Twist-4 Corrections to Polarized Structure Functions in the Flavor Singlet Sector

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We summarize findings of infrared renormalon calculations to model twist-4 corrections to the polarized structure function $g_1(x, Q^2)$.

Data on polarised structure functions are taken at relative small values of the photon virtuality Q^2. This is especially the case for the HERMES and SLAC experiments. In this kinematical domain special attention should be paid to corrections that fall off in powers of the photon virtuality Q^2 and become important here. To extract parton distribution functions from experimental data these corrections have to be included into data fits. For inclusive cross sections power corrections are defined in the framework of Wilson's operator product expansion. They can be divided into kinematical target-mass corrections which can be calculated to all orders in $(M^2/Q^2)^k$ and consist of power suppressed twist-2 operators [1]. On the other hand dynamical higher twist corrections are given by multiparton correlations which at present cannot be calculated directly. Although this is known for decades, only recently there has been progress in the phenomenology of dynamical higher twist effects in DIS. The phenomenology rests on the observation that twist-2 and twist-4 contributions cannot be regarded as independent (see [2] and references therein). The perturbative expansion of the leading twist is ill defined and suffers from ambiguities. Also the corresponding twist-4 operator appearing in the OPE is not well defined, only the sum of twist-2 and twist-4 has a physical meaning.

The polarized structure function g_1 appears in the parameterization of the antisymmetric part of the hadronic scattering tensor:

$$W_{\mu\nu} = \frac{\Im}{2\pi} \int d^4 z e^{iqz} \langle pS | T\left(J_\mu(z) J_\nu(0)\right) | pS \rangle$$
$$= -i\epsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda S^\sigma}{p \cdot q} g_1(x, Q^2) + \dots \quad . \tag{1}$$

Applying OPE to the time ordered product of the two currents one gets for the leading term contributing to the first moment of g_1

$$\begin{aligned} & i \int d^4 z\; e^{iqz}\; T\left\{J_\mu(z) J_\nu(0)\right\} \\ & = 2i\varepsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda}{q^2} \bigg(C^A_{t_2}(\mu^2/Q^2, g^2)(\bar q \gamma^\sigma \gamma^5 q)(0) \\ & - \frac{8}{9q^2} C^A_{t4}(\mu^2/Q^2, g^2)(\bar q g \tilde G^{\sigma\beta} \gamma_\beta q)(0) \bigg) \\ & + \dots \; . \end{aligned} \tag{2}$$

Note that we have not explicitly written down the spin-2, twist-3 operator $(\bar q g \tilde G^{\sigma\alpha} \gamma^\beta q)(0)$ which also contributes at order $1/Q^2$. Its matrix element $d^{(2)}$ not only contributes to the first moment of $g_1(x, Q^2)$ but also determines the twist-3 contribution to the third moment of $g_2(x, Q^2)$. $C^A_{t_n}(\mu^2/Q^2, g^2)$ are the Wilson coefficients corresponding to the operator of twist-n and can be calculated in perturbation theory.

Perturbative expansions in QCD diverge and are not summable in the Borel-sense. Therefore they can at best be considered as asymptotic expansions. The approximation to some physical quantity $R(Q^2)$ in perturbation theory therefore is plagued by ambiguities. The ambiguity is estimated as being of the order of the minimal term in the perturbative expansion.

$$\left| R(Q^2) - \sum_{n=0}^{N} a_n \alpha_S^{n+1}(Q^2) \right| < K_{N+1} \alpha_S^{N+1}(Q^2) \tag{3}$$

In QCD the series is found to diverge factorially with $N \sim \log(Q^2/\Lambda^2)$ and therefore the error estimate takes the form of a power correction:

$$K_{N+1}\alpha_S^{N+1}(Q^2) \sim \frac{A_2'}{Q^2} \quad . \tag{4}$$

The twist-4 matrix element itself is not well defined but contains a corresponding ambiguity which manifests itself as quadratic UV divergence

$$C_{t4}^A(\mu^2/Q^2, g^2)(\bar{q}g\tilde{G}^{\sigma\beta}\gamma_\beta q)(0) \sim \mu^2(\bar{q}\gamma^\sigma\gamma^5 q)(0) \tag{5}$$

so that only the sum of twist-4 and twist-2 contributions in Eq. (2) is well defined. By explicit calculation it can be shown, that the ambiguities in Eq. (3) and Eq. (5) cancel when inserted in Eq. (2) [3].

As approximation to an exact all order expression of the Wilson coefficient, calculations are performed in the large N_F limit. Subsequently this is converted via the naive non-abelianization (NNA) by the substitution $N_F \to N_F - 33/2$ to an approximation to real QCD. The resulting behavior of the fixed order expansion of the the Wilson coefficient of the Bjorken sum rule is shown in Fig. 1. Calculating the minimal term in the

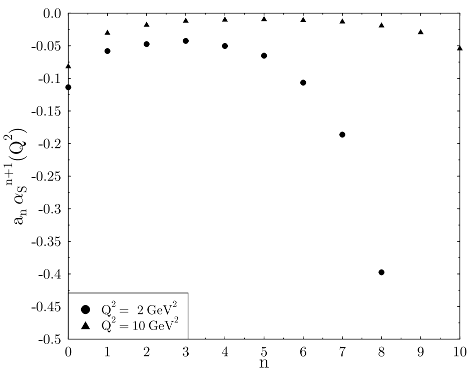

Figure 1. NNA-Approxmimants to the Bjorken Sum Rule: $\int dx \quad g_1^{p-n}(x,Q^2)/\left(\frac{1}{6}\frac{g_A}{g_V}\right) = 1 + \sum_n a_n \alpha_S^{n+1}(Q^2)$ at $Q^2 = 2\text{GeV}^2$ and $Q^2 = 10\text{GeV}^2$.

expansion, a scale of the power corrections can be extracted. Equivalently this can be done by taking the imaginary part (divided by π) of the corresponding Borel transform.

$$\frac{A_2'}{Q^2} = \pm\frac{2C_F}{(11 - 3/2N_F)}\frac{\Lambda_{\overline{MS}}^2 e^{+5/3}}{Q^2} \tag{6}$$

The calculation sketched above can be performed for all moments of the structure function $g_1(x,Q^2)$ in the non-singlet (NS) and in the singlet channel. Writing the polarized structure function up to $\mathcal{O}(\frac{1}{Q^4})$ accuracy as

$$\begin{aligned} xg_1(x,Q^2) &= xg_1^{\text{tw}-2}(x,Q^2) \\ &+\frac{1}{Q^2}\left(h_\Delta(x,Q^2) + h^{\text{tw}-3}(x,Q^2)\right) \\ &+\frac{1}{Q^2}h^{\text{TMC}}(x,Q^2) + \mathcal{O}(\frac{1}{Q^4}) \end{aligned} \tag{7}$$

the IR-renormalon model will provide a model for h_Δ which contains the genuine twist-4 corrections and is sensitive to multi-parton correlations in the target. h_i^{TMC} describes the target mass corrections which are directly related to twist-2 matrix elements, $h^{\text{tw}-3}$ on the other hand are twist-3 corrections that cannot be traced by the renormalon since they do not mix with the leading twist contribution.

The resulting model for the twist-4 contributions h_Δ (c.f. Eq. (7)), then has the form of a Mellin convolution [3] :

$$\begin{aligned} &h_\Delta^{(i)}(z,\mu^2) = \Lambda_{(i)}^2\, x\, \times \\ &\int_x^1 \frac{\mathrm{d}z}{z}\, A_{(i)}^{(2)}(z, \log(\Lambda_{(i)}^2/\mu^2))\, g_1^{\text{tw}-2,(\text{i})}(x/z,\mu^2) \end{aligned} \tag{8}$$

where the index (i) refers to the gluon, non-singlet and quark-singlet channel. Non-singlet contributions have been calculated in [4,5]. Singlet contributions in [6]. The function $A_{(i)}^{(2)}(z,\log(\Lambda_{(i)}^2/\mu^2)$ is exactly calculable, the normalization $\Lambda_{(i)}^2$ should be fitted to the data or can be taken from the NNA-procedure.

Fig. 2 shows the renormalon model prediction for the twist-4 part of the polarised structure function $xg_1^{\text{tw}-4}(x,Q^2)$. The resulting NS correction is negative at small x, changes sign around $x = 0.02$, and reaches a maximum around $x \sim 0.4$. The pure singlet contribution is significant only in the small x region but, contrary to the unpolarized case [6], even there it does not dominate over the NS part.

Twist-4 contributions give important corrections to scaling at low values of Q^2, and thus data fits should include these corrections. The renormalon model calculations are available and cover the whole domain of Bjorken-x even though care has to be taken at very small x where soft gluons have to be resummed. So far the model has been applied in data fits to unpolarized structure functions F_2, F_3 and F_L in the non singlet channel for $x > 0.1$. The observed x-dependence is surprisingly well predicted by the model and even the absolute normalization - as it comes out of the NNA-procedure - is not inconsistent with experimental data [7–9]. The absolute magnitude however decreases when going from LO to NLO and further to NNLO order fits as observed in [10]. A corresponding analysis of the power corrections to $g_1(x, Q^2)$ would give valuable insight into the relative importance of higher twist corrections since in this case not only twist-4 operators and target mass corrections contribute but also twist-3 operators. These operators do not mix with leading twist contributions and therefore cannot be traced in the renormalon model.

In conclusion we have provided a model for the x-dependence of twist-4 corrections to $g_1(x, Q^2)$. Assuming for the moment that the absolute normalization as it is provided by the procedure of naive non-abelianization gives a correct estimate of the absolute size of the power corrections (which is supported by the analysis of unpolarized structure functions), we can conclude that twist-4 corrections to $g_1^d(x, Q^2)$ at 4 GeV^2 are of the order of 6%.

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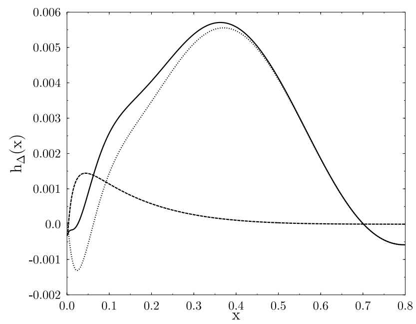

Figure 2. The shape of the $1/Q^2$ power correction $h_\Delta(x)$ to the polarized structure function $xg_1(x, Q^2)$ for a deuteron target. The dashed line corresponds to the pure-singlet quark contribution, the dotted line to the contribution of the non-singlet part. The full line is the sum of both. $\Lambda_{\overline{MS}} = 231$ MeV, $Q^2 = 4$ GeV^2 and $n_f = 4$. [6]

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