# Rare Z Decays

Gad Eilam (Technion, Haifa, Israel)

RADCOR02

Work done in collaboration with:
D. Atwood, S. Bar-Shalom and A. Soni
PRD (2002), in press hep-ph/0203200

# Outline

1: Introduction: The importance of rare decays

2: Generic calculation

3: Predictions in various models

3I: 2HDM

3IA: 2HDMII

3IB: T2HDM

3II: Supersymmetry with squark mixing

3IIA: $\tilde{b} - \tilde{s}$ mixing

3IIB: $\tilde{t} - \tilde{c}$ mixing

3III: Supersymmery with $\not{R}_P$

3IIIA: Trilinear−trilinear terms

3IIIB: Trilinear−bilinear terms

4: Experimental feasability

5: Summary and conclusions

*************************************************************************

## 1: Introduction: The importance of rare decays

For the sake of this introduction:
"rare" $\equiv$ Flavor Changing Neutral Currents (FCNC) which are either small or 0 in the SM.

Very good experience with FCNC, for example:

- In $K$ physics: The first appearance of charm in loops Gaillard & Lee '73.

- In $B$ physics: The importance of $b \to s\gamma$ for extracting paramreters,
  see *e.g.* Hewett '93, Masiero, ...

- The top quark FCNC provide an excellent tool to investigate various beyond the SM scenarios: see *e.g.* Atwood, Bar-Shalom, GE and Soni '01 for a review.

In the cases of $K$ and $B$, FCNC were measured and found to be small in accordance with their SM value, while the FCNC top quark decays in the SM Hewett, GE and Soni '91, and erratum are beyond the reach of any conceivable experiment, with:
$10^{-14} \lesssim \mathrm{BR}(\text{FCNC top decays}) \lesssim 10^{-10}$.

FCNC of $Z$ into quarks, which is the subject of this talk, reside somewhere in between $K$, $B$ and $t$, with:

$$\boxed{\mathrm{BR}(Z \to b\bar{s}) \approx 3 \times 10^{-8} \qquad \text{SM}}$$

Mann and Riemann '82, Clements *et al.* '83, Ganapathi *et al.* '83, Hou, Deshpande, GE and Soni '86, Bernabeu, Gavela and Santamria '86.

As we'll see, in the extensions above $Z \to b\bar{s}$ can be either smaller, the same or above the SM with a maximal value of $\mathrm{BR}(Z \to b\bar{s}) \approx 10^{-6}$.

**A comment**: $Z$ decays that are 0 in the SM, like $Z \to l_I \bar{l}_J$, which are actually $\lesssim 10^{-54}$ in the SM with small Dirac neutrino masses, will be discussed by Illana.

Experimentally, the attention devoted to FCNC in $Z$ decays at LEP and SLD has been close to 0. The best upper limit is DELPHI '99

$$\sum_{q=d,s} \mathrm{BR}(\mathrm{Z} \to \mathrm{b}\bar{\mathrm{q}}) \leq 1.8 \times 10^{-3} \text{ at } 90\% \text{ CL.}$$

Preliminary, based on about $3.5 \times 10^6$ hadronic decays. LEPists should be encouraged to look in their combined data, and improve the above limit.

Two possible venues in which at least Giga-$Z$ bosons will be produced:

1. At LHC with L$=$ 100 fb$^{-1}$: one expects $5.5 \times 10^9$ $Z$ bosons.

2. Cleaner environment in a future $e^+e^-$ linear collider. *E.g.*, it may be possible to lower TESLA to $\sqrt{s} = m_Z$ ("Giga-Z" option.) L$=$ 30 fb$^{-1}$, more than $10^9$ $Z$ bosons will be produced.
   This is $\approx$ 2 orders of magnitude larger than in LEP.

To grasp the improvement in going from LEP to Giga-Z option of TESLA:
The sensitivity of LEP to $Z \to \tau\mu$ was $\approx 10^{-5}$, it is expected to be $\approx 10^{-8}$ in Giga-Z TESLA.

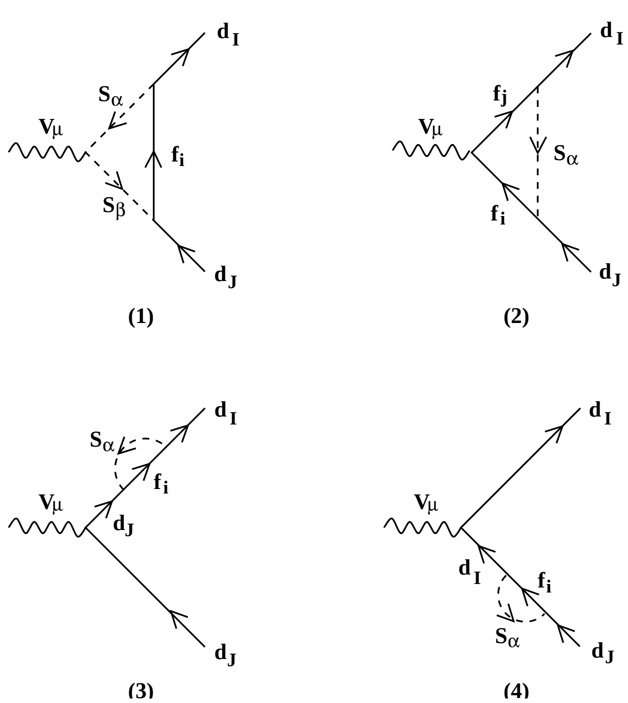
∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗

2: Generic calculation

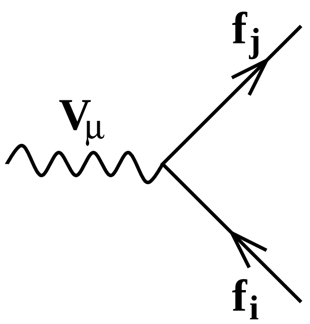
We outline a generic calculation of the one-loop flavor changing vertex $V d_I \bar{d}_J$. We specialize here to: $V = Z$ and $d_I = b$, $\bar{d}_J = \bar{s}$. Actually, we mean:

$$\Gamma(Z \to b\bar{s}) \equiv \Gamma(Z \to b\bar{s} + \bar{b}s).$$

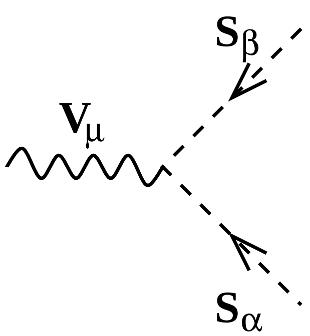
The diagrams that modify (at one loop) the $V d_I \bar{d}_J$ coupling due to charged or neutral scalar exchanges are:



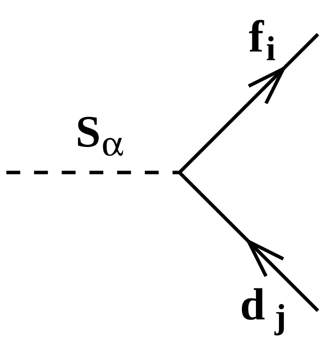
The Feynman rules are:




$$i\gamma_\mu \left(a^{ij}_{L(Vf)} L + a^{ij}_{R(Vf)} R\right)$$




$$i g_V^{\alpha\beta} \left(p_\alpha - p_\beta\right)_\mu$$




$$i \left(b^{ij}_{L(\alpha)} L + b^{ij}_{R(\alpha)} R\right)$$

Where $i, j$ and $\alpha, \beta$ indicate which fermions and scalars we're considering, respectively, $L(R) = [1-(+)\gamma_5]/2$, $V = Z$. If no confusion can arise, as many indices as possible will become invisible.

There are 4 one-loop amplitudes, each corresponding to one of the 4 one-loop diagrams. Each amplitude is proportional to:

$$\epsilon_\mu(q)\bar{u}(p_b)\left[\gamma^\mu\left(A_L L + A_R R\right) + \left(B_L L + B_R R\right)p_\mu\right]v(p_s).$$

$A_{L,R}$, $B_{L,R}$ are momentum dependent form factors, calculable from the diagrams.
There are 4/diagram $\Rightarrow$ 16 form factors. With most indices resurrected, $A_L$ for diagram (1) is:

$$A_L = -2\sum_{\alpha,\beta,i} g_Z^{\alpha\beta} b^{iI}_{L(\alpha)} b^{iJ}_{L(\beta)} C_{24},$$

where $I, J = b, s$. Similary for the other 15 form factors. $C_{24}$ is one of the usual Passarino Veltman scalar functions, at $m^2_{f_i}, m^2_{S_\alpha}, m^2_{S_\beta}, m^2_{d_I}, q^2, m^2_{d_J}$.

Finally:

$$\Gamma(Z \to b\bar{s}) = 2\frac{N_C}{3}\left(\frac{1}{16\pi^2}\right)^2 \frac{M_Z}{16\pi} \times \left[2\left(\mid A_L^T \mid^2 + \mid A_R^T \mid^2\right) + \frac{M_Z^2}{4}\left(\mid B_L^T \mid^2 + \mid B_R^T \mid^2\right)\right]$$

$A_L^T$ is the Total sum of $A_L$'s from the 4 diagrams, and similarly for $A_R^T$, $B_L^T$ and $B_R^T$.

## 3: Predictions in various models

### 3I: 2HDM

In a 2HDM with flavor diagonal couplings of the neutral Higgs to down-quarks, the FCNC $Z \to b\bar{s}$ goes through the one-loop diagrams in the 1st figure. They are:

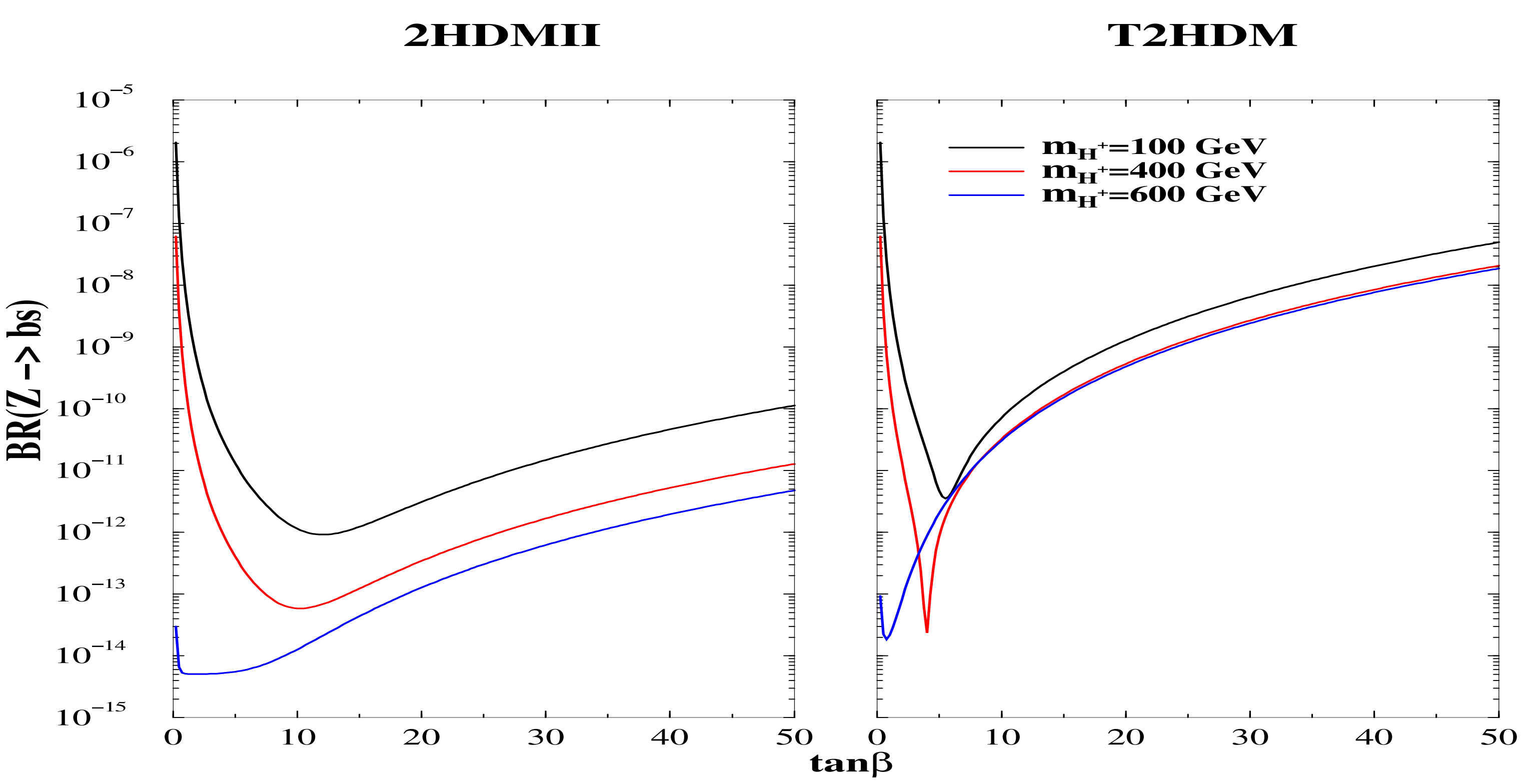
$$
\begin{array}{rl}
Z_\mu u_i \bar{u}_j & \text{as in the SM,} \\
Z_\mu H^+ H^- & \text{from } (D^\mu \Phi_i)^\dagger (D_\mu \Phi_i)\,, \\
H^+ \bar{u}_i d_j & \text{from the Yukawa :}
\end{array}
$$

$$
\begin{aligned}
\mathcal{L}_Y \;=\; & -\sum_{i,j} \bar{Q}_L^i \left[ \left( U_{ij}^1 \tilde{\Phi}_1 + U_{ij}^2 \tilde{\Phi}_2 \right) u_R^j \right. \\
& \left. + \left( D_{ij}^1 \Phi_1 + D_{ij}^2 \Phi_2 \right) d_R^j \right]
\end{aligned}
$$

$Q_L$: SU(2) L doublet,
$u_R$, $d_R$: SU(2) singlets,
$\Phi_{1,2}$: The 2 SU(2) Higgs doublets,
$\tilde{\Phi}_{1,2} = i\sigma_2 \Phi_{1,2}^*$ .
$U^1, U^2, D^1, D^2$: General Yukawa 3$\times$3 matrices in flavor space. Their choice $\Rightarrow$ a specific 2HDM.

3IA: 2HDMII

Choose: $U^1 = 0$, $D^1 = 0$ Froggatt, Moorhouse, and Knowles '92 $\Rightarrow$ $m_u$ from $\Phi_2$, $m_d$ from $\Phi_1$. This is realized in MSSM.
$Z \to b\bar{s}$ in 2HDMII in the lhs was done by Busch, Hou and Stuart, Grzadkowki+Gunion+Krawczyk (all in '89). Results are in the lhs of the figure:




For $\tan\beta \sim \mathcal{O}(1)$: Dominated by $t$ couplings $\propto 1/\tan\beta$ $\Rightarrow$ BR $\propto (m_t/\tan\beta)^4 \times (V_{CKM}^{tb})^2 (V_{CKM}^{ts})^2$. $\Rightarrow$ BR $\uparrow$ as $\tan\beta \downarrow$.

At $\tan\beta \sim 13$: $c \sim t$ since $c$ contribution $\propto m_s^2 m_b^2 \tan^4\beta$ $(V_{CKM}^{cs})^2 (V_{CKM}^{cb})^2$.

For $\tan\beta > 13$ $c \sim t$ dominated by $b_R^{ij} \propto \tan\beta \Rightarrow$ BR $\uparrow$ $\tan\beta \uparrow$.

The curves for 2HDMII pass through unrealistic values in the $\tan\beta - m_{H+}$ plane $\Rightarrow$

$$\boxed{\mathrm{BR}(\mathrm{Z} \to \mathrm{b}\bar{\mathrm{s}}) \lesssim 10^{-10} \qquad\qquad \text{2HDMII.}}$$

2 orders of magnitude below the SM.

3IB: T2HDM

In this 2HDM "for the top" Das+Kao, $t$ is rewarded for its "fatness" by getting its mass from the large $v_2$ $\Rightarrow \tan\beta >> 1.$ All other quarks acquire mass from $v_1$.

$$\begin{aligned} U_{ij}^1 &\to G_{ij} \times (\delta_{j1} + \delta_{j2}) \ , \\ U_{ij}^2 &\to G_{ij} \times \delta_{j3} \ , \\ D_{ij}^2 &\to 0. \end{aligned}$$

where $G$ is an unknown Yukawa $3 \times 3$ matrix in quark flavor space. All the couplings of 2HDMII and THDM are summarized in the table:

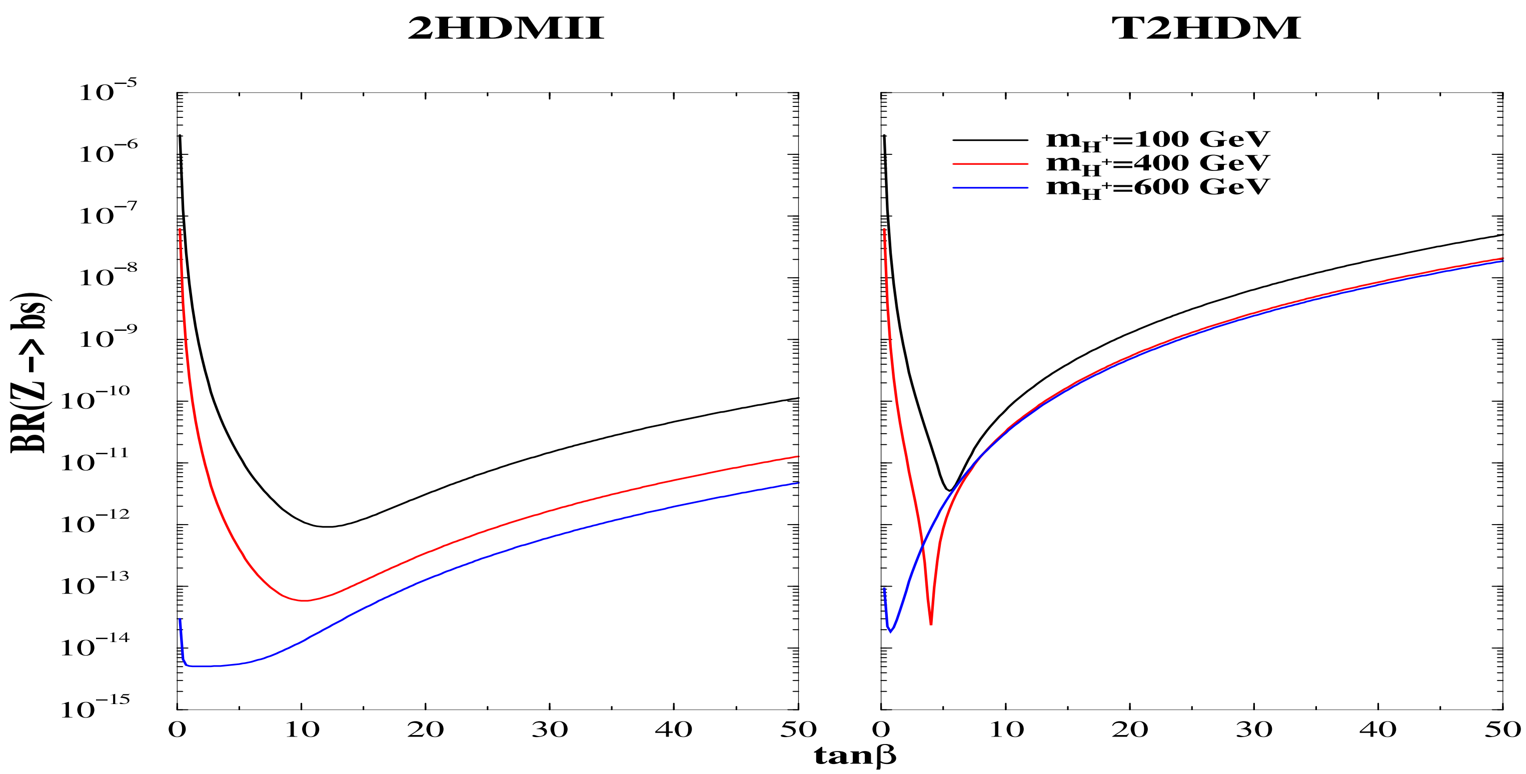
| | 2HDMII | T2HDM |
|---|---|---|
| scalar ($S_{\alpha=1}$) | $H^+$ | $H^+$ |
| fermion ($f_i$) | $u_i,\ i=1,2,3$ | $u_i,\ i=1,2,3$ |
| $a^{ij}_{L(Zf)}$ | $a_{L(Zu)}$ | $a_{L(Zu)}$ |
| $a^{ij}_{R(Zf)}$ | $a_{R(Zu)}$ | $a_{R(Zu)}$ |
| $b^{ij}_{L(\alpha=1)}$ | $Y_{ij}\times\frac{1}{\tan^2\beta}\frac{m_{u_i}}{m_t}$ | $Y_{ij}\times\left[\frac{\Sigma^\dagger_{ik}V^{kj}_{CKM}}{m_t V_{ij}}\left(1+\frac{1}{\tan^2\beta}\right)-\frac{m_{u_i}}{m_t}\right]$ |
| $b^{ij}_{R(\alpha=1)}$ | $Y_{ij}\times\frac{m_{d_j}}{m_t}$ | $Y_{ij}\times\frac{m_{d_j}}{m_t}$ |
| $g_Z^{\alpha=1\beta=1}$ | $-e\frac{1-2s_W^2}{2s_W c_W}$ | $-e\frac{1-2s_W^2}{2s_W c_W}$ |

$$Y_{ij}=-\frac{e}{\sqrt{2}s_W}\frac{m_t}{M_W}\tan\beta V^{ij}_{CKM}\ .$$

$$\Sigma=\begin{pmatrix} 0 & 0 & 0\\ 0 & m_c\varepsilon_{ct}^2|\xi|^2 & m_c\varepsilon_{ct}\xi^\star\\ 0 & m_c\xi\sqrt{1-|\varepsilon_{ct}\xi|^2} & m_t\left(1-|\varepsilon_{ct}\xi|^2\right)\end{pmatrix},$$

$\varepsilon_{ct}=m_c/m_t$ and $\xi$ is an unknown parameter. We assume: $\xi=1$.

The results for $Z\to b\bar{s}$ in T2HDM are on the rhs of the figure:




In 2HDMII and T2HDM: $\bar{t}_R d^i_L \propto m_t V^{td^i}_{CKM}/\tan\beta$.

In T2HDM: $\bar{c}_R b_L H^+ \propto m_c V^{tb}_{CKM}\tan\beta$,

$\bar{c}_R s_L H^+ \propto m_c V^{cs}_{CKM}\tan\beta$.

$\Rightarrow$ These $c$ couplings are enhanced in T2HD vs 2HD-MII by: $\tan^2\beta \times (V^{tb}_{CKM}/V^{cb}_{CKM})$, $\tan^2\beta$, respectively.

For large $\tan\beta$ $c$ dominates, and in view of the constraints from data Kiers+Soni+Wu ×2 '99 & '00, Soni+Wu '00,

$$\boxed{\mathrm{BR}(Z \to b\bar{s}) \lesssim 10^{-8} \qquad \text{T2HDM.}}$$

3II: Supersymmetry with squark mixing

FCNC in SUSY can emanate from squark mixing in:

$$\begin{aligned}\mathcal{L}_{\text{soft}}^{\text{squark}} &= -\tilde{Q}_i^\dagger (M_Q^2)_{ij}\tilde{Q}_j - \tilde{U}_i^\dagger (M_U^2)_{ij}\tilde{U}_j - \tilde{D}_i^\dagger (M_D^2)_{ij}\tilde{D}_j \\ &+ A_u^{ij}\tilde{Q}_i H_u \tilde{U}_j + A_d^{ij}\tilde{Q}_i H_d \tilde{D}_j \ ,\end{aligned}$$

$\tilde{Q}$: SU(2) squark doublet
$\tilde{U}$, $\tilde{D}$: SU(2) squark singlets.
$i, j$ are generation indices.

$$M_{U,D}^2 = \begin{pmatrix} (m^2_{\tilde{U},\tilde{D}})_{LL} & (m^2_{\tilde{U},\tilde{D}})_{LR} \\ (m^2_{\tilde{U},\tilde{D}})^\dagger_{LR} & (m^2_{\tilde{U},\tilde{D}})_{RR} \end{pmatrix},$$

where $(m^2_{\tilde{U},\tilde{D}})_{LL}$ ..., are $3 \times 3$ matrices.
Under certain asssumptions, $m^2$ are related to quantities in $\mathcal{L}$ above, via $V_{L,R}^{D,U}$ (where $V_{CKM} = V_L V_L^{D\dagger}$)
Misiak+Pokorski+Rosiek '98:

$$\begin{aligned}(m^2_{\tilde{U},\tilde{D}})_{LL} &= V_L^{U,D} M_Q^2 V_L^{U,D\dagger}, \\ (m^2_{\tilde{U},\tilde{D}})_{RR} &= V_R^{U,D} M_{U,D}^{2T} V_R^{U,D\dagger},\end{aligned}$$

and

$$(m^2_{\tilde{U},\tilde{D}})_{LR} = \mp \frac{v \cdot (\sin\beta, \cos\beta)}{\sqrt{2}} V_L^{U,D} A^*_{u,d} V_R^{U,D\dagger}.$$

Assume: Only $\tilde{b}-\tilde{s}$ or $\tilde{t}-\tilde{c}$ mixing. Write:

$$(m^2_{\tilde{U},\tilde{D}})_{LL,RR} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \delta^{U,D(23)}_{LL,RR} \\ 0 & \delta^{U,D(32)}_{LL,RR} & 1 \end{pmatrix} m_0^2 \; .$$

The above $\delta$'s represent squark mixing from non-diagonal bilinears in $\mathcal{L}$. $m_0$ is a common squarks mass scale, obeying: $m_0 >> M_Z$.

$\delta_{LR}$'s stand for squark mixing from non-diagonal trilinears in $\mathcal{L}$. For them we adopt the Ansatz Diaz-Cruz+He+Yuan '02, where $V_L^{U,D} A^*_{u,d} V_R^{U,D\dagger} \propto$ to a common $A$ multiplied by $3\times3$ matrices with a block of 0's on the upper left-hand corner.

$M^2_{D,U}$ become $4\times4$ matrices in the weak bases:

$$\Phi^0_{D,U} = (\tilde{s}_L, \tilde{s}_R, \tilde{b}_L, \tilde{b}_R),\ (\tilde{c}_L, \tilde{c}_R, \tilde{t}_L, \tilde{t}_R).$$

They are diagonalized to obtain the mass eigenstates:

$$\Phi_{D,U} = (\tilde{s}_1, \tilde{s}_2, \tilde{b}_1, \tilde{b}_2),\ (\tilde{c}_1, \tilde{c}_2, \tilde{t}_1, \tilde{t}_2),$$

with the help of $R_{U,D}$:

$$\Phi^0_{D,U,i} = R_{D,U,ik} \Phi_{D,U,k} \; .$$

3IIA: $\tilde{b}-\tilde{s}$ mixing

Applying to $Z\to b\bar{s}$: $\tilde{b}-\tilde{s}$ admixture states in $\Phi_{D,\alpha}$, run in the loops. There's one less diagram (out of the generic 4), since: $Z\tilde{g}\tilde{g}=0$. The relevant sparticles and form-factors are Rosiek '90, '95:

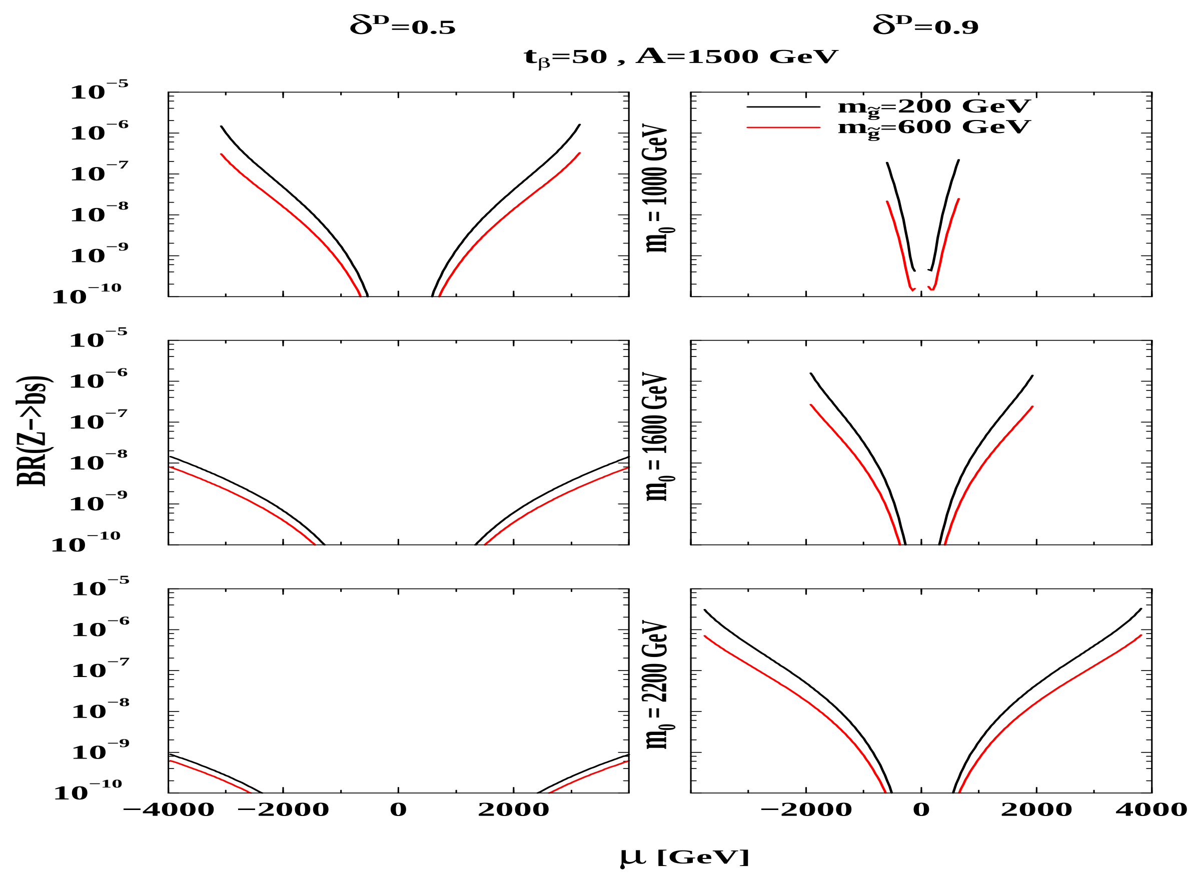
| | SUSY with $\tilde{b}-\tilde{s}$ mixing |
|---|---|
| scalar ($S_\alpha$) | $\Phi_{D,\alpha}, \quad \alpha=1,2,3,4$ |
| fermion ($f_i$) | $\tilde{g}$ |
| $a^{ij}_{L(Zf)}$ | 0 |
| $a^{ij}_{R(Zf)}$ | 0 |
| $b^{ij}_{L(\alpha)}$ | $-\sqrt{2}g_sT^a\left(R^\star_{D,3\alpha}\delta_{3j}+R^\star_{D,1\alpha}\delta_{2j}\right)$ |
| $b^{ij}_{R(\alpha)}$ | $-\sqrt{2}g_sT^a\left(R^\star_{D,4\alpha}\delta_{3j}+R^\star_{D,2\alpha}\delta_{2j}\right)$ |
| $g_Z^{\alpha\beta}$ | $\frac{-e}{2s_Wc_W}\left(R^\star_{D,1\alpha}R_{D,1\beta}+R^\star_{D,3\alpha}R_{D,3\beta}-\frac{2}{3}s^2_W\delta_{\alpha\beta}\right)$ |

The parameters:

I. From $b\to s\gamma$ Misiak+Pokorski+Rosiek '98, ... Besmer+Greub+Hurth '01: $\delta^{D(23)}_{LR}$, $\delta^{D(32)}_{LR}\lesssim\mathcal{O}(10^{-2})$.
We'll neglect them and use: $\delta^{D(23)}_{LL}=\delta^{D(32)}_{LL}=\delta^{D(23)}_{RR}=\delta^{D(32)}_{RR}=\delta^D$, and vary $0<\delta^D<1$.

II. Parameters needed for for masses, mixing and $\Gamma(Z \to b\bar{s})$: $m_0,\ \mu,\ A,\ \tan\beta,\ m_{\tilde{g}}$ and $\delta^D$. We vary them subject to: $m_{\text{squarks}} > 150$ GeV.

We have plots of everything as a function of everything. *E. g*:




$\boxed{\text{BR}(\text{Z} \to \text{b}\bar{\text{s}}) \lesssim 10^{-6} \qquad \tilde{b} - \tilde{s} \text{ mixing.}}$

The highest value: $m_{\tilde{g}}$ and one $m_{\tilde{d}_i} \approx$ the EW scale, while $m_{\tilde{d}_j}$, $j \neq i \approx$ few TeV. Such splitting requires "heavy" SUSY mass scale with soft breaking parameters $\mathcal{O}$ few TeV. This is somewhat motivated by the non-observability of SUSY particles in past and present colliders.

3IIB: $\tilde{t}-\tilde{c}$ mixing

In this scenario as in previous one we have the particles and form factors (from the Feynman rules of Rosiek):

| | SUSY with $\tilde{t}-\tilde{c}$ mixing |
|---|---|
| scalar ($S_\alpha$) | $\Phi_{U,\alpha}, \quad \alpha=1,2,3,4$ |
| fermion ($f_i$) | $\chi_i^c, \quad i=1,2$ |
| $a^{ij}_{L(Zf)}$ | $-\frac{e}{2s_W c_W} a^{ij}_{L(Z\chi^c)}$ |
| $a^{ij}_{R(Zf)}$ | $-\frac{e}{2s_W c_W} a^{ij}_{R(Z\chi^c)}$ |
| $b^{ij}_{L(\alpha)}$ | $f_L^{R(\alpha ij)\star}\left(R^\star_{U,1\alpha}V^{2j}_{CKM}+R^\star_{U,3\alpha}V^{3j}_{CKM}\right)$ $+\frac{f_R^{R(\alpha ij)\star}}{m_{u_\alpha}}\left(m_c R^\star_{U,2\alpha}V^{2j}_{CKM}+m_t R^\star_{U,4\alpha}V^{3j}_{CKM}\right)$ |
| $b^{ij}_{R(\alpha)}$ | $f_L^{L(\alpha ij)\star}\left(R^\star_{U,1\alpha}V^{2j}_{CKM}+R^\star_{U,3\alpha}V^{3j}_{CKM}\right)$ |
| $g_Z^{\alpha\beta}$ | $\frac{e}{2s_W c_W}\left(R^\star_{U,1\alpha}R_{U,1\beta}+R^\star_{U,3\alpha}R_{U,3\beta}-\frac{4}{3}s_W^2\delta_{\alpha\beta}\right)$ |

The coefficients $f_{L,R}$ are proportional to the chargino mixing matrices. At the end of the day, running with the parameters over all values consistent with the data, and with $m_{\tilde{q}} > 150$ GeV and $m_\chi > 100$ GeV:

$$\boxed{\mathrm{BR}(Z\to b\bar{s}) \lesssim 10^{-8} \qquad \tilde{c}-\tilde{t}\ \text{mixing.}}$$

which we could have guessed ($\alpha/\alpha_s$).

3III: Supersymmetry with $\not{R}_P$

Assume: $\not{R}_P$ terms in the SUSY superpotential $\mathcal{W} \Rightarrow$ FCNC. $\lambda$ (pure $\not{L}$) in $\mathcal{W}$ is irrelevant at 1-loop and assume, for the pure $\not{B}$ terms: $\lambda'' << \lambda'$ and neglect the bilinear term in the $\not{R}_P$ $\mathcal{W}$ Grossman+ Haber '99, Ferrandis '99, Bisset, Kong, Macesanu and Orr '00. Then, the $\not{R}_P$ part of $\mathcal{W}$ is:

$$\mathcal{W}_{\not{R}_P} = \epsilon_{ab}\lambda'_{ijk}\hat{L}_i^a\hat{Q}_j^b\hat{D}_k^c,$$

In addition, if $\not{R}_P$ is OK then the $R_P$ conserving soft SUSY breaking is extended. We need only the bilinear:

$$V_{\not{R}_P} = \epsilon_{ab}b_i\tilde{L}_i^a H_u^b.$$

$\tilde{L}$, $H_u$: The scalar components of the hatted $L$ and $H_u$. We therefore have 2 types of FCNC:

**Type A:** Trilinear-trilinear: $\Gamma(Z \to b\bar{s}) \propto (\lambda'\lambda')^2$.

**Type B:** Trilinear-bilinear: $\Gamma(Z \to b\bar{s}) \propto (b\lambda')^2$.

3IIIA: Trilinear-trilinear terms

We subdivide type A into 6 subtypes according to the $S$ and $f$ in the loops, as summarized in:

| | type A1 | type A2 | type A3 |
|---|---|---|---|
| scalar ($S_\alpha$) | $\tilde{e}_{L,\alpha}$ | $\tilde{d}_{L,\alpha}$ | $\tilde{d}_{R,\alpha}$ |
| fermion ($f_i$) | $u_i$ | $\nu_i$ | $\nu_i^c$ |
| $a_{L(Zf)}^{ij}$ | $a_{L(Zu)}$ | $a_{L(Z\nu)}$ | $-a_{R(Z\nu)}$ |
| $a_{R(Zf)}^{ij}$ | $a_{R(Zu)}$ | $a_{R(Z\nu)}$ | $-a_{L(Z\nu)}$ |
| $b_{L(\alpha)}^{ij}$ | 0 | 0 | $\lambda'_{ij\alpha}$ |
| $b_{R(\alpha)}^{ij}$ | $-\lambda'^{*}_{\alpha ij}$ | $\lambda'^{*}_{i\alpha j}$ | 0 |
| $g_Z^{\alpha\beta}$ | $-e\frac{c_W^2-s_W^2}{2s_Wc_W}\delta_{\alpha\beta}$ | $-\frac{e}{2s_Wc_W}\left(1-\frac{2}{3}s_W^2\right)\delta_{\alpha\beta}$ | $\frac{1}{3}e\frac{s_W}{c_W}\delta_{\alpha\beta}$ |

and:

| | type A4 | type A5 | type A6 |
|---|---|---|---|
| scalar ($S_\alpha$) | $\tilde{\nu}_{L,\alpha}$ | $\tilde{\nu}^\star_{L,\alpha}$ | $\tilde{u}_{L,\alpha}$ |
| fermion ($f_i$) | $d_i$ | $d_i$ | $e_i$ |
| $a^{ij}_{L(Zf)}$ | $a_{L(Zd)}$ | $a_{L(Zd)}$ | $a_{L(Ze)}$ |
| $a^{ij}_{R(Zf)}$ | $a_{R(Zd)}$ | $a_{R(Zd)}$ | $a_{R(Ze)}$ |
| $b^{ij}_{L(\alpha)}$ | $\lambda'_{\alpha ji}$ | 0 | 0 |
| $b^{ij}_{R(\alpha)}$ | 0 | $\lambda'^*_{\alpha ij}$ | $-\lambda'^*_{i\alpha j}$ |
| $g^{\alpha\beta}_Z$ | $-\frac{e}{2s_W c_W}\delta_{\alpha\beta}$ | $\frac{e}{2s_W c_W}\delta_{\alpha\beta}$ | $\frac{e}{2s_W c_W}\left(1-\frac{4}{3}s_W^2\right)\delta_{\alpha\beta}$ |

$\alpha$ and $i = 1, 2, 3$. Unfortunately, due the existing limits on $\lambda'$'s, we find (in agreement with the special cases of Chemtob+Moreau '99)

$$\boxed{\mathrm{BR}(Z \to b\bar{s}) \lesssim 10^{-10} \qquad \not{R}_P \text{ trilinear-trilinear.}}$$

3IIIB: Trilinear-bilinear terms

This efect, called here type B, arises when Higgs exchanged in the loop mixes with a slepton, through $\epsilon_{ab} b_3 \tilde{L}_3^a H_u^b$, where we asssume that only $b_3 \neq 0$.
We choose to work in the "no VEV" basis $v_3 = 0$ :

$$H_u \equiv \left(h_u^+,\ (\xi_u^0 + v_u + i\phi_u^0)/\sqrt{2}\right),$$
$$H_d \equiv \left((\xi_d^0 + v_d + i\phi_d^0)/\sqrt{2},\ h_d^-\right),$$
$$\tilde{L}_3 \equiv \left((\tilde{\nu}_+^0 + i\tilde{\nu}_-^0)/\sqrt{2}\ , \tilde{e}_3^-\right).$$

$\tilde{\nu}_+^0$, $\tilde{\nu}_-^0$, $\tilde{e}_3^-$: SU(2) CP-even, CP-odd, $\tau$-sneutrinos, $\tilde{\tau}_L$, respectively.
In the basis $\Phi_C^0 = (h_u^+, h_d^+, \tilde{e}_3^+)$ we wrote the squared mass matrix in the charged scalar sector,
In the basis $\Phi_E^0 = (\xi_d^0, \xi_u^0, \tilde{\nu}_+^0)$ we wrote the squared mass matrix in the CP-even neutral scalar sector,
In the basis $\Phi_O^0 = (\phi_d^0, \phi_u^0, \tilde{\nu}_-^0)$ we wrote the squared mass matrix in the CP-odd neutral scalar sector.
The new charged scalar and CP-even and CP-odd neutral scalar mass-eigenstates (physical states):
By diagonalizing the above-mentioned matrices.
The physical states:

$$\begin{aligned}\Phi_C &= \left(H^+, G^+, \tilde{\tau}^+\right),\\ \Phi_E &= \left(H, h, \tilde{\nu}_+^\tau\right),\\ \Phi_O &= \left(A, G^0, \tilde{\nu}_-^\tau\right).\end{aligned}$$

$b_3 \to 0$: $H,\ h,\ A,\ H^+$ become the usual ones.
Rotating with the diagonalizing $R_{C,E,O}$ (for Charged, Even-CP, Odd-CP):

$$\begin{aligned} \Phi^0_{C,i} &= R_{C,ik}\Phi_{C,k} \ , \\ \Phi^0_{E,i} &= R_{E,ik}\Phi_{E,k} \ , \\ \Phi^0_{O,i} &= R_{O,ik}\Phi_{O,k} \ , \end{aligned}$$

All depends on 4 parameters: $m^0_A$, $m^0_{s\nu}$ the masses in the limit $b_3 \to 0$, $b_3$ and $\tan\beta$.
Subdividing:

$$\text{type B1}: \ S_\alpha = \Phi_{C,\alpha} \ ; \quad f_i = u_i,$$

with $\alpha = 1, 3$, $i = 1, 2, 3$ and

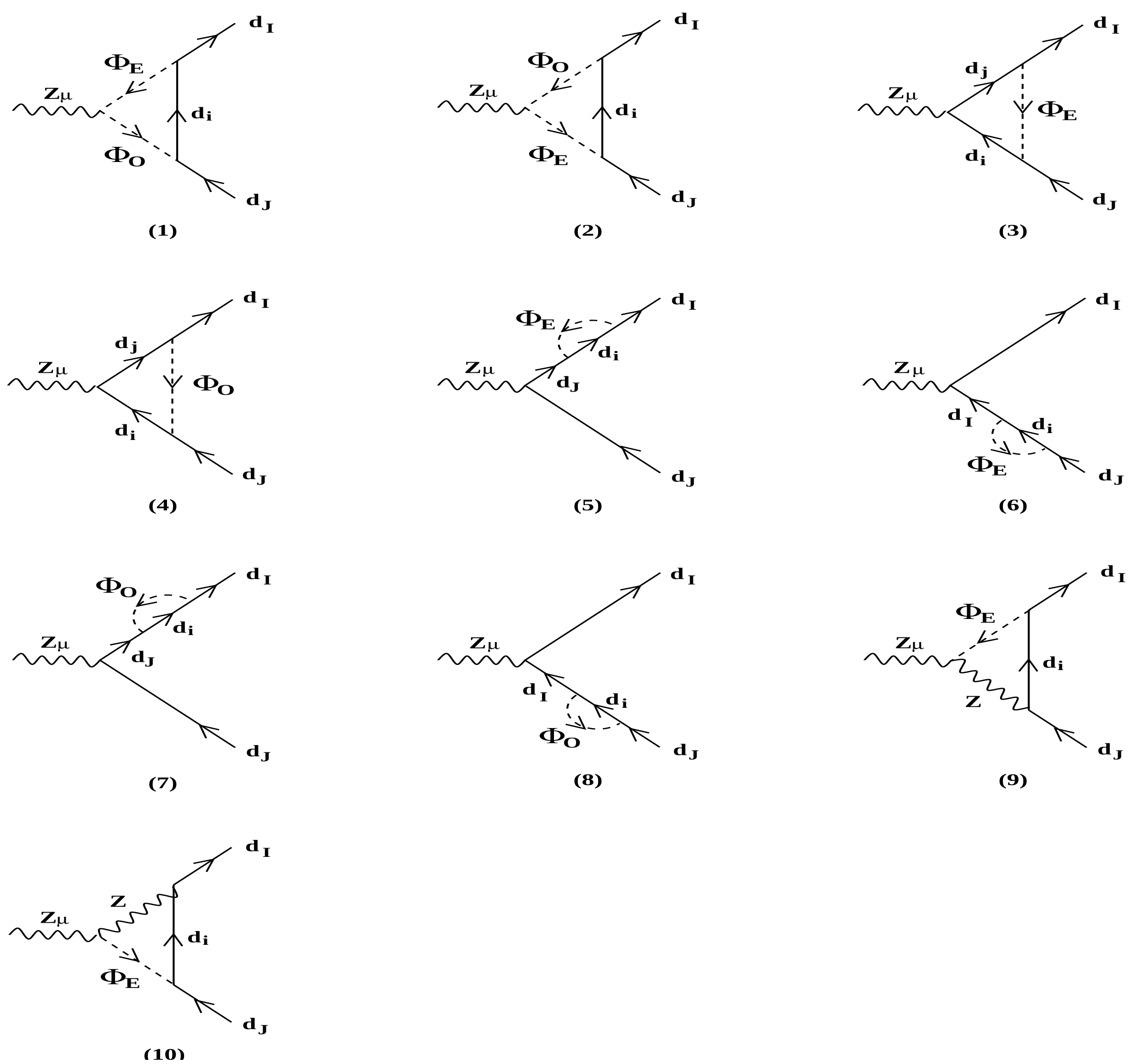
$$\text{type B2}: \ S_\alpha = \Phi_{E,\alpha} \text{ and } \Phi_{O,\beta} \ ; \quad f_i = d_i,$$

with $\alpha = 1, 2, 3$ , $\beta = 1, 3$, $i = 1, 2, 3$.

For type B1 of $\not{R}_P$ use the generic results with:

| | type B1 |
|---|---|
| scalar ($S_\alpha$) | $\Phi_{C,\alpha}, \quad \alpha = 1, 3$ |
| fermion ($f_i$) | $u_i, \quad i = 1, 2, 3$ |
| $a^{ij}_{L(Zf)}$ | $a_{L(Zu)}$ |
| $a^{ij}_{R(Zf)}$ | $a_{R(Zu)}$ |
| $b^{ij}_{L(\alpha)}$ | $\frac{e}{\sqrt{2}s_W}\frac{m_{u_i}}{m_W s_\beta} R_C^{1\alpha} V_{ij}$ |
| $b^{ij}_{R(\alpha)}$ | $\frac{e}{\sqrt{2}s_W}\frac{m_{d_j}}{m_W c_\beta} R_C^{2\alpha} V_{ij} - \lambda'^{\star}_{3ij} R_C^{3\alpha}$ |
| $g_Z^{\alpha\beta}$ | $-e \cot 2\theta_W \delta_{\alpha\beta}$ |

For type B1 $\not{R}_P$ there are 10 diagrams, 8 of which has the same generics as before + 2 diagrams which involve $ZZ\Phi_E$ coupling *i.e.*, the 2 last ones in the figure below:




The results for $R\!\!\!/_P$ B2 are summarized in the following table:

| | type B2 |
|---|---|
| scalar ($S_\alpha$) | $\Phi_{E,\alpha}, \quad \alpha = 1,2,3$ and $\Phi_{O,\alpha}, \quad \alpha = 1,3$ |
| fermion ($f_i$) | $d_i, \quad i = 1,2,3$ |
| $a^{ij}_{L(Zf)}$ | $a_{L(Zd)}$ |
| $a^{ij}_{R(Zf)}$ | $a_{R(Zd)}$ |
| $b^{(E)ij}_{L(\alpha)}$ | $-\frac{e}{2s_W}\frac{m_{d_i}}{m_W c_\beta} R_E^{1\alpha}\delta_{ij} + \frac{1}{\sqrt{2}}\lambda'_{3ji} R_E^{3\alpha}$ |
| $b^{(E)ij}_{R(\alpha)}$ | $-\frac{e}{2s_W}\frac{m_{d_i}}{m_W c_\beta} R_E^{1\alpha}\delta_{ij} + \frac{1}{\sqrt{2}}\lambda'^{\star}_{3ij} R_E^{3\alpha}$ |
| $b^{(O)ij}_{L(\alpha)}$ | $-i\frac{e}{2s_W}\frac{m_{d_i}}{m_W c_\beta} R_O^{1\alpha}\delta_{ij} + \frac{i}{\sqrt{2}}\lambda'_{3ji} R_O^{3\alpha}$ |
| $b^{(O)ij}_{R(\alpha)}$ | $i\frac{e}{2s_W}\frac{m_{d_i}}{m_W c_\beta} R_O^{1\alpha}\delta_{ij} + \frac{i}{\sqrt{2}}\lambda'^{\star}_{3ij} R_O^{3\alpha}$ |
| $g_Z^{\alpha\beta}$ | $i\frac{e}{\sin 2\theta_W}\left(R_E^{1\alpha}R_O^{1\beta} - R_E^{2\alpha}R_O^{2\beta} + R_E^{3\alpha}R_O^{3\beta}\right)$ |
| $g^{\alpha}_{ZZ\Phi_E}$ | $\frac{e}{s_W c_W} m_Z \left(c_\beta R_E^{1\alpha} + s_\beta R_E^{2\alpha}\right)$ |

Some points before presenting the results:

1. $(m_A^0)^2$ (the "bare" mass, *i.e.* for $b_3 \to 0$), can be approximated from the tree-level relation which, for $t_\beta^2 \gg 1$, gives $(m_A^0)^2 \sim b_0 t_\beta$. $b_0$ is the usual $R_P$ ($R$-Parity conserving) soft-breaking bilinear Higgs term in the scalar potential, i.e., $V_{R_P} \supset b_0 H_d H_u$.

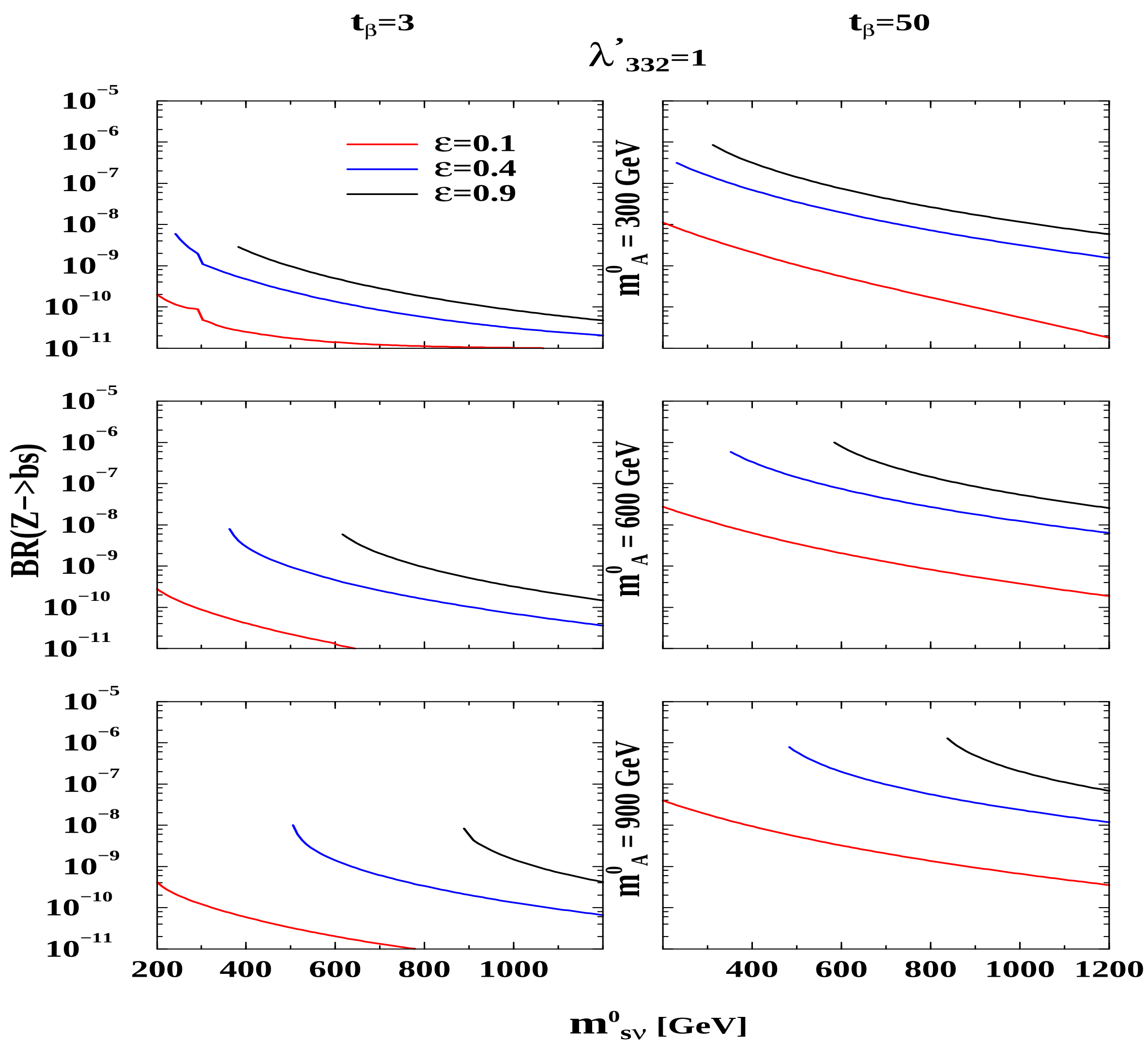
Thus, we trade the bilinear coupling $b_3$ with a dimensionless $\not{R}_P$ parameter, $\varepsilon$:

$$b_3 \equiv \varepsilon (m_A^0)^2 \cot \beta \ .$$

The experimental limits on $\varepsilon$ come from:
(a) A non-vanishing $b_3$ can generate a radiative (one-loop) tau-neutrino mass. The laboratory limit on the tau-neutrino mass allows, however, the quantity $\frac{b_3}{b_0} \sim \varepsilon$ to be of $\sim \mathcal{O}(1)$ Davidson+Losada '00.
(b) $\varepsilon$ affects the masses. We respect these limits Bar-Shalom+GE+Mele '01, Heinemeyer+Hollik+ Weigelin '00 when varying $m_A^0, m_{s\nu}^0, t_\beta, \varepsilon$.

2. The largest width in type B1: From $\lambda\prime_{332} \neq 0$. The limit at $2\sigma$ is 0.45, with squark masses $\approx$ 100 GeV. We use 1 for its value, which holds for $m_{\text{squarks}} \gtrsim 650$ GeV.

The results are:




Therefore, for type B1 $\not{R}_P$:

$\boxed{\mathrm{BR}(Z \to b\bar{s}) \lesssim 10^{-6} \qquad \not{R}_P \text{ trilinear-bilinear.}}$

4: Experimental feasability

Brief comment about the feasibility of observing (or limitting) a signal of $BR(Z \to b\bar{s}) \sim 10^{-6}$, at a Linear Collider producing $10^9$ Z-bosons.

Such a signal leads to one $b$-jet and one light-jet. In the spirit of the analysis made with the 1992 – 1995 LEP data on $Z \to b\bar{s}$ we concluded:

With their efficiecncies improved by a factor of $2 \sim 3$, a new physics signal $Z \to b\bar{s}$, with a branching ratio of order $10^{-6}$, can reach beyond the 3-sigma level.

We can also get a clue about how low one can go in the value (or limit) of $BR(Z \to b\bar{s})$ with $10^9$ Z-bosons, from the fact that the LEP preliminary result achieved $BR(Z \to b\bar{s}) < \mathcal{O}(10^{-3})$ with $\mathcal{O}(10^6)$ $Z$-bosons. Scaling this limit, especially with the expected advance in $b$-tagging and identification of non-$b$ jets methods, an $\mathcal{O}(10^{-6})$ branching ratios should be easily attained at a Giga-$Z$ factory.

## 5: Summary and conclusions

Summary:

| **model** | **scalars in the loops** | $\mathbf{BR(Z \to b\bar{s})}$ |
|---|---|---|
| SM | $W$-boson (no scalars) | $10^{-8}$ |
| 2HDMII | charged Higgs | $10^{-10}$ |
| T2HDM | charged Higgs | $10^{-8}$ |
| $\tilde{t} - \tilde{c}$ mixing | $\tilde{t} - \tilde{c}$ admixtures | $10^{-8}$ |
| $\tilde{b} - \tilde{s}$ mixing | $\tilde{b} - \tilde{s}$ admixtures | $10^{-6}$ |
| tri–tri $R_P$-violation | squarks and sleptons | $10^{-10}$ |
| tri–bi $R_P$-violation | slepton-Higgs admixtures | $10^{-6}$ |

Conclusions: **GO FOR IT!**