Scaling violations and off-forward parton distributions: leading order and beyond

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We give an outline of a formalism for the solution of the evolution equations for off-forward parton distributions in leading and next-to-leading orders based on partial conformal wave expansion and orthogonal polynomials reconstruction.

The off-forward parton distributions (OFPD) are new non-perturbative inputs used in exclusive electroproduction processes, like the hard diffractive production of mesons and the deeply virtual Compton scattering (DVCS), to parametrize hadronic substructure [1,2]. Their characteristic feature is a non-zero momentum transfer in the t-channel which results into different momentum fractions of constituents inside hadron.

The leading order amplitude, e.g. for the unpolarized DVCS, in Leipzig-Ji's conventions used throughout [1], looks like

$$\mathcal{A} \propto \int_{-1}^{1} dx \left\{ \frac{1}{x-\eta+i0} + \frac{1}{x+\eta-i0} \right\} \mathcal{O}(x,\eta), \quad (1)$$

with the quark OFPD $\mathcal{O}(x,\eta)$ given as the Fourier transform of the light-cone string operator (in the light-cone gauge $B_+ = 0$)

$$\mathcal{O}(x,\eta) = 2 \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p_2 | \bar\psi(-\lambda n) \gamma_+ \psi(\lambda n) | p_1 \rangle, \quad (2)$$

with the skewedness $\eta \equiv (p_1 - p_2)_+$ and the constraints $x \in [-1,1]$ coming form the support properties of the matrix element (2). Several peculiar properties can be learned from these: i) Translating the perturbative arguments used in the proof of factorization formula (1) to non-perturbative domain we immediately see that the $\mathcal{A}$ exists provided the $\mathcal{O}(\pm\eta,\eta)$ is continuous. ii) In different kinematical regions of the phase space OFPD's share common properties with the usual forward parton densities ($|x| > \eta$) and exclusive distribution amplitudes ($|x| < \eta$), and thus hybrids in this sense. iii) The amplitude (1) manifests Bjorken scaling.

The last property is violated once the QCD radiative corrections are taken into account. The Q-dependence of the amplitude appears via the scale dependence of the OFPD's which obey the generalized evolution equation [1,3–5]

$$\frac{d}{d\ln Q^2} \boldsymbol{\mathcal{O}}(x,\eta) = \int_{-1}^{1} dx' \, \boldsymbol{K}(x,x',\eta) \boldsymbol{\mathcal{O}}(x',\eta), \quad (3)$$

with the kernel given by the series in the coupling constant α_s. The diagonalization of leading order kernel can be achieved exploiting the consequences of conformal invariance of QCD at classical level. The eigenstates of one-loop non-forward evolution equation are given by Gegenbauer polynomials, C_j^ν, — with the numerical value of the index ν depending on the parton species, — which form an infinite dimensional irreducible representation of the conformal group in the space of bilinear composite operators. Starting from two loop order the conformal operators start to mix and the simple pattern of one-loop evolution is broken so that the eigenfunctions generalize to non-polynomial functions. In the basis of leading order conformal waves

$$\int_{-1}^{1} dx \, C_j^\nu\!\left(\frac{x}{\eta}\right) \boldsymbol{K}(x,x',\eta) = -\frac{1}{2} \sum_{k=0}^{j} \gamma_{jk} C_k^\nu\!\left(\frac{x'}{\eta}\right) \quad (4)$$

the anomalous dimension matrix is not diagonal and possesses non-diagonal entries

$$\boldsymbol{\gamma}_{jk} = \boldsymbol{\gamma}_j^{\rm D} \delta_{jk} + \boldsymbol{\gamma}_{jk}^{\rm ND}, \quad \text{with} \quad \boldsymbol{\gamma}_{jk}^{\rm ND} \propto \mathcal{O}(\alpha_s^2). \quad (5)$$

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The disadvantage of the standard approach to the study of scaling violation beyond leading order is the proliferation of Feynman graphs required for calculation of $\gamma^{(1)}_{jk}$. Our approach which allows for an extremely concise analytical solution of the problem [6] is mainly based on four major observations: i) The triangularity of the anomalous dimension matrix $\boldsymbol{\gamma}_{jk}$ implies that its eigenvalues are given by the diagonal elements and coincide with the well-known forward anomalous dimensions. ii) Tree-level special conformal invariance implies diagonal leading order matrix. One-loop violation of the symmetry induces non-diagonal elements, thus, one-loop special conformal anomaly will generate two-loop anomalous dimensions. iii) Scale Ward identity for the Green function with conformal operator insertion coincides with the Callan-Symanzik equation for the latter and thus the dilatation anomaly is the anomalous dimension of the composite operator $[\boldsymbol{\mathcal{O}}_j][\int dx\, \Theta_{\mu\mu}] \propto \frac{1}{\epsilon}\sum_{k=0}^{j}\boldsymbol{\gamma}_{jk}[\boldsymbol{\mathcal{O}}_k]$. iv) The four-dimensional conformal algebra provides a relation between the anomalies of dilatation and special conformal transformations via the commutator $[\mathcal{D},\mathcal{K}_-] = i\mathcal{K}_-$. Using these ideas we have deduced the form of the two-loop non-diagonal elements to be [6]

$$\boldsymbol{\gamma}^{\mathrm{ND}(1)} = [\boldsymbol{\gamma}^{\mathrm{D}(0)}, \boldsymbol{d}\,(\beta_0 - \boldsymbol{\gamma}^{\mathrm{D}(0)}) + \boldsymbol{g}]_-, \qquad (6)$$

where $\boldsymbol{d}$ is a simple matrix, $\gamma^{\mathrm{D}(0)}_j$ are the LO anomalous dimensions of the conformal operators and β_0 is the one-loop QCD Gell-Mann-Low function responsible for the violation of scale invariance. The most nontrivial information about the special conformal symmetry breaking is concentrated in the $\boldsymbol{g}$-matrices which are residues of the special conformal symmetry breaking counterterms at one-loop order $[\boldsymbol{\mathcal{O}}_j][\int dx\, x_- \Theta_{\mu\mu}] \propto \frac{\alpha_s}{\epsilon}\sum_{k=0}^{j} a(j,k)\left\{\boldsymbol{g}_{jk} + \dots\right\}[\boldsymbol{\mathcal{O}}_k]$.

Unfortunately the eigenfunctions of the evolution kernels cannot be used as a basis for expansion of OFPD since they do not form a complete set of functions outside the region $|x/\eta| > 1$ where, however, the OFPD's are nonvanishing in general. Our approach[2] [5] to the study of the scale dependence of the OFPD is based on the expansion of the distribution in series w.r.t. the complete set of orthogonal polynomials, $\mathcal{P}_j(x)$, on the interval $-1 \le x \le 1$ to preserve the support properties of the functions in question

$$\boldsymbol{\mathcal{O}}(x,\eta,Q) = \sum_{j=0}^{J_{\max}} \widetilde{\boldsymbol{\mathcal{P}}}_j(x)\boldsymbol{\mathcal{M}}_j(\eta,Q), \qquad (7)$$

where formally $J_{\max} = \infty$. The expansion coefficients can be reexpressed in terms of eigenstates of the evolution equation (3) according to

$$\begin{aligned}\boldsymbol{\mathcal{M}}_j(\eta,Q) &= \sum_{k=0}^{j} \boldsymbol{E}_{jk}(\eta)\sum_{l=0}^{k}\eta^{k-l}\boldsymbol{B}_{kl}(Q,Q_0)\\ &\quad\times\ \boldsymbol{\mathcal{E}}_l(Q,Q_0)\boldsymbol{\mathcal{O}}_l(\eta,Q_0),\end{aligned} \qquad (8)$$

where $\boldsymbol{E}_{jk}(\eta)$ is the overlap integral [5] between the one-loop eigenfunctions, C^{ν}_j, and the polynomials $\boldsymbol{\mathcal{P}}_j$. The conformal moments at a reference scale Q_0 are defined as

$$\boldsymbol{\mathcal{O}}_j(\eta,Q_0) = \eta^j \int_{-1}^{1} dx\, C^{\nu}_j\left(\frac{x}{\eta}\right)\boldsymbol{\mathcal{O}}(x,\eta,Q_0). \qquad (9)$$

All scale dependence in Eq. (8) is extracted to the usual evolution operator which obeys the standard first order differential equation

$$\frac{d}{d\ln Q}\boldsymbol{\mathcal{E}} + \boldsymbol{\gamma}^{\mathrm{D}}\boldsymbol{\mathcal{E}} = 0. \qquad (10)$$

Besides there is an additional dependence on the hard momentum transfer, Q, which appears due to the mixing of the conformal operators between themselves in two-loop approximation. This dependence is governed by a new evolution equation of the form [6]

$$\frac{d}{d\ln Q}\boldsymbol{B} + \left[\boldsymbol{\gamma}^{\mathrm{D}}, \boldsymbol{B}\right]_- + \boldsymbol{\gamma}^{\mathrm{ND}}\boldsymbol{B} = 0. \qquad (11)$$

Making use of these results we are in a position to study the evolution of OFPD explicitly. In order to save place let us address the non-singlet sector only. The rough features of the shape of OFPD can be gained from the perturbation theory itself. Assuming skewednessless input shown in Fig. 1 at a very low normalization point typical for phenomenological models of confinement we evolve it (with $J_{\max} = 80$) upwards with momentum scale $Q^2 = 100\,\mathrm{GeV}^2$ (Fig. 2). The relative size of next-to-leading order effects is shown

[2]Several LO methods are available on a market [7–10].

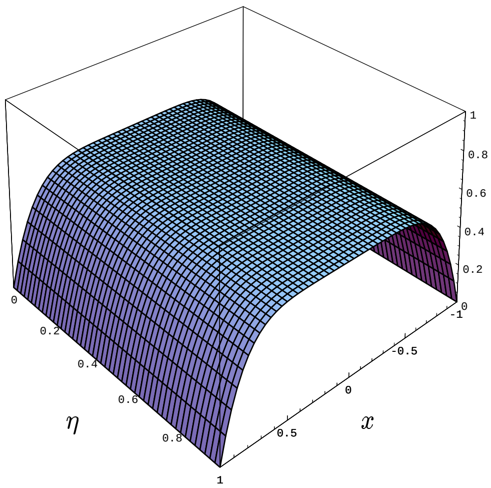

Figure 1. Sample η-independent valence-like input distribution $\mathcal{O}^{\mathrm{val}}(x,\eta) = \frac{7}{12}(1-x^6)$ at low scale $Q_0^2 = 0.2\,\mathrm{GeV}^2$.

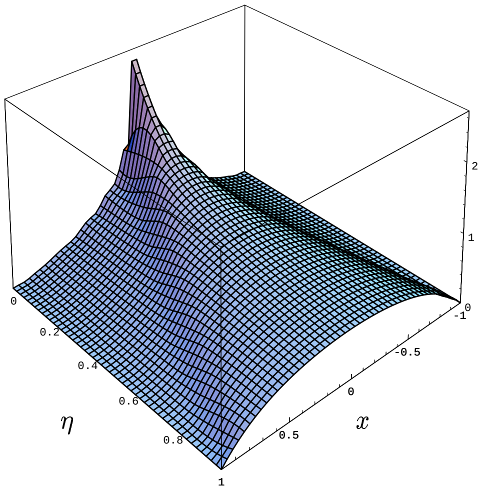

Figure 2. Distribution from Fig. 1 evolved with LO formulae up to $Q^2 = 100\,\mathrm{GeV}^2$.

for given η in Fig. 3. Clearly, NLO effects do not exceed the level of few percent in the non-singlet sector.

The specific feature of the evolution of the OFPD is that the partons with momentum fractions $\eta < |x| < 1$ tend to penetrate into the ER-BL-type region and once they do it they never return back from the domain $x \in [-\eta, \eta]$.

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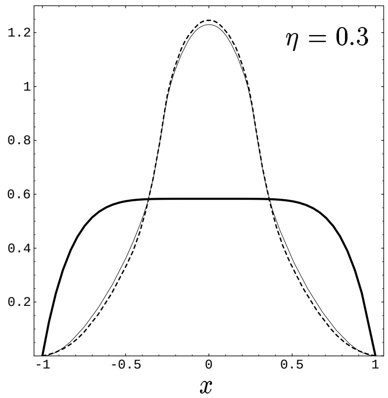

Figure 3. The input distribution $\mathcal{O}^{\mathrm{val}}$ (thick curve) evolved at LO (thin curve) and NLO (dashed curve) for $\eta = 0.3$ up to $Q^2 = 100\,\mathrm{GeV}^2$.

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