Asymptotic Properties of DVCS*

B.I. Ermolaev[a] F. Olness[b] and A.G. Shuvaev[c]

[a]A.F. Ioffe Physico-Technical Institute, St. Petersburg, 194021, Russia

[b]Southern Methodist University, Dallas, TX 75275, USA

[c]St. Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg, 188350, Russia

We compute the deeply virtual Compton scattering (DVCS) amplitude for forward and backward scattering in the asymptotic limit. We make use of the Regge calculus to resum important logarithmic contributions that are beyond those included by the DGLAP evolution. We find a power-like behavior for the forward DVCS amplitude.

1. DEEPLY VIRTUAL COMPTON SCATTERING

The deeply inelastic scattering (DIS) process, $\gamma^* P \to \gamma^* P$ has provided a wealth of information about the structure of the proton, or more specifically, the hadronic matrix element: $\langle P|...|P\rangle$. Recently, it was realized that one can actually investigate the non-diagonal hadronic matrix element $\langle P'|...|P\rangle$ via the Deeply Virtual Compton Scattering (DVCS) process $\gamma^* P \to \gamma P'$. This DVCS process differs from the DIS process in that the outgoing photon is on-shell, and hence the final hadron P' must absorb a small amount of momentum Δ to satisfy the kinematics [1–5]. The net result of the DVCS analysis is to obtain a generalized parton distribution function (PDF) which depends on two scaling variables, $\{x_1, x_2\}$, *cf.*, Fig.1. Various sum rules can be derived to relate these DVCS distributions $f(x_1, x_2)$ to the usual PDF's, $f(x)$ [4].

2. REGGE ANALYSIS

We want to analyze this new DVCS sub-field using some older techniques, namely Regge theory, to obtain analytic expressions for the scattering amplitudes. The key idea that we will use from Regge analysis is that if we confine our kinematics to the *strip regions* of the Mandelstam diagram (Fig.2), then the amplitude can accurately be described by a simple pole structure. If we stray beyond these strip regions into the rest of the physical region, then we are too far from low-energy crossed-channel poles to be dominated by them. Therefore, we will limit our analysis to one of two regions:

1) the Forward Region: $s \sim -u \gg -t$, and

2) the Backward Region: $s \sim -t \gg -u$,

with, of course $s \gg Q^2$.

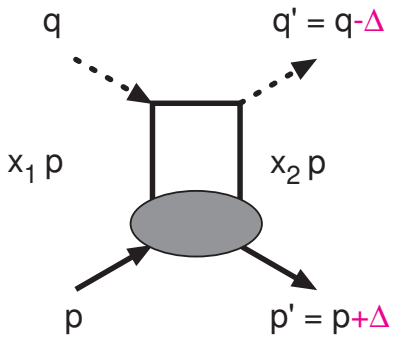

Figure 1. The kinematics of the DVCS process.

3. INFRARED EVOLUTION EQUATION

Since the higher order QCD corrections to the DVCS process (Fig.1) do not violate the Born spin structure, the general DVCS amplitude is: $M_F = M_F^{Born}\ \phi[z_s, z_Q, z_t]$, with:

$$z_s = \ln(s/\mu^2) \quad z_t = \ln(-t/\mu^2)$$
$$z_Q = \ln(Q^2/\mu^2) \quad z_u = \ln(-u/\mu^2)$$

*Presented by F. Olness. This work is supported in part by Grant INTAS-RFBR-95-311, Grant RFBR-98-02-17629, the US Department of Energy, and the Lightner-Sams Foundation. We thank V.Teplitz for helpful discussion.

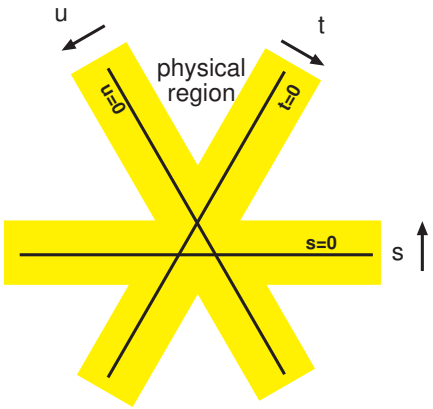

Figure 2. The Mandelstam diagram showing the *strip regions* (shaded) where the amplitude can be accurately described by a simple pole structure.

We differentiate w.r.t. μ to obtain the Infrared Evolution Equation (IREE):

$$-\mu^2 \frac{\partial M_F}{\partial \mu^2} = \frac{\partial M_F}{\partial z_s} + \frac{\partial M_F}{\partial z_Q} + \frac{\partial M_F}{\partial z_t}$$

Analysis of M_F yields the solution:

$$M_F = \phi(z_s - z_t, z_Q - z_t)\; e^{-\frac{\alpha_s C_F}{4\pi} z_t^2}$$

4. FORWARD REGION: $s > Q^2 > -t \gg \mu^2$

We decompose M_F into spin-dependent and spin-independent terms, $M_F = N_F + U_F$. We first obtain the spin-dependent asymptotic behavior in the Regge limit (large s) for the forward region ($t \simeq 0$):

$$N_F \simeq \left(\frac{s}{Q^2}\right)^a \left(\frac{Q^2}{\mu^2}\right)^{a/2} e^{-\frac{\alpha_s}{4\pi} \ln^2(-t/\mu^2)}$$

with $a \simeq 3.5[2\alpha_s N/(4\pi]^{1/2}$

In a similar manner, for the spin-independent part, we obtain:

$$U_F \simeq \left(\frac{s}{Q^2}\right)^{1+\Delta_P} \left(\frac{Q^2}{\mu_2}\right)^{\gamma_P} e^{-\frac{\alpha_s N}{4\pi} \ln^2(-t/\mu^2)}$$

where Δ_P is the Pomeron intercept, and γ_P is the Pomeron anomalous dimension.

Consequently, we find the asymptotic forward scattering amplitude takes the form of the Compton scattering amplitude times a Sudakov exponential factor.

5. BACKWARD REGION: $s > Q^2 > -u \gg \mu^2$

For the backward DVCS process ($u \simeq 0$), the analysis is simpler because there is no difference between asymptotic behavior of the polarized and unpolarized amplitude M_B. We obtain at the final expression:

$$M_B = M_B^{Born}\; e^{-\frac{\alpha_s}{4\pi} C_F \ln^2(s/Q^2)}$$

The DVCS amplitude in the backward scattering region is purely of the Sudakov type with an exponential factor.

6. RELATION TO DGLAP ANALYSIS

Here we highlight some differences of this work in comparison to the DGLAP analysis. For the calculation of the spin-dependent amplitude N_F in the $-t \sim 0$ limit, we define F via a Mellin transform:

$$N_F(z_s, z_Q) = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath}\; e^{z_s \omega}\; \xi(\omega)\; F(\omega, z_Q)$$

The IREE for the spin-dependent part takes the following form:

$$\left(\frac{\partial}{\partial z_Q} + \omega\right) F(\omega, z_Q) = \frac{1}{8\pi^2} F(\omega, z_Q) f_0(\omega)$$

where f_0 is the Mellin amplitude for quark-quark elastic scattering with all quarks on-shell. Putting all the pieces together, we obtain for N_F:

$$\frac{e^2}{g^2 C_F} \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} \left(\frac{s}{Q^2}\right)^{\omega} \frac{e^{-\frac{\alpha_s C_F}{4\pi} z_t^2}}{\omega - f_0/8\pi^2} \left(\frac{Q^2}{\mu^2}\right)^{\frac{f_0}{8\pi^2}}$$

Note that if we expand f_0 keeping the leading powers of $1/\omega$, we arrive at the standard leading-order DGLAP leading order expression for the DVCS. The DGLAP incorporates both singular and non-singular contributions from a finite number of terms. Here, we instead resum only the singular contributions from an infinite number of terms to double-log accuracy. While we neglect the nonsingular terms, this does not affect the asymptotic behavior [6].

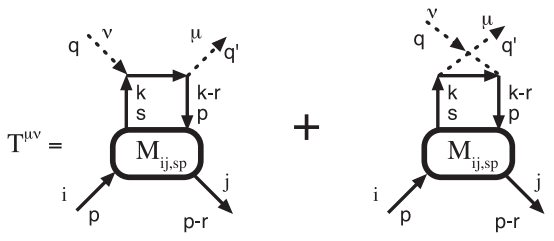

Figure 3. Total amplitude for the DVCS process.

7. NON-FORWARD STRUCTURE FUNCTIONS

Now that we have expressions for the asymptotic forward and backward amplitudes, M_F and M_B, we can relate these quantities to the total amplitude which, for $t \simeq 0$ is given by:

$$T^{\mu\nu} \;=\; \frac{g_\perp^{\mu\nu}}{2p\cdot q'}\,T_1 \;-\; \frac{i\varepsilon^{\mu\nu\rho\eta}}{p\cdot q'}\frac{p_\rho\, q'_\eta}{p\cdot q'}\,T_2,$$

cf., Fig.3. Using the Sudakov representation for the kinematic variables, we can easily evaluate T_1 and T_2 in the asymptotic limit, and then invert these equations to obtain analytic expressions for the non-forward structure functions $\mathcal{F}_\zeta^{q,\bar q}(x)$ and $G_\zeta^{q,\bar q}(x)$.

We find the following non-forward structure functions: [7]

$$\mathcal{F}_\zeta^{q}(\beta) = \tfrac{C_F}{8\pi^2}\tfrac{1}{1-\beta}\int\limits_{-\imath\infty}^{+\imath\infty}\tfrac{d\omega}{2\pi\imath}R_S(\omega)\tfrac{1}{\omega}\times$$

$$\left[\theta\left(\beta-\tfrac{\zeta}{2}\right)\left(\tfrac{1-\beta}{\beta}\right)^{\omega}+\theta\left(\tfrac{\zeta}{2}-\beta\right)\left(\tfrac{1-\beta}{\zeta-\beta}\right)^{\omega}\right]$$

$$\mathcal{F}_\zeta^{\bar q}(\beta) = \tfrac{C_F}{8\pi^2}\tfrac{1}{1+\beta-\zeta}\int\limits_{-\imath\infty}^{+\imath\infty}\tfrac{d\omega}{2\pi\imath}R_U(\omega)\tfrac{1}{\omega}\times$$

$$\left[\theta\left(\beta-\tfrac{\zeta}{2}\right)\left(\tfrac{1+\beta-\zeta}{\beta}\right)^{\omega}+\theta\left(\tfrac{\zeta}{2}-\beta\right)\left(\tfrac{1+\beta-\zeta}{\zeta-\beta}\right)^{\omega}\right]$$

The corresponding $G_\zeta^{q,\bar q}(\beta)$ structure functions are expressed via analogous formulas. Here, $R_{S,U}$ are the standard s- and u-channel Regge amplitudes. The Regge limit implies the argument of the Mellin transform is be asymptotically large, which corresponds to the the region where $\zeta \ll 1$ and $\beta \sim \zeta$. Note that the pair of θ-functions yields the full support in the variable β.

8. CONCLUSIONS

Deeply Virtual Compton Scattering (DVCS) provides a means to expand our knowledge of the hadron structure and extract information on new quantities such as the non-diagonal hadronic matrix element: $\langle P'|...|P\rangle$. Via the IREE, we have resummed singular terms in $1/\omega$ to double logarithmic accuracy. While we have neglected the non-singular terms (*eg.*, as included in the DGLAP approach), these do not contribute in the asymptotic region. This allows us to obtain expressions for the structure functions in the forward and backward regions in the Regge limit. As we have not used the DGLAP evolution equations, which assume the transverse momenta to be strictly ordered in virtuality, we include important logarithmic contributions which extend the realm of applicability.

REFERENCES

1. X.Ji, Phys. Rev. Lett. 78 (1997) 610; Phys. Rev. D55 (1997) 7114; X.Ji, J. Nucl. Part. Phys. G24 (1998) 1181, also references therein.
2. A.V.Radyushkin, Phys. Lett. B380 (1996) 417; B385 (1996) 333; Phys.Rev. D56 (1997) 5524.
3. J.Blumlein and A.Vogt, Phys. Lett. B370 (1996) 149; Acta Phys. Pol. B27 (1996) 1309; Phys. Lett. B386 (1996) 350; J.Blumlein, B.Geyer and D.Robaschik, Phys. Lett. B406 (1997) 161; hep-ph/9711405.
4. S.Brodsky, L.Frankfurt, J.Gunion, A.Mueller and M.Strikman, Phys. Rev. **D50** (1994) 3134; J.Collins, L.Frankfurt and M.Strikman, Phys. Rev. **D56** (1997) 2982; Z.Chen, Nucl. Phys. **B525** (1998) 369.
5. *Cf.*, DIS'99 Presentations by: A. Belitsky; A. Freund; K. Golec-Biernat; A. Martin.
6. V.S.Fadin, E.A.Kuraev and L.N.Lipatov, Sov. Phys. JETP **44** (1976) 443; *ibid* 45 (1977) 199; Y.Y.Balitskij and L.N.Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822.
7. B.I. Ermolaev, F. Olness, A.G. Shuvaev. hep-ph/9812271, also references therein.