# Structure Functions in Polarized Proton-Deuteron Drell-Yan Processes

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Polarized proton-deuteron (pd) Drell-Yan processes are investigated for studying new spin structure of a spin-1 hadron. Our formalism indicates that there exist many structure functions: 108 in general and 22 in the $\vec{Q}_T$-integrated case. A naive parton model suggests that at least one of them should be related to the tensor polarized distributions, which have not been measured at all. There are some experimental possibilities at FNAL, HERA, and RHIC to study the polarized pd reactions.

## 1. INTRODUCTION

Spin structure of the spin-1/2 nucleon has been investigated extensively. On the other hand, spin structure of spin-1 hadrons has not been well investigated in connection with new spin physics, namely the tensor structure. We know that the tensor structure function $b_1$ exists for a spin-1 hadron and it will be measured in the polarized electron-deuteron scattering. However, it is known in the unpolarized reactions that we can not determine the antiquark distributions in the medium-$x$ region by the electron scattering data. They are determined by the Drell-Yan measurements. In the same way, the studies of polarized proton-deuteron (pd) Drell-Yan processes should be valuable for finding the tensor-polarized antiquark distributions. We discuss a general formalism [1] and a parton-model analysis [2] of the pd Drell-Yan processes. In particular, we explain how a quadrupole spin asymmetry is related to the $b_1$-type distributions.

## 2. POSSIBLE STRUCTURE FUNCTIONS

The general formalism of the proton-proton (pp) Drell-Yan process was completed many years ago, and it is the foundation of the RHIC-SPIN project. On the other hand, the polarized pd formalism was studied only recently [1]. Because the space is limited, only the major points are explained in this section. Two independent methods are employed for finding possible structure functions.

In the first method, the Jacob-Wick helicity formalism is used by introducing the spin density matrix. The important point in the formalism is that there exist rank-two tensors in the density matrix because of the spin-1 nature of the deuteron. Imposing Hermiticity, parity conservation, and time-reversal invariance, we find that 108 structure functions exist in the pd Drell-Yan processes. In comparison with the 48 functions in the pp Drell-Yan, there are 60 new structure functions. Of course, all of them are associated with the rank-two terms, namely the tensor structure of the deuteron. The 108 functions are too many to be investigated seriously. In order to extract the essential ones, the cross section is integrated over the virtual-photon transverse momentum $\vec{Q}_T$. Even in this case, we find that there are 22 structure functions. Because only 11 functions exist in the pp reactions, there are 11 additional ones. Therefore, the interesting point of studying the pd Drell-Yan is to investigate these new structure functions.

In the second method, the hadron tensor is expanded in terms of possible combinations of momentum and spin vectors with the conditions of Hermiticity, parity conservation, time-reversal invariance, and current conservation. It is not useful to list all the 108 combinations; therefore, only the $Q_T \to 0$ limit is considered in this formalism. We have to be particularly careful about including spin-dependent tensor terms in the expansion. Assigning a structure function for each expansion coefficient, we find also the 22 structure functions.

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It means that the possible functions are confirmed by the two independent methods. The details of these formalisms are found in Ref. [1].

The new structure functions are characterized by the polarizations given by the spherical harmonics $Y_{20}$, $Y_{21}$, and $Y_{22}$. We express these quadrupole polarizations as $Q_0$, $Q_1$, and $Q_2$:

$$\begin{aligned} &Q_0 \quad \text{for the term} \quad 3\,cos^2\beta - 1 \sim Y_{20} \ , \\ &Q_1 \quad \text{for} \quad sin\beta\, cos\beta \sim Y_{21} \ , \\ &Q_2 \quad \text{for} \quad sin^2\beta \sim Y_{22} \ , \end{aligned} \tag{1}$$

where $\beta$ is the polar angle of the spin polarization. They are related to the quadrupole spin asymmetries in the $xz$, $yz$, and $xy$ planes, respectively. A $Q_0$ type structure function is measured by the difference between the longitudinally and transversely-polarized cross sections. A $Q_2$ one is measured by the difference between the cross sections with the polarizations of $x$ and $y$ directions. The $Q_1$ type structure functions are interesting in the sense that they cannot be measured in the longitudinally-polarized ($\beta = 0$) and transversely-polarized ($\beta = \pi/2$) reactions. The optimum way of measuring them is to choose the polarization angle in between ($\beta = \pi/4$). In this sense, it may be called "intermediate" polarization. It is an important finding of our studies that there exist intermediate structure functions which are not related to the longitudinal and transverse polarizations.

The polarized structure functions could be obtained by polarization asymmetry measurements. It is well known that five spin combinations should exist in the pp Drell-Yan: unpolarized cross section $<\sigma>$, longitudinal (transverse) double spin asymmetry $A_{LL}$ ($A_{TT}$), longitudinal-transverse spin asymmetry $A_{LT}$, and transverse single spin asymmetry $A_T$ (or denoted as $A_N$). We should be careful in defining the spin asymmetries in the deuteron reactions so that the tensor distributions should be excluded from the denominator [1]. In the pd Drell-Yan, there are additional quadrupole asymmetries and we have the following fifteen spin combinations

$$\begin{aligned} &<\sigma>, A_{LL}, A_{TT}, A_{LT}, A_{TL}, A_{UT}, A_{TU}, A_{UQ_0}, \\ &A_{TQ_0}, A_{UQ_1}, A_{LQ_1}, A_{TQ_1}, A_{UQ_2}, A_{LQ_2}, A_{TQ_2}, \end{aligned} \tag{2}$$

where $U$ denotes the unpolarized case. For example, the asymmetry $A_{UQ_0}$ indicates that the proton is unpolarized and the quadrupole $Q_0$ spin combination is taken for the deuteron. The precise definitions of these asymmetries should be found in Ref. [1]. The new asymmetries are those with the subscript $Q_0$, $Q_1$, or $Q_2$.

### 3. PARTON-MODEL ANALYSIS

The dependence of the structure functions on the proton and deuteron polarizations is revealed by the formalism of the previous section. However, it is not obvious how these structure functions are related to the parton distributions. In particular, the meaning of the new quadrupole structure functions is not obvious at all. In order to clarify it, the hadron tensor is analyzed in a naive parton model. Because the polarized pd Drell-Yan had not been discussed before Ref. [2], we should content ourselves at this stage with the naive analysis: $O(1/Q)$ contributions are neglected in the course of calculations.

The hadron tensor due to the annihilation process, $q$(in A)+$\bar{q}$(in B)$\rightarrow \ell^+ + \ell^-$, is given in the parton model as

$$\begin{aligned} W^{\mu\nu} = &\frac{1}{3} \sum_{a,b} \delta_{b\bar{a}}\, e_a^2 \int d^4k_a\, d^4k_b\, \delta^4(k_a + k_b - Q) \\ &\times Tr[\Phi_{a/A}(P_A S_A; k_a)\gamma^\mu \bar{\Phi}_{b/B}(P_B S_B; k_b)\gamma^\nu] \ , \end{aligned} \tag{3}$$

where $k_a$ and $k_b = k_{\bar{a}}$ are the quark and antiquark momenta, the color average is taken by the factor $1/3 = 3 \cdot (1/3)^2$, $e_a$ is the charge of a quark with the flavor $a$, and $\Phi$ is a correlation function. Of course, the opposite process $\bar{q}$(in A)+$q$(in B)$\rightarrow \ell^+ + \ell^-$ should be taken into account in order to compare with the experimental cross section. The correlation function $\Phi(PS;k)$ is a matrix with sixteen components, so that it can be expanded in terms of the sixteen $4 \times 4$ matrices: $\mathbf{1}$, $\gamma_5$, $\gamma^\mu$, $\gamma^\mu\gamma_5$, $\sigma^{\mu\nu}\gamma_5$ together with the possible Lorentz vectors and pseudovector: $P^\mu$, $k^\mu$, and $S^\mu$. Of course, the expansion terms should satisfy the conditions of Hermiticity, parity conservation, and time-reversal invariance. However, the most important point is that the second rank tensors exist in the deuteron although the spin dependent terms are allowed up to the linear spin ones in the proton. It is shown in Ref. [2] that the additional terms give rise to the tensor distribution $b_1$. We anticipated to have $b_1$; however, we also find a new one which is related to the intermediate polarization. It is the first time that we encounter such a distribution, so that it is simply named a $c_1$ distribution.

Even in the naive analysis, there are still 19 structure functions. In order to find the most essential ones, the cross section is integrated over $\vec{Q}_T$. Then, only four finite structure functions exist. Noting that there are three functions in the pp Drell-Yan, we find a new structure function which is specific to the deuteron. Furthermore, we find that it is expressed by the combinations of unpolarized distributions in the proton with the tensor polarized distributions in the deuteron. This structure function can be investigated by the unpolarized-quadrupole $Q_0$ asymmetry:

$$A_{UQ_0} = \frac{\sum_a e_a^2 \, [\, f_1(x_A)\, \bar{b}_1(x_B) + \bar{f}_1(x_A)\, b_1(x_B)\,]}{\sum_a e_a^2 \, [\, f_1(x_A)\, \bar{f}_1(x_B) + \bar{f}_1(x_A)\, f_1(x_B)\,]} . \quad (4)$$

The advantage of using the hadron reaction is that the tensor-polarized antiquark distributions could be obtained rather easily. In the electron scattering, the antiquark distributions cannot be determined precisely. For example, the violation of the Gottfried sum rule suggested $\bar{u} \neq \bar{d}$ [3]; however, the precise $x$ dependence of $\bar{u}/\bar{d}$ is determined only recently by the E866 Drell-Yan experiments. If the large $x_F$ region is considered in Eq. (4), it becomes

$$A_{UQ_0}(\text{large } x_F) \approx \frac{\sum_a e_a^2 \, f_1(x_A)\, \bar{b}_1(x_B)}{\sum_a e_a^2 \, f_1(x_A)\, \bar{f}_1(x_B)} . \quad (5)$$

It indicates that the antiquark tensor distributions $\bar{b}_1$ can be determined if the unpolarized distributions are well known in the proton and deuteron.

Another advantage of studying the polarized pd Drell-Yan is that it becomes possible to extract the flavor asymmetry $\Delta_T \bar{u}/\Delta_T \bar{d}$ in the transversity distributions by comparing the pp and pd cross sections [1, 2, 4].

It is rather difficult to attain the longitudinal polarization for the deuteron in the collider experiment (e.g. a possible next-generation RHIC project) because of its small magnetic moment. However, we could combine the transversely polarized cross sections with the unpolarized one for investigating the tensor structure. If a fixed target can be used for the deuteron, there is no such difficulty. In fact, there are possibilities to study the polarized pd Drell-Yan at FNAL and also in the HERA-N project. However, significant theoretical and experimental efforts are necessary for proposing such an experiment. At this stage, our numerical analysis is in progress [4] in order to find the experimental possibilities.

## 4. CONCLUSION

We discussed first what kinds of structure functions could be studied in the polarized proton-deuteron Drell-Yan processes. Because of the new spin structure for the deuteron, there exist 108 structure functions. Among them, there are 22 finite ones if the cross section is integrated over $\vec{Q}_T$. The new structure functions are associated with the tensor structure of the deuteron. The parton-model analysis indicated that there are only four structure functions in the $\vec{Q}_T$-integrated case. They are unpolarized, longitudinally-polarized, transversity, tensor-polarized structure functions. The last one does not exist in the proton-proton reactions. It could be measured in the quadrupole $Q_0$ polarization asymmetry with the unpolarized proton. We expect that the tensor structure will become one of the exciting topics in high-energy spin physics in the near future.

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