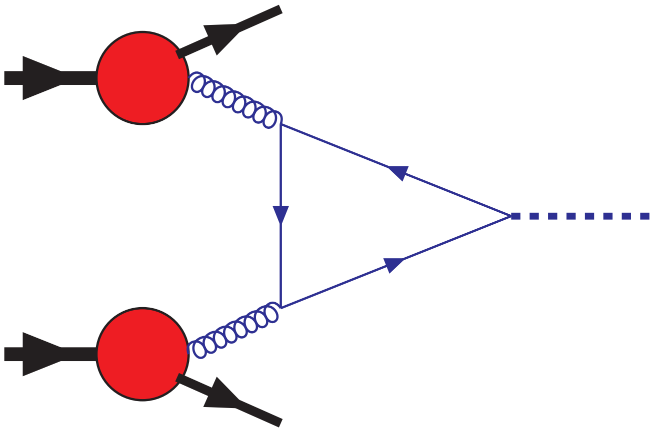
“Inclusive” phase-space integrations
and
Higgs production by gluon fusion at NNLO

Babis Anastasiou

SLAC

in collaboration with Kirill Melnikov



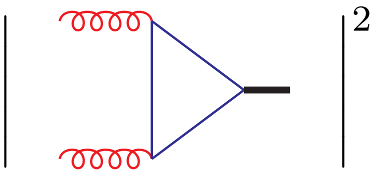
RADCOR 2002

Motivation

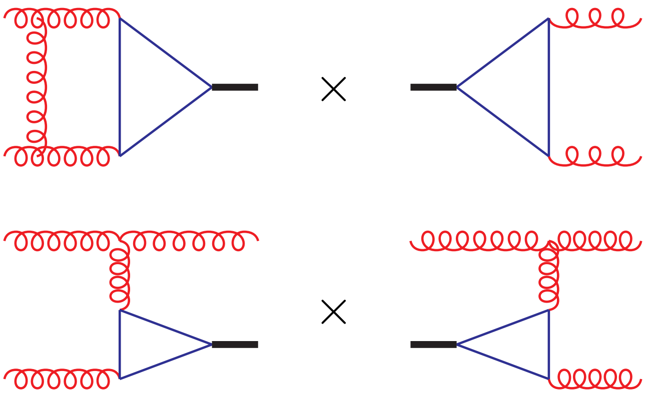
- Main production mechanism at Tevatron, LHC.
- Large NLO corrections
- Available NNLO techniques were not generalizable.
- Ideal test-process for a new method to integrate over phase-space.

Components

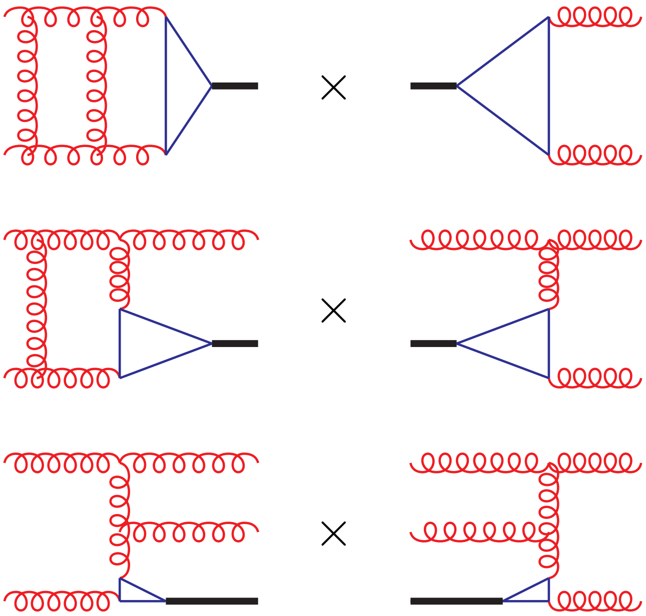
- At LO:



- At NLO:



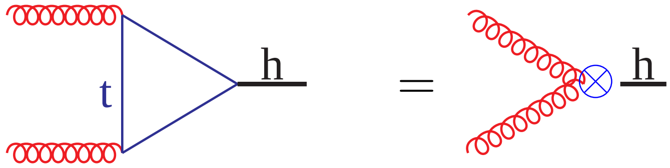
- At NNLO:



Effective Lagrangian

APPROXIMATION: $M_{\rm Higgs} << 2M_{\sf top}$

- Top-quark loops shrink into vertices



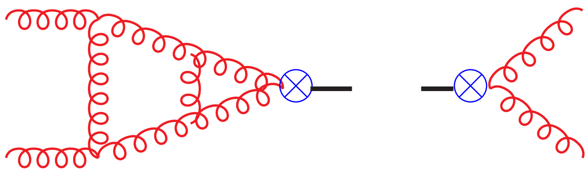
$+\,\mathcal{O}(m_h^2/m_t^2)$

- Works within 10% at NLO for $M_{\sf Higgs} < 1$TeV.

$$\sigma \approx \frac{\sigma_{\rm NLO}^{m_t\to\infty}}{\sigma_{\rm Born}{}^{m_t\to\infty}}\sigma_{\rm Born}^{\rm exact}$$

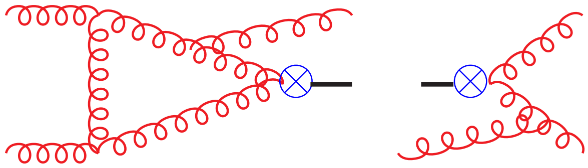
NNLO contributions

- Double Virtual (V-V)



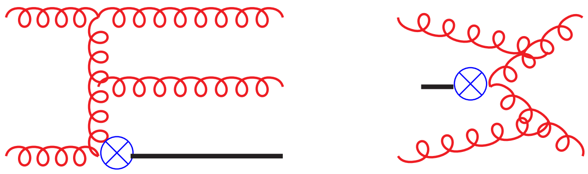
$+$ 148 terms

- Real - Virtual (V-R)



$+$ 559 terms

- Double Real (R-R)



$+$ 675 terms

Method

- We write phase-space integrals as loop integrals

$$\int \frac{d^d p}{(2\pi)^{d-1}} \theta(p^0)\delta(p^2 - M^2) \rightarrow$$
$$\int \frac{d^d p}{(2\pi)^d} \left[\frac{i}{p^2 - M^2 - i0} - \frac{i}{p^2 - M^2 + i0} \right]$$

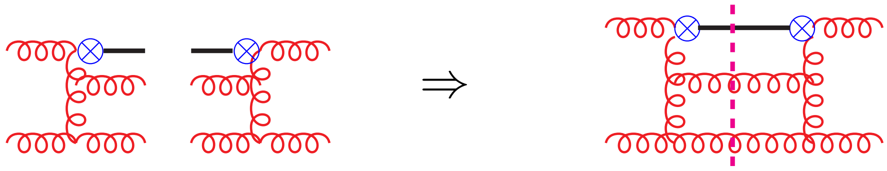
Traditionally, the cutting rules are used in the reverse direction

BUT:

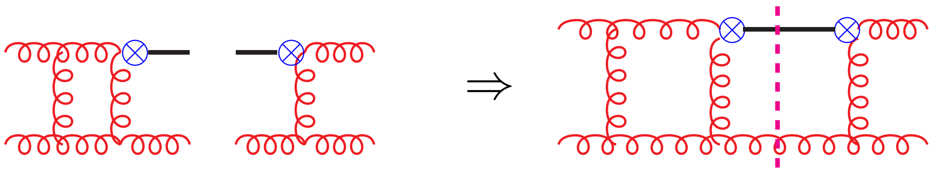
- phase-space integrals are usually solved on a "case by case" basis

- loop-integrals can be evaluated with ALGORITHMIC / UNIVERSAL methods

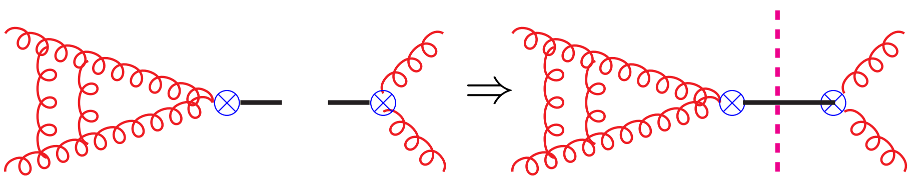
- Double-Real $\Rightarrow$ Triple-Cut



- Real-Virtual $\Rightarrow$ Double-Cut



- Double-Virtual $\Rightarrow$ Single-Cut



Integration By Parts

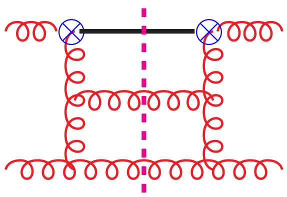
$$\int d^d k \frac{\partial}{\partial k_\mu} \frac{p^\mu}{k^2 (k+p)^2 \cdots} = 0$$

- IBP connect different integrals with simple algebraic relations.

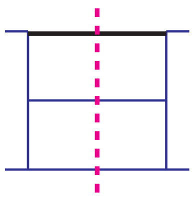
- They commute with the cuts (derivatives act in the same manner on a propagator and its conjugate)

- System of equations can be solved algorithmically.

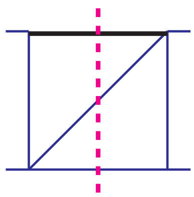
A few (29) independent MASTER INTEGRALS remain to be evaluated.



$= A_1$



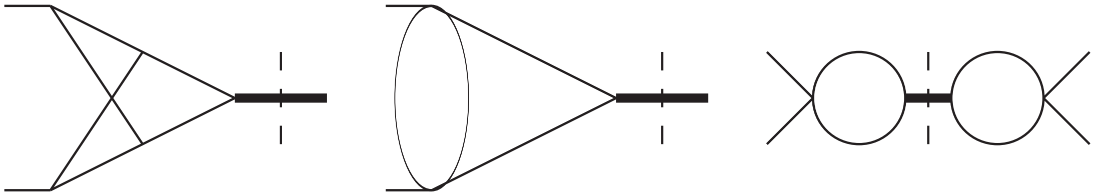
$+ \quad A_2$



$+ \dots$

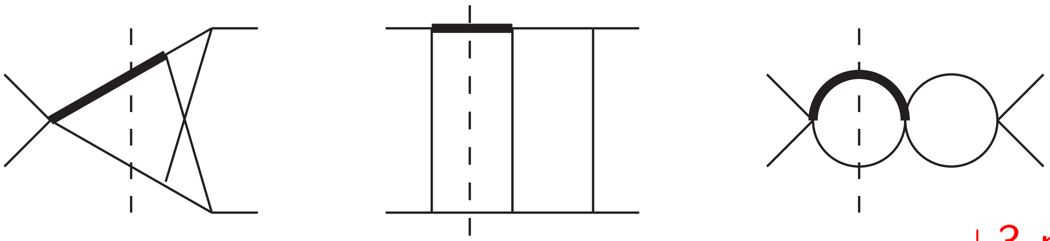
Master integrals

- Double-Virtual



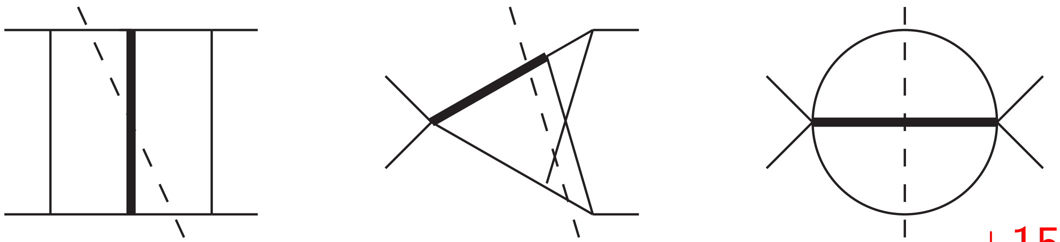
+2 more

- Real-Virtual



+3 more

- Double-Real



+15 more

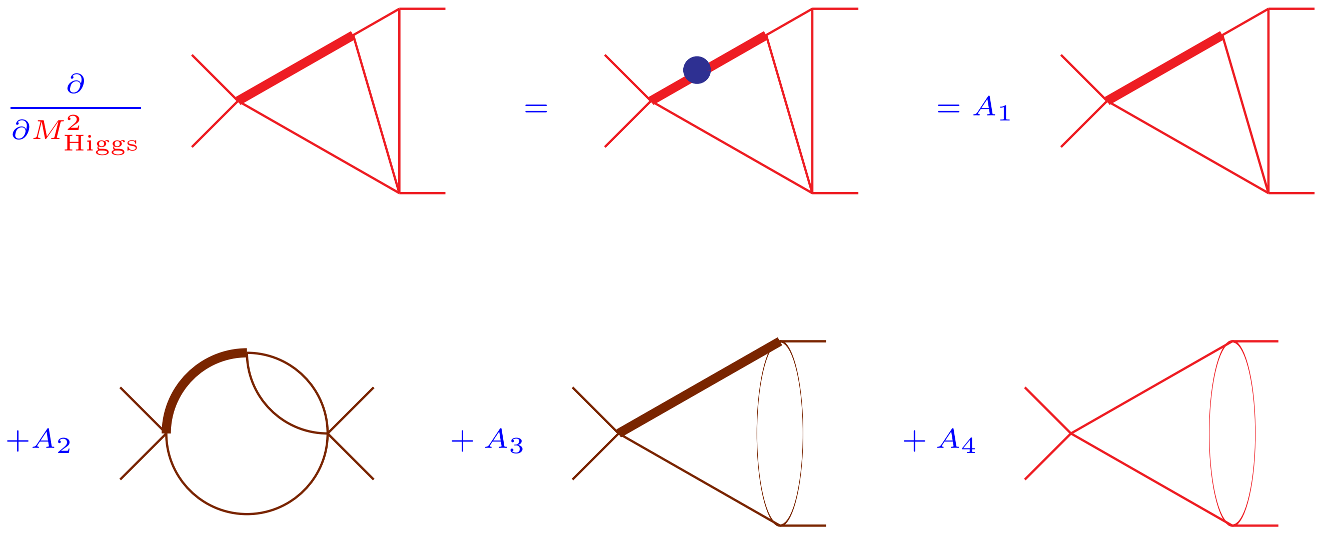
Evaluating master integrals

We can(?) perform the phase-space integrations directly...

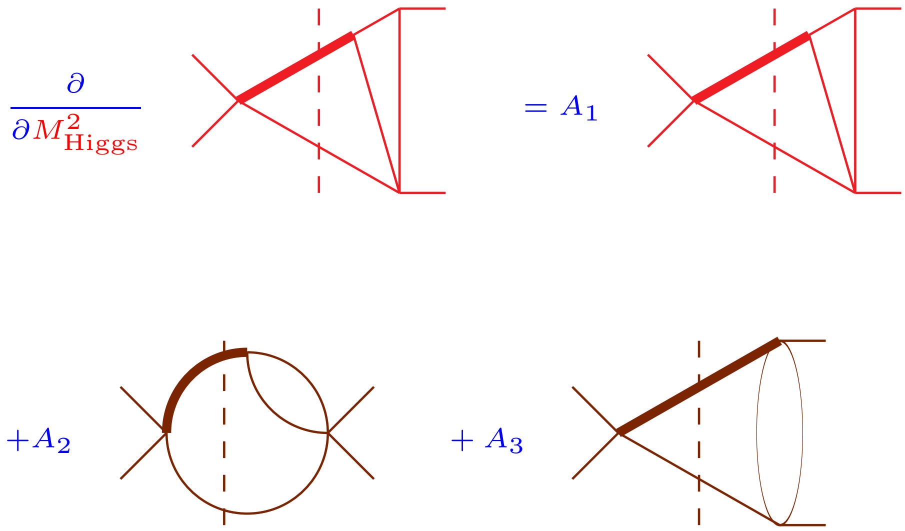
OR

Solve differential equations:

- Write differential equations w.r.t the kinematic variables:



- Then select cuts:



Solving the D. Equations for MI

$$\frac{\partial f}{\partial x} = a(x) f(x) + b(x)$$

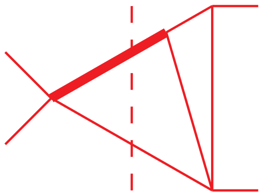
- Extract behavior at threshold $(x \equiv \frac{m_h^2}{s} = 1)$

$$(1-x)^{-1+\epsilon} \to \frac{1}{\epsilon}\delta(1-x) + \epsilon \left[\frac{1}{1-x}\right]_+ + \ldots$$

- Solve DE order by order in ϵ

- Move recursively to the more complicated master integrals

Answer in terms of logs, polylogs, ζ:



$$= \Gamma(1+\epsilon)\frac{x^{-2\epsilon}}{1-2\epsilon}\left\{\frac{1}{\epsilon}\left[\mathrm{Li}_2(x) + \frac{\log^2(x)}{2} - \zeta_2\right]\right.$$
$$+5\mathrm{S}_{12}(x) - 4\mathrm{Li}_3(x) + 4\log(x)\mathrm{Li}_2(x)$$
$$\left.+\frac{\log^3(x)}{2} + 2\zeta_2\log(x) - \zeta_3 + \mathcal{O}(\epsilon)\right\},$$

Results

$$\sigma_{ij}^{(2)} = \Delta_{ij}^{(2)\mathrm{A}} + n_f \Delta_{ij}^{(2)\mathrm{F}}.$$

For $i = j = g$, we find:

$$\Delta_{gg}^{(2)\mathrm{F}} = (-\frac{1189}{144} + \frac{5}{6}\zeta_3 - \frac{5}{3}\zeta_2 + \frac{2}{3}L_t)\delta(1-x) - \frac{10}{3}\left[\frac{\log(1-x)}{1-x}\right]_+$$
$$+\left(\frac{14}{9} - 2\zeta_2\right)\left[\frac{1}{1-x}\right]_+ + 2\left[\frac{\log(1-x)^2}{1-x}\right]_+ + \left(-\frac{1}{12}x^2 + \frac{1}{6}x - \frac{1}{6}\right)\zeta_2 \ln(1-x)$$
$$+\left(\frac{31}{6}x + \frac{1}{6} + \frac{65}{12}x^2\right)\mathrm{S}_{12}(x) + \left(-\frac{31}{12}x^2 + \frac{1}{6} - \frac{17}{6}x\right)\mathrm{Li}_3(x) + \left(\frac{47}{12}x^2 + \frac{25}{6}x - \frac{1}{6}\right)\mathrm{Li}_2(x)\ln(x)$$
$$+\left(-\frac{1}{6}x + \frac{1}{6} + \frac{1}{12}x^2\right)\mathrm{Li}_2(x)\ln(1-x) + \left(\frac{1}{12} - \frac{1}{12}x + \frac{1}{24}x^2\right)\ln(1-x)\ln(x)\ln\left(\frac{(1-x)}{x}\right)$$
$$+\frac{5}{9}x(1+x)\ln^3 x + \left(-\frac{17}{6}x^2 - \frac{7}{3}x - \frac{1}{3}\right)\zeta_3 - 4x(1+x)\zeta_2\ln(x)$$
$$+\left(-\frac{34}{9}x^3 + \frac{2}{3}x^2 - \frac{8}{3}x + \frac{16}{9}\right)\ln^2(1-x) - \frac{2}{9}\frac{(21x^2 + 7x + 25x^4 + 17 - 61x^3)}{1-x}\ln(x)\ln(1-x)$$
$$+\left(\frac{785}{54}x^3 - \frac{83}{36}x^2 + \frac{49}{18}x - \frac{461}{54}\right)\ln(1-x) + \frac{1}{72}\frac{(-351x^3 + 117x^2 + 68 + 132x^4 + 52x)}{1-x}\ln^2(x)$$
$$+\frac{1}{216}\frac{(333x^2 + 2384x^4 - 598x - 3041x^3 + 1282)}{1-x}\ln(x)$$
$$+\frac{1}{36}\frac{(227x^3 + 68 + 4x^4 - 302x + 21x^2)}{1-x}\mathrm{Li}_2(1-x)$$
$$+\left(\frac{34}{9}x^3 - \frac{2}{3}x^2 + \frac{8}{3}x - \frac{16}{9}\right)\zeta_2 - \frac{8887}{648}x^3 + \frac{1267}{432}x^2 - \frac{497}{216}x + \frac{12923}{1296}.$$

Cross-Section for the LHC

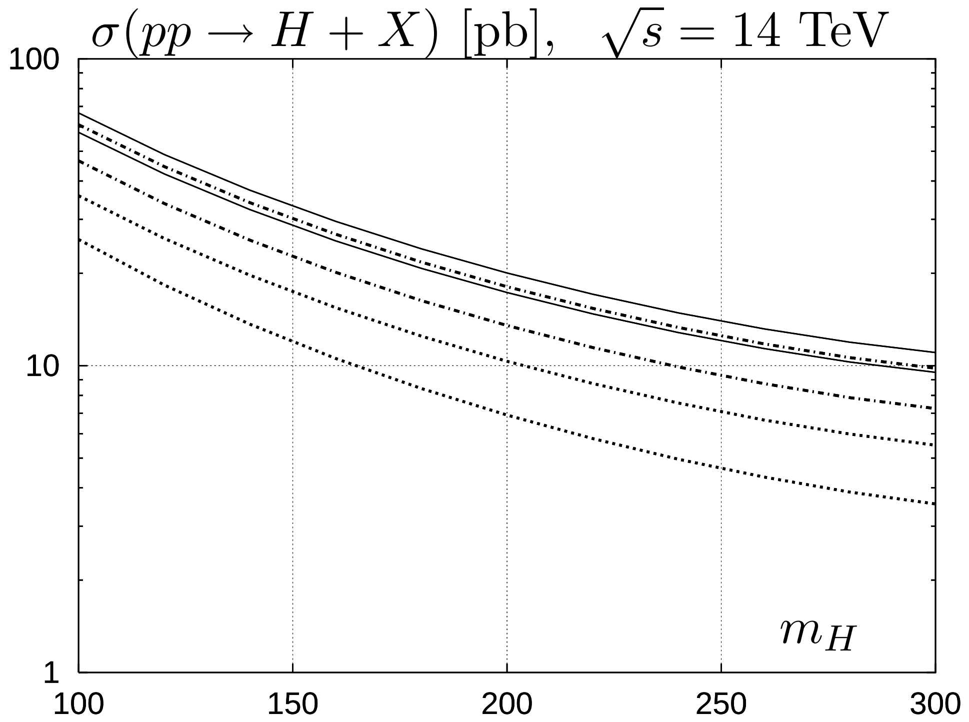


Figure 1: The Higgs boson production cross section at the LHC at leading (dotted), next-to-leading (dashed-dotted) and next-to-next-to-leading (solid) order. The two curves for each case correspond to $\mu_{\rm ren} = \mu_{\rm fac} = m_H/2$ (upper) and $\mu_{\rm ren} = \mu_{\rm fac} = 2m_H$ (lower).

Cross-Section for the Tevatron

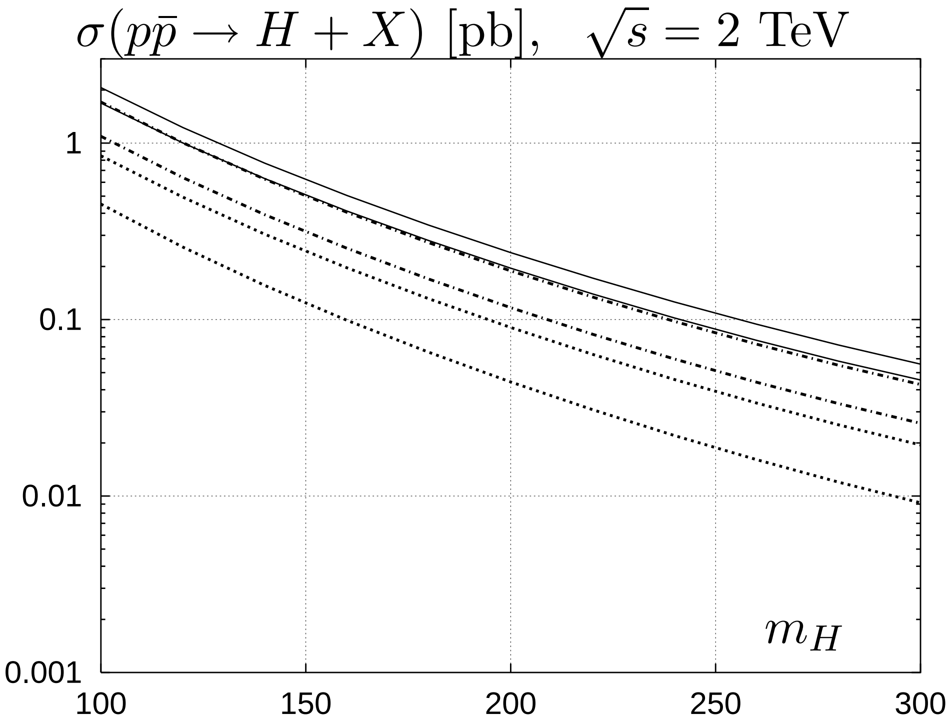


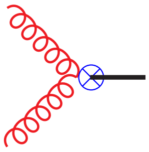
Figure 2: The Higgs boson production cross section at the Tevatron at leading (solid), next-to-leading (dashed) and next-to-next-to-leading (dotted) order. The two curves for each case correspond to $\mu_{\rm ren} = \mu_{\rm fac} = m_H/2$ (upper) and $\mu_{\rm ren} = \mu_{\rm fac} = 2m_H$ (lower).

Advantages of the Method

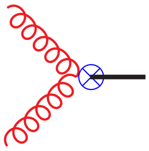
- All contributions (VV, VR, RR) are calculated in the same manner
- Utilizes the expertise obtained from multi-loop calculations to tackle phase-space problems.
- An algorithmic approach, automated at a very high degree.

CP-odd Higgs boson production

- Topologically equivalent process.



$$h \sim p_1^{\mu_1} p_2^{\mu_2} - g^{\mu_1\mu_2} p_1 \cdot p_2$$



$$A \sim \epsilon^{\sigma\mu_1\rho\mu_2} p_1^{\sigma} p_2^{\rho}$$

- complication: ϵ-tensor in dimensional regularisation

$$\epsilon^{\alpha\beta\gamma\delta}\epsilon_{\mu\nu\rho\sigma} \longrightarrow g_{\mu}^{\alpha} g_{\nu}^{\beta} g_{\rho}^{\gamma} g_{\sigma}^{\delta} - \ldots$$

We verify that this prescription is compatible with the employed renormalisation of the effective Lagrangian by computing the NNLO CP-odd Higgs decay rate in the same manner.

Partonic cross-sections for the CP-odd Higgs boson

$$\sigma_A^{ij} = \sigma_h^{ij} + \Delta\sigma^{ij}$$

where:

$$\begin{aligned}
\Delta\sigma_{gg}^{(2)} \quad = \quad & \left[\left(\frac{L_t}{3} - \frac{21}{16}\right)\delta\left(1-x\right) + \frac{2}{3}x\ln^2\left(x\right) + x\ln\left(x\right)\right. \\
& \left. -\frac{1}{6}\left(10x-1\right)\left(x-1\right)\right] n_f + 6\left[\frac{\ln(1-x)}{(1-x)}\right]_+ \\
& +\left(3\zeta_2 + \frac{1939}{144} - \frac{19}{8}L_t\right)\delta\left(1-x\right) \\
& -6x\left(x^2-x+2\right)\ln\left(1-x\right) - 9x\ln^2\left(x\right) \\
& +\frac{3}{2}\left(2x^4-4x^3+13x^2+x-10\right)\frac{\ln\left(x\right)}{\left(x-1\right)} \\
& +\frac{1}{4}\left(x-1\right)\left(11x^2+35x-154\right),
\end{aligned}$$

...

Cross-Section for the LHC

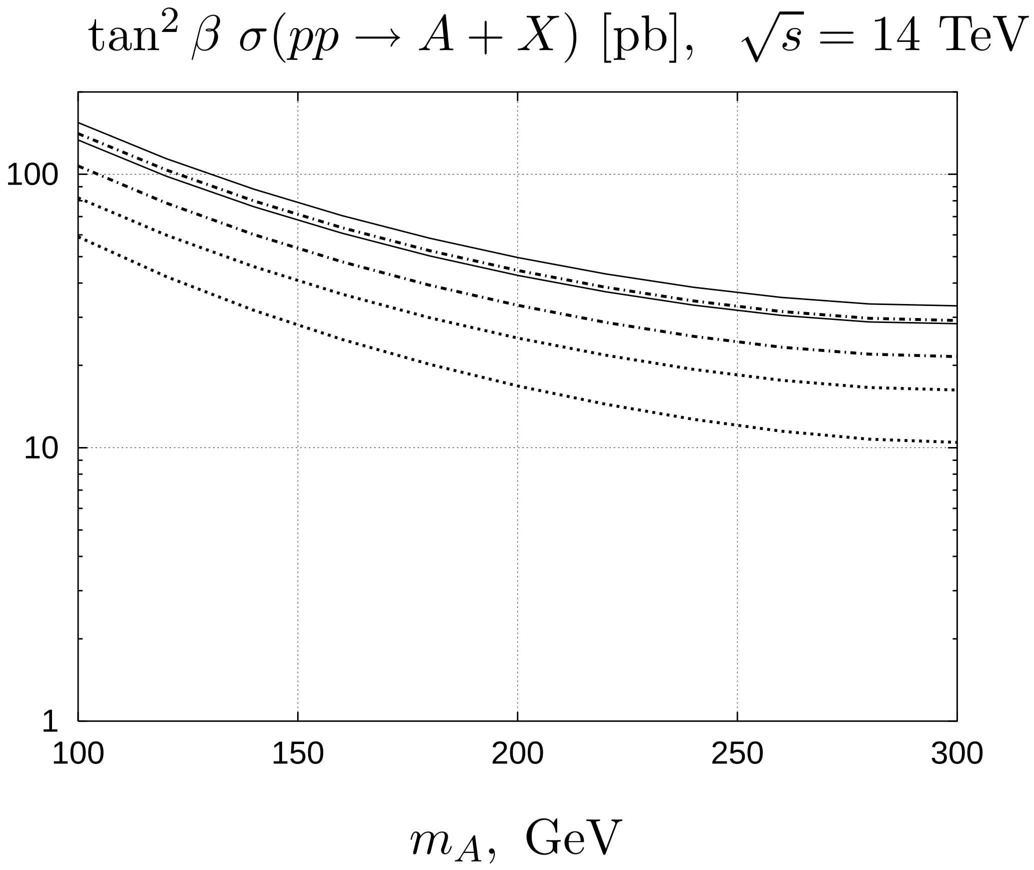


Figure 3: The pseudoscalar Higgs boson production cross-section at the LHC at leading (dotted), next-to-leading (dashed-dotted) and next-to-next-to-leading (solid) order. The two curves for each case correspond to $\mu_r = \mu_f = m_A/2$ (upper) and $\mu_r = \mu_f = 2m_A$ (lower).

Cross-Section for the Tevatron

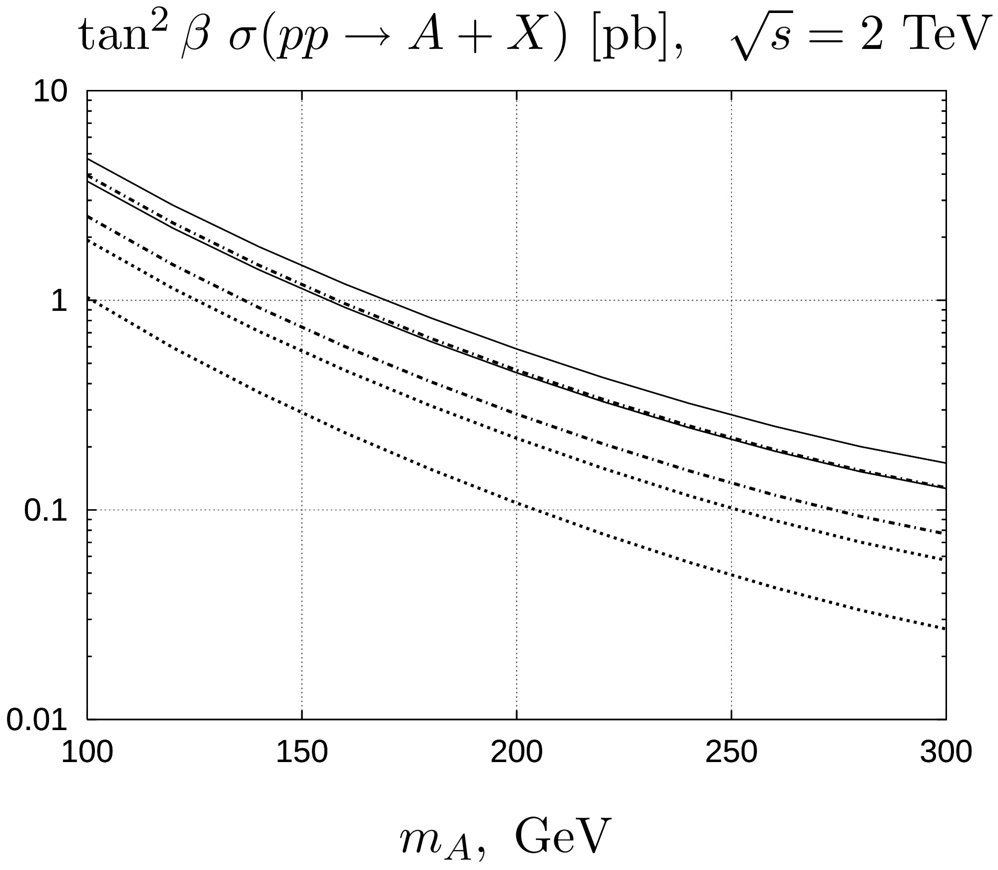


Figure 4: The pseudoscalar Higgs boson production cross-section at the Tevatron at leading (solid), next-to-leading (dashed) and next-to-next-to-leading (dotted) order. The two curves for each case correspond to $\mu_r = \mu_f = m_A/2$ (upper) and $\mu_r = \mu_f = 2m_A$ (lower).

Next Step: Differential Distributions

Requires additional constrains of the phase-space:

$$\frac{d\sigma}{dx} = \int d\Pi \, |\mathcal{M}|^2 \, \delta\left(x - f(\{p_f\})\right)$$

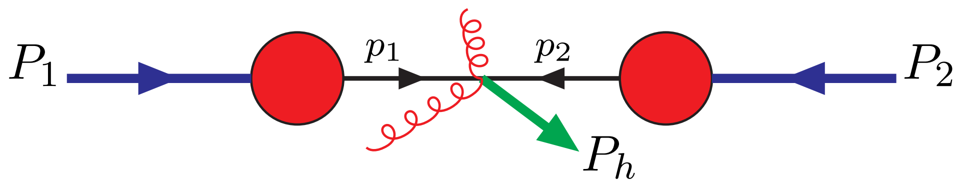
- We use the same trick:

$$\delta\left(x - f(\{p_f\})\right) \rightarrow \frac{1}{x - f\{p_f\}}$$

- We then reduce to master integrals with:
  - Partial Fractioning
  - Integration By Parts
- We evaluate the master integrals with the differential equation method

Higgs boson rapidity distribution

work in progress with Lance Dixon and Kirill Melnikov



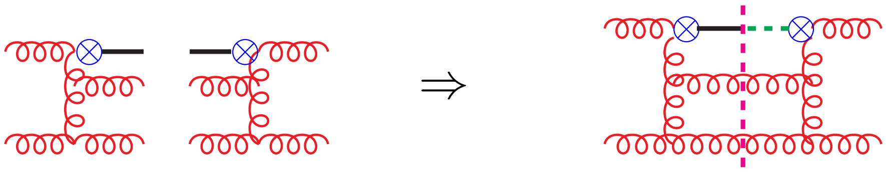
$$P_1 = \tfrac{1}{2}(1, \mathbf{0}, 1) \qquad p_1 = x_1 P_1$$
$$P_2 = \tfrac{1}{2}(1, \mathbf{0}, -1) \qquad p_2 = x_2 P_2$$
$$P_h = (E, \mathbf{p_T}, p_z)$$

Rapidity: $Y = \frac{1}{2}\log\left(\frac{E-p_z}{E+p_z}\right)$

$$\frac{d\sigma}{dY} \sim \int d\Pi\, |\mathcal{M}|^2\, \delta\left(\frac{x_1 e^{2Y}}{x_2} - \frac{p_1 \cdot P_h}{p_2 \cdot P_h}\right)$$

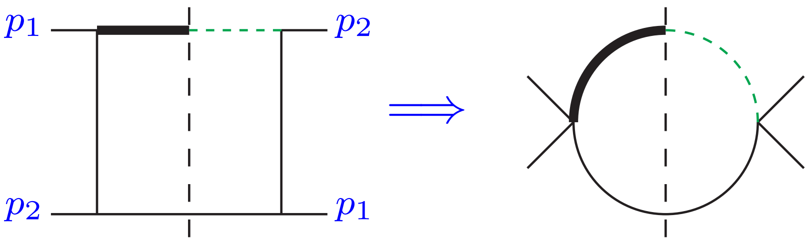
Replace δ functions with "propagators":

$$\delta\left(u - \frac{p_1 \cdot P_h}{p_2 \cdot P_h}\right) \to \frac{p_2 \cdot P_h}{(p_1 - u p_2) \cdot P_h}$$



NLO scalar Higgs boson rapidity distribution

- Partial Fractioning $\Longrightarrow$ 1 Master Integral



- Partonic NLO differential cross-sections

$$
\begin{aligned}
\frac{d\sigma_1^{gg}}{dy} = & \Bigg[\left(\frac{11}{4} + \frac{9}{2}\zeta_2\right) \delta\left(1-x\right) + 3\,\frac{\left(x^2 - x + 1\right)^2 \ln\left(x\right)}{x\left(-1+x\right)} \\
& -6\,\frac{\left(x^3 - x^2 + 2\,x - 1\right)\ln\left(1-x\right)}{x} + 6\left[\frac{\ln\left(1-x\right)}{1-x}\right]_+ \Bigg] \left[\delta\left(y\right) + \delta\left(1-y\right)\right] \\
& -\frac{3}{x}\left[x^3 - x^2 - \frac{x}{\left(1-x\right)_+} + 2\,x - 1\right]\left[\frac{1}{\left(1-y\right)_+} + \frac{1}{y_+}\right] \\
& +\frac{3}{2}\,\delta\left(1-x\right)\left[\frac{\ln\left(1-y\right)}{y} + \frac{\ln\left(y\right)}{1-y}\right] + 3\,\frac{\left(1-x\right)^5\left(y^4 - 2\,y^3\right)}{x^2} \\
& -3\,\frac{\left(-1+x\right)^3\left(3\,x^2 - 7\,x + 3\right)y^2}{x^2} + 3\,\frac{\left(-1+x\right)^3\left(2\,x-1\right)\left(x-2\right)y}{x^2} \\
& -3\,\frac{\left(-1+x\right)\left(x^4 - 4\,x^3 + 7\,x^2 - 4\,x + 1\right)}{x^2}
\end{aligned}
$$

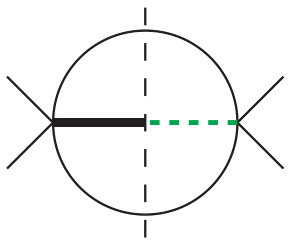
- Hadronic NLO differential cross-section

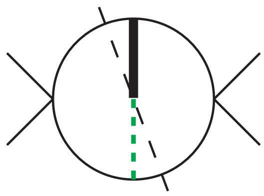
...

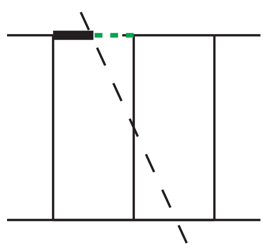
NNLO rapidity master integrals for DRELL-YAN type processes

- IBP reduction is completed for major topologies without significant difficulties.

- Master Integrals:
  - Depend on an additional variable
  - Involve polylogs of lower rank: $\mathrm{Li}_3(z) \rightarrow \mathrm{Li}_2(f(z,y))$







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Summary

- We introduce a new method for inclusive phase-space integrations, setting virtual and real corrections on the same footing.

- We have calculated the full NNLO partonic cross-sections for inclusive scalar/pseudoscalar Higgs boson hadroproduction

- We extend the same method for the computation of differential distributions at NNLO.

- Many interesting problems can be now tackled.