$\boldsymbol{\mathcal{O}(m_q^2\alpha_s^4 n_f^2)}$ corrections

non-diagonal currents:
$\boldsymbol{J_\mu^{V/A} = \bar{\psi}_a \gamma_\mu(\gamma_5)\psi_b}$ and $\boldsymbol{J_\mu^{S/P} = \bar{\psi}_a(\gamma_5)\psi_b}$

possible $O(m_q^2)$ terms: $m_a^2, m_b^2, \bar{m}^2 = \sum_q m_q^2$ in structure functions $\Pi^{[1]}$ and $\Pi^{[2]}$ as defined through

$$\Pi_{\mu\nu}^{(V/A)} = g_{\mu\nu}\Pi^{[1]} + q_\mu q_\nu \Pi^{[2]}$$

and $\Pi^{(S/P)}$ have been computed at $\boldsymbol{\mathcal{O}(\alpha_s^4 n_f^2)}$

Case of relevance for the τ decay: $m_b = 0, m_a = m, \bar{m} = m$

$$\Pi^{[1]} = \Pi_0^{[1]} + \Pi_2^{[1]} m^2 \qquad \Pi^{[2]} = \Pi_0^{[2]} + \Pi_2^{[2]} m^2/Q^2$$

corresponding multiplicatively renormalized Adler functions:

$$D_2^{[1]}(Q^2) \equiv -\frac{1}{2} Q^2 \frac{\mathrm{d}}{\mathrm{d}\, Q^2}\Pi_2^{[1]} \quad \text{and} \quad \Pi_2^{[2]}$$