)} F_{151}^\gamma$$

Vertex (Z-pole):

$$V_Z = \gamma_\mu^{(1)} \times \gamma^{\mu(2)} F_{111}^Z + \gamma_\mu^{(1)} \times \gamma^{\mu(2)} \gamma_5^{(2)} F_{115}^Z + \gamma_\mu^{(1)} \gamma_5^{(1)} \times \gamma^{\mu(2)} F_{151}^Z + \gamma_\mu^{(1)} \gamma_5^{(1)} \times \gamma^{\mu(2)} \gamma_5^{(2)} F_{155}^Z$$

Selfenergies

Photon selfenergy:

$$S_\gamma = \gamma_\mu^{(1)} \times \gamma^{\mu(2)} Q_e Q_t e^4 \Pi_{\gamma\gamma} \frac{1}{s},$$

Z selfenergy:

$$S_Z = \gamma_\mu^{(1)} (Gv_t - Ga_t \gamma_5^{(1)}) \times \gamma_\mu^{(2)} (Gv_e - Ga_e \gamma_5^{(2)}) (\frac{e^2}{2 s_W c_W})^2 \; \Pi_{ZZ} \; \frac{1}{(s - m_Z^2)^2},$$

$\gamma - Z$ mixing:

$$S_{\gamma Z} = -\gamma_\mu^{(1)} (Gv_t - Ga_t \gamma_5^{(1)}) \times \gamma_\mu^{(2)} \; Q_e \; (\frac{e^2}{2 s_W c_W})^2 \; \Pi_{\gamma Z} \; m_Z^2 \; (\frac{1}{s - m_Z^2} - \frac{1}{s}),$$

$$S_{Z\gamma} = -\gamma_\mu^{(1)} \times \gamma_\mu^{(2)} (Gv_e - Ga_e \gamma_5^{(2)}) \; Q_t (\frac{e^2}{2 s_W c_W})^2 \; \Pi_{\gamma Z} \; m_Z^2 \; (\frac{1}{s - m_Z^2} - \frac{1}{s}),$$

With the abbreviations:

$$\begin{aligned}
Gv_e &= -2(Q_e \; s_W^2 + 1/4) \\
Ga_e &= -1/2 \\
Gv_t &= -2(Q_t \; s_W^2 - 1/4) \\
Ga_t &= 1/2 \\
Q_e &= -1 \\
Q_t &= 2/3
\end{aligned}$$

Counterterms:

[1] external wavefunction renormalization factors Z $= za + zb\gamma_5$ for electron and top.

[2] parameter renormalization: mass counterterms for photon (=0) and Z;
coupling counterterms: electron charge e and sinus of the Weinberg angle s_W.

Counterterms from the photon-Born:

$$C_\gamma = -\left[\,\gamma_\mu^{(1)} \times \gamma_\mu^{(2)}\ (za_e + zb_e\gamma_5^{(2)}) + \gamma_\mu^{(1)}\ (za_t + zb_t\gamma_5^{(1)}) \times \gamma_\mu^{(2)} - \gamma_\mu^{(1)} \times \gamma^{\mu(2)}\ 2\frac{\delta(e)}{e}\right] Q_e Q_t \frac{e^4}{s}$$

$\frac{\delta(e)}{e}$:taking off e^2

Counterterms from the Z-Born:

$$\begin{aligned}
C_Z = \big[&-\gamma_\mu^{(1)}(Gv_t - Ga_t\gamma_5^{(1)}) \times \gamma_\mu^{(2)}(Gv_e - Ga_e\gamma_5^{(2)})(za_e + zb_e\gamma_5^{(2)}) - \\
&\gamma_\mu^{(1)}(Gv_t - Ga_t\gamma_5^{(1)})(za_t + zb_t\gamma_5^{(1)}) \times \gamma_\mu^{(2)}(Gv_e - Ga_e\gamma_5^{(2)}) + \\
&\gamma_\mu^{(1)}(Gv_t - Ga_t\gamma_5^{(1)}) \times \gamma_\mu^{(2)}(Gv_e - Ga_e\gamma_5^{(2)})[2\frac{\delta(e)}{e} + (c_W^{-2} - s_W^{-2})\delta(s_W^2)] - \\
&\gamma_\mu^{(1)}(Gv_t - Ga_t\gamma_5^{(1)}) \times \gamma_\mu^{(2)}\ 2Q_e\delta(s_W^2) - \\
&\gamma_\mu^{(1)} \times \gamma_\mu^{(2)}(Gv_e - Ga_e\gamma_5^{(2)})2Q_t\delta(s_W^2)\big]\ (\frac{e^2}{2s_Wc_W})^2\ \frac{1}{s - m_Z^2}.
\end{aligned}$$

$\delta(s_W^2)$:taking off e^2

Some UV divergent terms (in dimensional regularization, $d = 4 - \varepsilon$):

$$F_{111}^\gamma = (-\frac{82}{27} - \frac{62}{27}c_W^{-2} - \frac{m_t^2}{m_W^2} - \frac{1}{3}\frac{m_b^2}{m_W^2})(\frac{e^2}{2s_Wc_W})^2\ c_W^2\ \frac{1}{s}\frac{1}{\varepsilon}$$

$$F_{111}^Z = (\frac{973}{216} - \frac{25}{18}c_W^{-2} - \frac{9}{16}\frac{m_t^2}{m_W^2} - \frac{1}{16}\frac{m_b^2}{m_W^2} - c_W^2\frac{m_t^2}{m_Z^2} - \frac{1}{3}c_W^2\frac{m_b^2}{m_Z^2} - \frac{157}{108}c_W^2 - \frac{82}{27}c_W^4 + \frac{3}{2}\frac{m_t^2}{m_Z^2} + \frac{1}{3}\frac{m_b^2}{m_Z^2})\ (\frac{e^2}{2s_Wc_W})^2\ \frac{1}{s_W^2}\ \frac{1}{s - m_Z^2}\ \frac{1}{\varepsilon}$$

$$\Pi_{\gamma\gamma} = -\frac{23}{3}\frac{1}{\varepsilon},\quad \Pi_{\gamma Z} = (\frac{46}{3}c_W^2\ s - 4m_W^2 - \frac{41}{3}s)\frac{1}{\varepsilon},\quad \Pi_{ZZ} = (\frac{23}{3} + 6s_W^{-2} - \frac{41}{6}c_W^{-2}s_W^{-2})(s - m_Z^2)\frac{1}{\varepsilon}$$

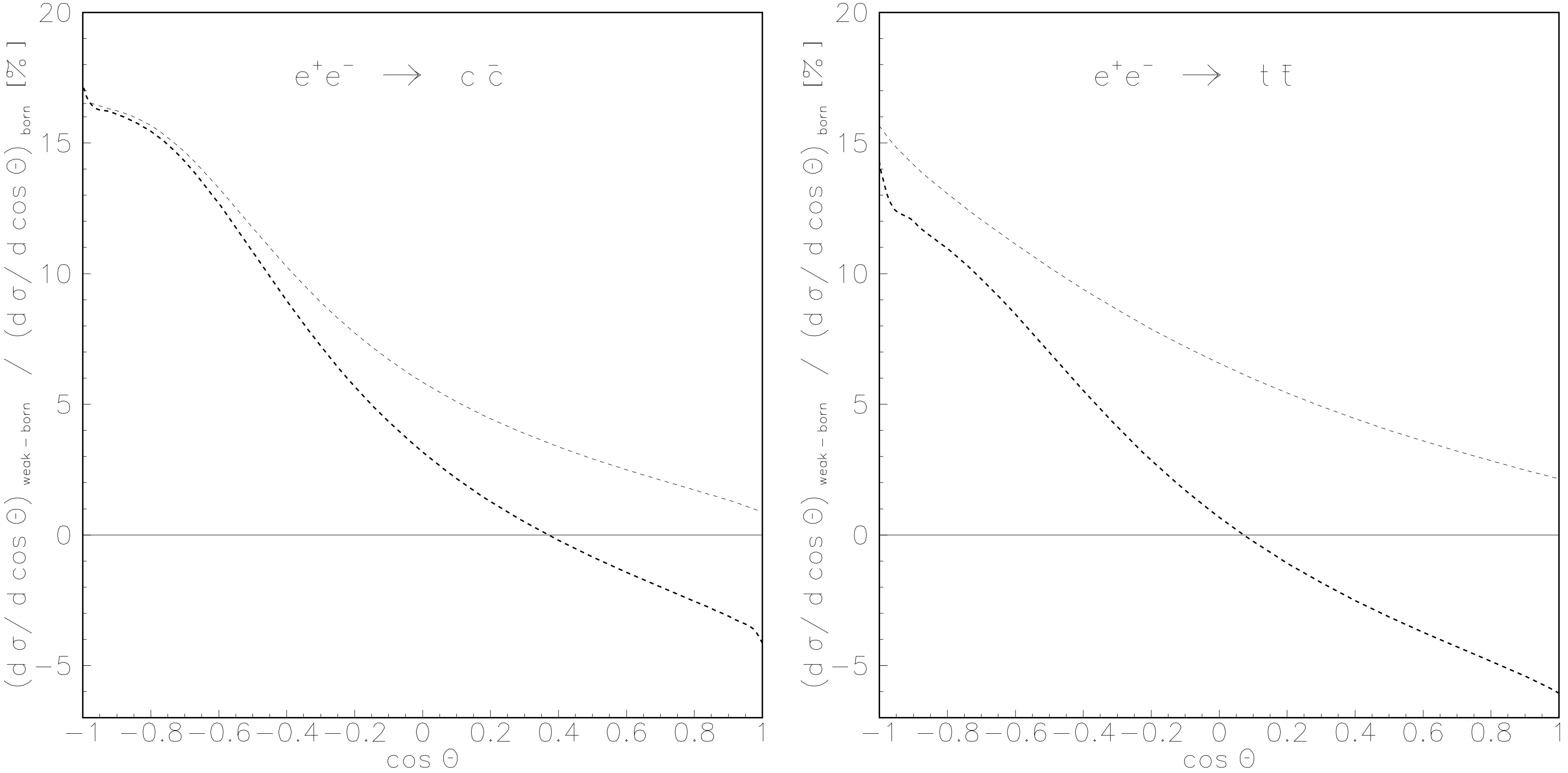
$$za_e = (-\frac{1}{4} + \frac{5}{8}c_W^{-2})s_W^{-2}\frac{1}{\varepsilon},\qquad zb_e = (-\frac{3}{4} + \frac{3}{8}c_W^{-2})s_W^{-2}\frac{1}{\varepsilon}$$

$$\frac{\delta(e)}{e} = \frac{11}{6}\frac{1}{\varepsilon},\qquad \delta(s_W^2) = (\frac{41}{6} - \frac{21}{2}c_W^2 + \frac{11}{3}c_W^4)s_W^{-2}\frac{1}{\varepsilon}$$

12

Top-quark Pair-production Cross section

$\frac{d\sigma}{dcos\theta}$ for $e^+ + e^- \to c + \bar{c} + \gamma$ and $e^+ + e^- \to t + \bar{t} + \gamma$ at TESLA



One-loop percentage corrections with respect to Born.
Thin lines: $\sqrt{s} = 500$ GeV, thick lines: $\sqrt{s} = 1000$ GeV.

Top pair production: Feynarts and topfit

$\sqrt{s} = 500$ GeV: $e^+ + e^- \to \bar{t} + t$ at TESLA

$\cos\theta$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{Born}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{QED}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{weak}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{SM}}$
-0.9	0.1088 3919 4075	0.09866 4253	0.1242 59037 1	0.1140 8410
	0.1088 3919 4075	0.09866 4253	0.1242 59037 6	0.1140 8410
-0.5	0.1422 7506 9392	0.1285 0790	0.1568 48371 9	0.1430 8121
	0.1422 7506 9392	0.1285 0790	0.1568 48371 8	0.1430 8121
0.	0.2254 7046 4032	0.2023 9167	0.2402 6680 4	0.2171 8801
	0.2254 7046 4033	0.2023 9167	0.2402 6680 3	0.2171 8801
0.5	0.3546 6647 0331	0.3151 1724	0.3688 8650 7	0.3293 3727
	0.3546 6647 0332	0.3151 1723	0.3688 8650 5	0.3293 3727
0.9	0.4911 4371 5766	0.4307 1437	0.5033 3751 2	0.4429 0817
	0.4911 4371 5767	0.4307 1437	0.5033 3750 8	0.4429 0816

Agreement confirms accuracy of electroweak calculations of two groups :

upper line: Feynarts, lower line: topfit

Top pair production: topfit and Grace, angular distribution

$\cos\theta$	$\omega/\sqrt{s}$	$\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\,\theta}\right]_{\mathsf{Born}}$	$\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\,\theta}\right]_{\mathsf{QED}}$	$\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\,\theta}\right]_{\mathsf{SM}}$	$\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\,\theta}\right]_{\mathsf{tot}}$
-0.9	T : 0.1	0.108839194075	+0.098664253	+0.11408410	0.13144
	T : 0.00001	0.108839194075	−0.017474702	−0.002054858	0.13229
	G : 0.00001	0.108839194076		−0.002054859	0.13206(12)
-0.5	T : 0.1	0.142275069392	+0.12850790	+0.14308121	0.15973
	T : 0.00001	0.142275069392	−0.029702340	−0.015129038	0.16029
	G : 0.00001	0.142275069393		−0.015129039	0.16013(13)
$+0.0$	T : 0.1	0.225470464033	+0.20239167	+0.21718801	0.23638
	T : 0.00001	0.225470464033	−0.058010508	−0.043214169	0.23476
	G : 0.00001	0.225470464033		−0.043214168	0.23513(14)
$+0.5$	T : 0.1	0.354666470332	+0.31511723	+0.32933727	0.35651
	T : 0.00001	0.354666470332	−0.109721291	−0.095501257	0.35062
	G : 0.00001	0.354666470332		−0.095501252	0.35104(17)
$+0.9$	T : 0.1	0.491143715767	+0.43071437	+0.44290816	0.48796
	T : 0.00001	0.491143715767	−0.179672655	−0.16747886	0.47768
	G : 0.00001	0.491143715767		−0.16747886	0.47709(21)

Various differential cross sections. The upper and lower numbers correspond to the `topfit` (T) and GRACE (G) approach, respectively, $\sqrt{s} = 500$ GeV.

Factorizing one-loop contributions to two-loop Bhabha scattering

J. F., O.V. Tarasov, T. Riemann, A. Werthenbach

We are moving in the direction of **two-loop** Bhabha scattering, which is extremely important, in particular at higher energies, for the luminosity determination of the coming accellerators.

Factorizing one-loop contributions enter directly by 'squaring':

$$1\text{-loop} \times 1\text{-loop} = 2\text{-loop contribution}$$

and via **renormalization.** Mass-renormalization: in any Feynman diagram we replace

$$\frac{1}{k^2-(m^2+\delta(m^2))} \to \frac{1}{k^2-m^2}\left(1+\frac{\delta(m^2)}{k^2-m^2}\right),$$

thus obtaining 'dotted' diagrams.

Decomposition of the diagrams into Amplitudes $\mathbf{A_j}$ (here diagram 8)

$$\begin{array}{lcccccc}
O_1 = & \overline{U}(-p_2) & I & V(-p_3) & \cdot\ \overline{V}(p_4) & I & U(p_1) \\
O_2 = & \overline{U}(-p_2) & \hat{p}_4 & V(-p_3) & \cdot\ \overline{V}(p_4) & I & U(p_1) \\
O_3 = & \overline{U}(-p_2) & I & V(-p_3) & \cdot\ \overline{V}(p_4) & \hat{p}_2 & U(p_1) \\
O_4 = & \overline{U}(-p_2) & \hat{p}_4 & V(-p_3) & \cdot\ \overline{V}(p_4) & \hat{p}_2 & U(p_1) \\
O_5 = & \overline{U}(-p_2) & \gamma_\mu & V(-p_3) & \cdot\ \overline{V}(p_4) & \gamma^\mu & U(p_1) \\
O_6 = & \overline{U}(-p_2) & \gamma_\mu\hat{p}_4 & V(-p_3) & \cdot\ \overline{V}(p_4) & \gamma^\mu & U(p_1) \\
O_7 = & \overline{U}(-p_2) & \gamma_\mu & V(-p_3) & \cdot\ \overline{V}(p_4) & \gamma^\mu\hat{p}_2 & U(p_1) \\
O_8 = & \overline{U}(-p_2) & \gamma_\mu\gamma_\nu & V(-p_3) & \cdot\ \overline{V}(p_4) & \gamma^\nu\gamma^\mu & U(p_1) \\
O_9 = & \overline{U}(-p_2) & \gamma_\mu\gamma_\nu\hat{p}_4 & V(-p_3) & \cdot\ \overline{V}(p_4) & \gamma^\nu\gamma^\mu & U(p_1) \\
O_{10} = & \overline{U}(-p_2) & \gamma_\mu\gamma_\nu & V(-p_3) & \cdot\ \overline{V}(p_4) & \gamma^\nu\gamma^\mu\hat{p}_2 & U(p_1) \\
O_{11} = & \overline{U}(-p_2) & \gamma_\mu\gamma_\nu\hat{p}_4 & V(-p_3) & \cdot\ \overline{V}(p_4) & \gamma^\nu\gamma^\mu\hat{p}_2 & U(p_1) \\
O_{12} = & \overline{U}(-p_2) & \gamma_\mu\gamma_\nu\gamma_\rho & V(-p_3) & \cdot\ \overline{V}(p_4) & \gamma^\rho\gamma^\nu\gamma^\mu & U(p_1)
\end{array}$$

On-shell diagram $= \sum_{j=1}^{12} A_j O_j,$

Further relations between amplitudes (obtained by solving a system of equations):

$$A_3 = A_2, \quad A_7 = A_6, \quad A_{10} = A_9$$

$$A_8 = \frac{1}{4}A_1 - \frac{m_e}{2}A_2 + A_{12}, \quad A_9 = \frac{1}{4m_e}A_1 - \frac{1}{2}A_2 - \frac{1}{2}A_6, \quad A_{11} = \frac{1}{4m_e^2}A_1 - \frac{1}{2m_e}A_2$$

Thus: there are **only 6 independent amplitudes**.

Diagram 1; topology u1_

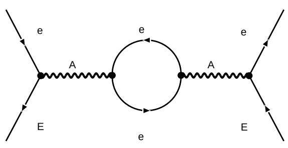


Diagram 2; topology u2_

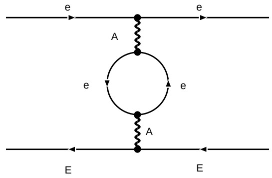


Diagram 3; topology u3_

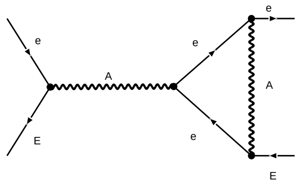


Diagram 4; topology u4_

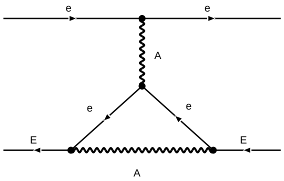


Diagram 5; topology u5_

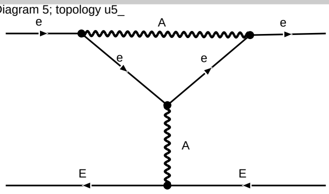


Diagram 6; topology u6_

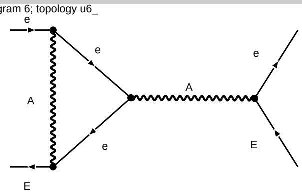


Diagram 7; topology u7_

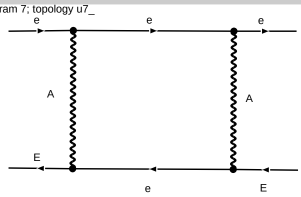


Diagram 8; topology u7_

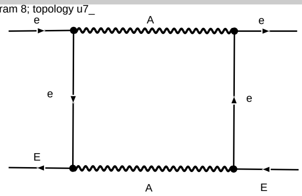


Diagram 9; topology u8_

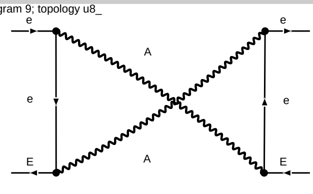
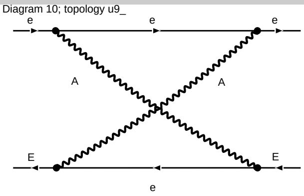


Diagram 10; topology u9_



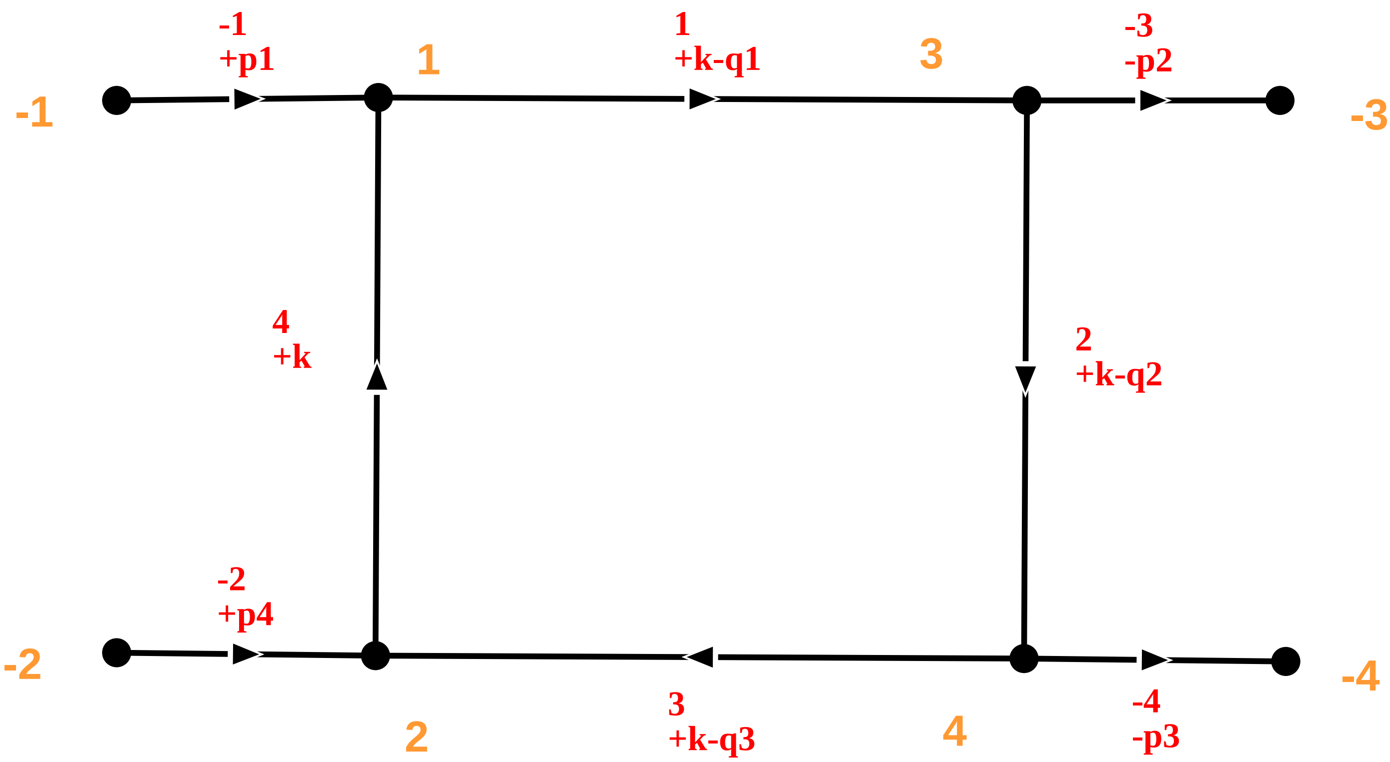
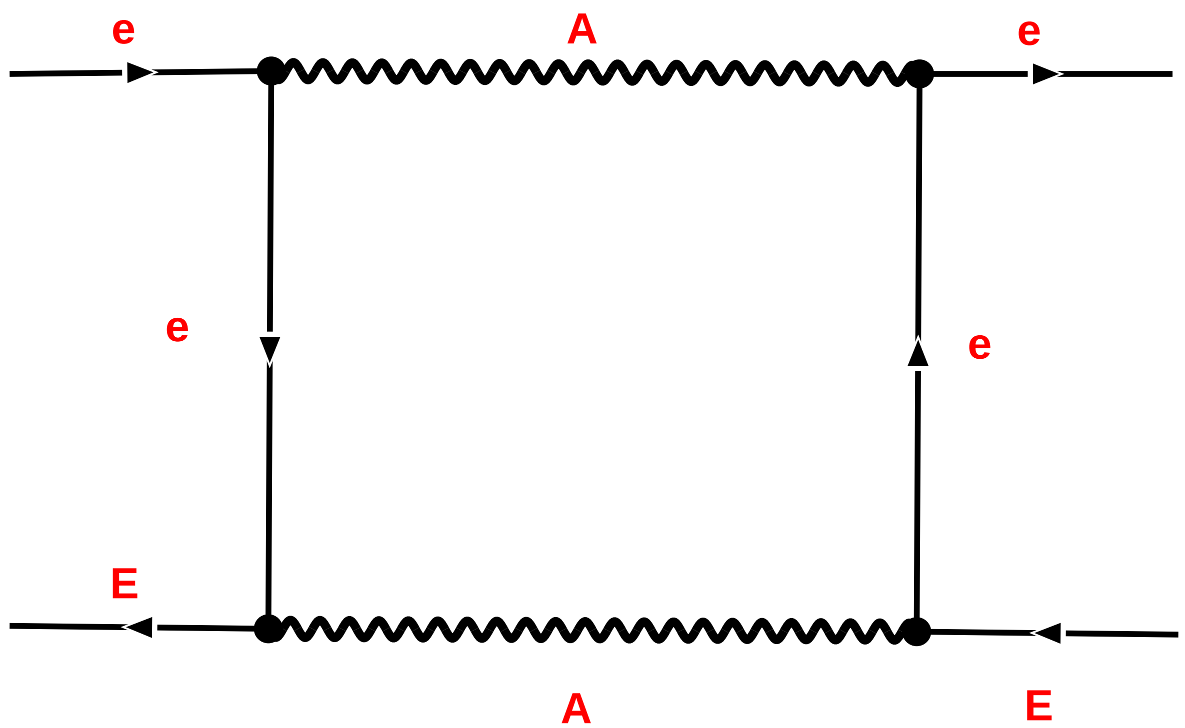


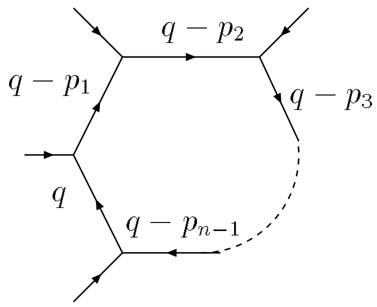
Diagram 8 topology u7_ (unique u7_) momentaset 1 (of 1)



8

Calculation of 1-loop integrals

J.F. , F. Jegerlehner and O.V. Tarasov, Nucl.Phys. B566 (2000) p.423



One-loop diagram with n external legs.

Tensor integrals

$$I_{n,r}^{(d)} = \int \frac{d^d q}{\pi^{d/2}} \prod_{j=1}^{n} \frac{q_{\mu_1} \dots q_{\mu_r}}{c_j^{\nu_j}},$$

$$= T_{\mu_1 \dots \mu_r}(\{p_s\}, \{\partial_j\}, \mathbf{d}) I_n^{(d)}$$

where $\nu_j =$ 'index'

$$c_j = (q-p_j)^2 - m_j^2 + i\epsilon \quad \text{for} \quad j<n \quad \text{and} \quad c_n = q^2 - m_n^2 + i\epsilon.$$

Tensor operator:

$$T_{\mu_1 \dots \mu_r}(\{p_s\}, \{\partial_j\}, \mathbf{d}^+) = \frac{1}{i^r} \prod_{j=1}^{r} \frac{\partial}{\partial a_{\mu_j}} \exp\left[i \left(\sum_{k=1}^{n-1} (a p_k) \alpha_k - \frac{1}{4} a^2 \right) \rho \right] \Bigg|_{\substack{a_j=0 \\ \alpha_j = i\partial_j \\ \rho = i\mathbf{d}^+}} .$$

with

$$\partial_j = \frac{\partial}{\partial m_j^2}, \qquad \mathbf{d}^+ I^{(d)} = I^{(d+2)},$$

Only scalar integrals remain to be evaluated !

Integrals with non-zero Gram determinants

$$\Delta_n = \begin{vmatrix} Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{12} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{1n} & Y_{2n} & \dots & Y_{nn} \end{vmatrix}.$$

where

$$Y_{ij} = -(p_i - p_j)^2 + m_i^2 + m_j^2,$$

"Modified Cayley determinant" of the diagram with internal lines $1 \dots n$

$$()_n \equiv \begin{vmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & Y_{11} & Y_{12} & \dots & Y_{1n} \\ 1 & Y_{12} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & Y_{1n} & Y_{2n} & \dots & Y_{nn} \end{vmatrix},$$

"Signed minors"

$$\begin{pmatrix} j_1 j_2 \cdots \\ k_1 k_2 \dots \end{pmatrix}_n$$

will be labeled by the rows $j_1, j_2, \dots$ and columns $k_1, k_2, \dots$ excluded from $()_n$. E.g. we have

$$\Delta_n = \begin{pmatrix} 0 \\ 0 \end{pmatrix}_n$$

Recursion Relations:

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}_n \nu_j \mathbf{j}^+ I_n^{(d)} = \sum_{k=1}^{n} \begin{pmatrix} 0j \\ 0k \end{pmatrix}_n \left[d - \sum_{i=1}^{n} \nu_i (\mathbf{k}^- \mathbf{i}^+ + 1) \right] I_n^{(d)}.$$

This relation **reduces the indices and leaves the dimension** (operators $\mathbf{j}^\pm$ etc. shift the indices $\nu_j \to \nu_j \pm 1$).

The following one **reduces simultaneously indices and dimension**:

$$()_n \, \nu_j \mathbf{j}^+ I_n^{(d+2)} = \left[- \begin{pmatrix} j \\ 0 \end{pmatrix}_n + \sum_{k=1}^{n} \begin{pmatrix} j \\ k \end{pmatrix}_n \mathbf{k}^- \right] I_n^{(d)},$$

A relation **reducing the space time dimension** is given by:

$$(d-\sum_{i=1}^{n}\nu_i+1)\,()_n\,I_n^{(d+2)}=\left[\binom{0}{0}_n-\sum_{k=1}^{n}\binom{0}{k}_n\mathbf{k}^-\right]I_n^{(d)}.$$

Integrals with zero kinematic determinants

$$(d-\sum_{i=1}^{n}\nu_i+1)\,()_n\,I_n^{(d+2)}=-\sum_{k=1}^{n}\binom{0}{k}_n\mathbf{k}^-I_n^{(d)}.$$

To increase again the dimension d one uses the relation

$$\sum_{j=1}^{n}\nu_j\mathbf{j}^+I_n^{(d+2)}\ =-I_n^{(d)}\ .$$

As an example we show the relation for the three-point integral C_0 with zero Gram determinant:

$$(s-4m_e^2)C_0(m_e,0,m_e,m_e^2,m_e^2,s)=-\frac{2(d-3)}{(d-4)}B_0(m_e^2,m_e^2,s)-\frac{(d-2)}{m_e^2(d-4)}A_0(m_e^2)$$

or

$$(s-4m_e^2)\frac{\Gamma(2-\frac{d}{2})}{2(m_e^2)^{3-\frac{d}{2}}}\ {}_2F_1(1,3-\frac{d}{2},\frac{3}{2};\frac{s}{4m_e^2})=$$

$$-\frac{2(d-3)}{d-4}\frac{\Gamma(2-\frac{d}{2})}{(m_e^2)^{2-\frac{d}{2}}}\ {}_2F_1(1,2-\frac{d}{2},\frac{3}{2};\frac{s}{4m_e^2})-\frac{d-2}{d-4}\frac{\Gamma(1-\frac{d}{2})}{2(m_e^2)^{2-\frac{d}{2}}}\ {}_2F_1(0,2-\frac{d}{2},\frac{3}{2};\frac{s}{4m_e^2}),$$

which is a Gauss relation between contiguous hypergeometric functions.

'Undotted' Diagrams; common factor $\frac{e^2}{(4\pi)^{d/2}}$ (normalized to Born term)

Selfenergy - diagrams:

$$A_5 = A_0 \frac{4}{s^2}[\frac{1}{d-1} - 1] + B_e \frac{2}{s}[\frac{1}{d-1}(1 - z^2) - 1]$$

Vertex - diagrams:

$$A_2 = A_0 \frac{2}{m_e s^2} x^2 [2\frac{1}{d-3} - (d-4)] + B_e \frac{4m_e}{s^2} x^2 [1 - (d-4)]$$

$$A_5 = 2(-A_0 \frac{1}{m_e^2 s}[2\frac{1}{d-3} - x^2 z^2] + B_e \frac{1}{s}[x^2(1+z^2) + d - 4] + F_s \frac{2}{(d-4)s}[1 + x^2])$$

'Dotted' Diagrams; common factor $\delta(m_e)/m_e\ e^4$

Selfenergy - diagrams:

$$A_5 = A_0 \frac{4}{s^2}(d-2)x^2 z^2 - B_e \frac{2}{s} z^2 [1 - (d-3)x^2]$$

Vertex - diagrams:

$$A_2 = -A_0 \frac{4}{m_e s^2} x^2 [\frac{1}{d-3} x^2 + (d-4)(1 + (d-4)(1 - x^2/2)) - 3 + 2x^2]$$
$$-B_e \frac{4m_e}{s^2} x^2 [(d-4)(x^2 - (d-4)x^2 z^2) + 2x^2 z^2] + F_s \frac{8m_e}{(d-4)s^2} x^2 [1 + x^2]$$

$$A_5 = 2(A_0 \frac{2}{s^2} x^2 [(\frac{1}{d-3} + 6\frac{1}{d-5}) - (d-4)(2x^2 - 1 + (d-4)) + 7 - 4x^2]$$
$$+A_0 \frac{2}{m_e^2 s}[1 + 3\frac{1}{d-5}] - B_e \frac{1}{s} x^2 z^2 [(d-4)(2x^2 + (d-4)) + 2x^2] - F_s \frac{2}{(d-4)s} x^2 z^2)$$

Notation:

$$A_0 = A_0(m_e), \quad B_e = B_0(m_e, m_e, s), \quad F_s = A_0(m_e)/{m_e}^2 + B_0(m_e, m_e, s).$$

Further abbreviations:

$$x^2 = 1/(1 - 4m_e^2/s), \quad z^2 = \frac{4m_e^2}{s}\ (x^2 - 1 = x^2 z^2).$$

Contribution to the differential cross section

$$\frac{d\sigma}{dcos\theta} = \frac{\pi\alpha^2}{2s}\sum_{j=1}^{12}\left[\frac{B_j(s,t)}{4}A_j(s,t) - \frac{B_j(t,s)}{4}A_j(t,s).\right]$$

$$B_1 = -\frac{4}{t}s^2 + (2d - 8 + \frac{24}{t}m_e^2)s - \frac{32}{t}m_e^4 - 8dm_e^2 - 32\frac{m_e^2}{s}(t - 2m_e^2),$$

$$B_2 = \frac{4}{t}m_e(s^2 + 8m_e^4) + (2(4-d)m_e - \frac{24}{t}m_e^3)s - 4(d+2)m_e t + 8(d-2)m_e^3 - 16\frac{m_e}{s}(t - 2m_e^2)^2,$$
$$B_3 = B_2,$$
$$B_4 = -\frac{4}{t}(m_e^2 + t)s^2 + ((-4 - 2d)t + 2dm_e^2 + \frac{24}{t}m_e^4)s - 2(d+4)t^2 + 8(1+d)m_e^2 t - 8(d-4)m_e^4$$
$$-\frac{32}{t}m_e^6 - \frac{8}{s}(t - 2m_e^2)^3,$$
$$B_5 = -\frac{1}{m_e}B_6 + 4td + \frac{8(st + 2t^2 + 2m_e^2 s - 4m_e^2 t)}{s},$$
$$\frac{B_6}{m_e} = -4\frac{(d-2)}{t}(s^2 - 2tm_e^2) + (2(d-2)(d-10) + 8(d-4)\frac{m_e^2}{t})s + 32\frac{m_e^4}{t} + 32\frac{m_e^2}{s}(t - 2m_e^2),$$

$$B_7 = B_6,$$
$$B_8 = -4\frac{(d-2)^2}{t}s^2 + (2(d-2)(d-4)(d-10) + 8(3d^2 - 8d + 8)\frac{m_e^2}{t})s - 8(d-2)(d-4)t$$
$$-8(d-2)(d^2 - 10d + 20)m_e^2 - 32(d^2 - 2d + 2)\frac{m_e^4}{t} - 32d\frac{m_e^2}{s}(t - 2m_e^2),$$

$$\frac{B_9}{m_e} = 4(d-2)^2\frac{s^2}{t} + (2(2-d)(d-4)(d-10) - 8(3d^2 - 8d + 8)\frac{m_e^2}{t})s - 4(d^3 + 34d$$
$$-10d^2 - 28)t + 8(d^3 - 14d^2 + 52d - 60)m_e^2 + 32(d^2 - 2d + 2)\frac{m_e^4}{t} - 16d\frac{(t - 2m_e^2)^2}{s},$$
$$B_{10} = B_9,$$

$$B_{11} = (-4(d-4)^2 t - 4(d-2)^2 m_e^2)s^3 + (2(4-d)(d^2 - 4d - 2)t^2 + 2(d^3 + 20d - 12d^2 + 32)m_e^2 t$$
$$+8(3d^2 - 8d + 8)m_e^4)s^2 + ((20d^2 - 68d + 64 - 2d^3)t^3 + 8(d-4)(d^2 - 8d + 10)m_e^2 t^2 - 8(d-2)$$
$$(d^2 - 14d + 28)m_e^4 t - 32(d^2 - 2d + 2)m_e^6)s - 8(d-2)t^3(t - 6m_e^2) - 32(3d-4)m_e^4 t^2 + 64dm_e^6 t,$$

$$B_{12} = 4(d-2)^3 s^3 + (-2(d-2)(d^3 - 20d^2 + 110d - 208)t - 8(24d - 16 + d^3 - 12d^2)m_e^2)s^2$$
$$+((12d^3 - 120d^2 - 352 + 400d)t^2 - 8(3d^3 - 42d^2 + 148d - 160)m_e^2 t - 32(4 - 6d + 3d^2)m_e^4)s$$
$$+16(3d-2)t(t^2 - 4m_e^2 t + 4m_e^4).$$

One loop integrals occurring in Bhabha scattering

1. Two - point integrals

$$I_G^{(d)} = I_2^{(d)}\Big|_{m_1=0,m_2=0,p^2} = \frac{\sqrt{\pi}}{(-p^2)^{(2-\frac{d}{2})}} \frac{\Gamma\left(2-\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)}{2^{d-3}\ \Gamma\left(\frac{d-1}{2}\right)}$$

$$I_F^{(d)} = I_2^{(d)}\Big|_{m_1=m,m_2=m} = (m^2)^{-(2-\frac{d}{2})}\ \Gamma\left(2-\frac{d}{2}\right)\ {}_2F_1\left[\begin{matrix} 1, 2-\frac{d}{2}\,; \\ \frac{3}{2}\,; \end{matrix}\ \frac{p^2}{4m^2}\right].$$

2. Three - point integrals

For **two zero masses** and final on-shell momenta we have (from **J. F., F. Jegerlehner and O.V Tarasov, unpublished**)

$$\frac{J_G}{\Gamma\left(2-\frac{d}{2}\right)} = \frac{(m^2)^{\frac{d}{2}-3}}{2(d-3)}\ {}_2F_1\left[\begin{matrix} 1, 1\,; \\ \frac{d-1}{2}\,; \end{matrix}\ 1-\frac{p^2}{4m^2}\right] - \frac{\sqrt{\pi}\Gamma\left(\frac{d-2}{2}\right)(-p^2)^{\frac{d-4}{2}}}{2^{d-2}\Gamma\left(\frac{d-1}{2}\right)m^2}\ {}_2F_1\left[\begin{matrix} 1, \frac{d-2}{2}\,; \\ \frac{d-1}{2}\,; \end{matrix}\ 1-\frac{p^2}{4m^2}\right],$$

where

$$J_G = -C_0(0, me, 0, m_e^2, m_e^2, p^2).$$

For the other master three-point integral with **two equal masses** and **one zero mass** and final on-shell momenta the **Gram determinant is zero**; a compact result is (see above)

$$J_F = \frac{\Gamma\left(2-\frac{d}{2}\right)}{2\ m^{6-d}}\ {}_2F_1\left[\begin{matrix} 1, 3-\frac{d}{2}\,; \\ \frac{3}{2}\,; \end{matrix}\ \frac{p^2}{4m^2}\right],$$

where

$$J_F = -C_0(m_e, 0, m_e, m_e^2, m_e^2, p^2).$$