Construction of Nonlocal Light–Cone Operators with Definite Twist*

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1. INTRODUCTION

With the growing precision of data for light–cone dominated hard scattering processes like DIS, DVCS and various (semi) exclusive processes a better perturbative understanding of QCD concerning higher twist contributions is required. Thereby the *nonlocal* light–cone expansion (LCE) is optimally adapted since the *same* nonlocal LC operator, and its anomalous dimension, are related to *different* phenomenological distribution amplitudes, and their Q^2–evolution kernels [1,2]. But, 'geometric' twist = dimension − spin, $\tau = d - j$, introduced for the *local* LC–operators [3] cannot be extended directly to the nonlocal LC–operators. On the other hand, motivated by LC–quantization where the quark fields may be decomposed into 'good' and 'bad' components [4] and by kinematic phenomenology [5] the notion of 'dynamic' twist (t) was introduced counting powers Q^{2-t} of the momentum transfer. However, this notion is defined only for *matrix elements* of operators, is *not* Lorentz invariant and its relation to 'geometric' twist is quite complicated, cf. [6].

Here, we introduce a systematic procedure to uniquely decompose nonlocal LC–operators into harmonic operators of well defined geometric twist, cf. Ref. [7]. This will be demonstrated for the case of (pseudo)scalar, (axial) vector and (skew) tensor bilocal quark light–ray operators.

2. GENERAL PROCEDURE

Let us shortly state the consecutive steps which are used to decompose a bilocal light–ray operator into operators of definite twist:
(1) *Expansion* of the nonlocal operators for *arbitrary* values of x into a Taylor series of *local* tensor operators (rank n, dimension d); cf. Eq. (2).
(2) *Decomposition* of these local operators w.r.t *irreducible tensor representations* of the (orthochronous) Lorentz group having definite rank, dimension and spin (n, d, j).
(3) *Resummation* of the infinite series (for any n) of irreducible tensor operators of equal twist τ to nonlocal *harmonic operators of definite twist.*
(4) *Projection* onto the light–cone, $x \to \tilde{x}$, with

$$\tilde{x} = x + \eta(x\eta)\big(\sqrt{1 - x^2/(x\eta)^2} - 1\big),\ \eta^2 = 1,$$

leading to the required twist decomposition:

$$O_\Gamma(\kappa_1\tilde{x}, \kappa_2\tilde{x}) \ = \ \sum_{\tau_i=\tau_{\min}}^{\tau_{\max}} O_\Gamma^{\tau_i}(\kappa_1\tilde{x}, \kappa_2\tilde{x}). \qquad (1)$$

Generically the nonlocal quark operators with tensor structure $\Gamma = \{1, \gamma_\alpha, \sigma_{\alpha\beta}, \gamma_5\gamma_\alpha, \gamma_5\}$ for arbitrary x are represented by

$$O_\Gamma(\kappa_1 x, \kappa_2 x) = :\overline{\psi}(\kappa_1 x)\Gamma U(\kappa_1 x, \kappa_2 x)\psi(\kappa_2 x): \\ = \sum_{n=0}^{\infty}(n!)^{-1} x^{\mu_1}\ldots x^{\mu_n} O^\Gamma_{\mu_1\ldots\mu_n}(\kappa_1, \kappa_2), \qquad (2)$$

with the corresponding local operators

$$O^\Gamma_{\mu_1\ldots\mu_n} \equiv \big[\bar{\psi}(x)\Gamma \mathsf{D}_{(\mu_1}\ldots \mathsf{D}_{\mu_n)}\psi(x)\big]_{x=0}, \qquad (3)$$

where the brackets $(\ldots)$ denote total symmetrization; thereby we used the phase factors

$$U(\kappa_1 x, \kappa_2 x) \ = \ \mathcal{P}\exp\Big\{-ig\int_{\kappa_2}^{\kappa_1} d\kappa'\, x^\mu A_\mu(\kappa' x)\Big\},$$

and the generalized covariant derivatives

$$\mathsf{D}_\mu(\kappa_1, \kappa_2) \ \equiv \ \kappa_1(\overleftarrow{\partial}_\mu - igA_\mu) + \kappa_2(\overrightarrow{\partial}_\mu + igA_\mu).$$

Concerning the decomposition of the local tensor operators (3) into irreducible tensors w.r.t. the

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Lorentz group $\mathcal{L}^{\uparrow}$ we remember that these are uniquely given by *traceless* tensors being classified according to their *symmetry class* w.r.t. the symmetric group. A symmetry class is determined by a (normalized) *Young-Operator* $\mathcal{Y}_{[m]} = f_{[m]}\mathcal{P}\mathcal{Q}/n!$, where $[m] = (m_1 \geq m_2 \geq \ldots \geq m_r)$ with $\sum_{i=1}^{r} m_i = n$ defines a Young pattern, $\mathcal{P} = \sum_{p\in H_{[m]}} p$ and $\mathcal{Q} = \sum_{q\in V_{[m]}} \delta_q\, q$ are the symmetrizations and antisymmetrizations, respectively, related to the horizontal ($H_{[m]}$) and vertical ($V_{[m]}$) permutation w.r.t. a standard tableau obtained by putting into $[m]$ the numbers $1, \ldots, n$ raising from left to right and from top to bottom; $f_{[m]}$ is the number of standard tableau's to $[m]$.

For the Lorentz group (in 4 dimensions) the allowed Young patterns are restricted by $\ell_1 + \ell_2 \leq 4$ (ℓ_i : lenght of columns). Since the local operators (3) are totally symmetric w.r.t. μ_i's, depending to the additional tensor structure Γ, only the following Young patterns are of relevance (the spin j is related to the various trace terms):

(i) $[m] = (n)$ $\quad j = n, n-2, n-4, \ldots$
(ii) $[m] = (n-1,1)$ $\quad j = n-1, n-2, \ldots$
(iii) $[m] = (n-2,1,1)$ $\quad j = n-2, n-3, \ldots$

Because the decomposition into irreducible representations does not depend on the specific values of the κ–variables we may choose $(\kappa_1, \kappa_2) = (0, \kappa)$ and $\mathsf{D}_\mu(\kappa_1, \kappa_2) \Rightarrow \kappa\overrightarrow{D}_\mu(A)$; afterwards we generalize to arbitrary values of κ_i.

3. HARMONIC TENSOR FUNCTIONS

For constructing traceless tensors $\overset{\circ}{T}_{\Gamma(\mu_1\ldots\mu_n)}$ let us start with a generic tensor, not being traceless, whose symmetrized indices are contracted by x^{μ_i}'s: $T_{\Gamma n}(x) = x^{\mu_1}\ldots x^{\mu_n} T_{\Gamma(\mu_1\ldots\mu_n)}$. The conditions for these tensors to be traceless read:

$$\Box \overset{\circ}{T}_{\Gamma\ n}(x) = 0, \tag{4}$$

and in addition

$$\partial^\alpha \overset{\circ}{T}_{\alpha\ n}(x) = 0 \quad \text{resp.} \quad \partial^\alpha \overset{\circ}{T}_{[\alpha\beta]\ n}(x) = 0 \tag{5}$$

for (axial) vectors resp. skew tensors.
The solutions of these equations ($D = 4$) are:
(1) *Scalar harmonic polynomials*:

$$\overset{\circ}{T}_n(x) = H_n^{(4)}\left(x^2|\Box\right) T_n(x) \tag{6}$$

with the harmonic projection operator

$$H_n^{(4)}\left(x^2|\Box\right) = \sum_{k=0}^{[\frac{n}{2}]} \frac{(n-k)!}{k!n!}\left(\frac{-x^2}{4}\right)^k \Box^k. \tag{7}$$

(2) *Vectorial harmonic polynomials*:

$$\overset{\circ}{T}_{\alpha n}(x) = \Big\{\delta_\alpha^\beta - \frac{2}{(n+1)^2}\Big(x_\alpha\partial^\beta(x\partial) - \frac{x^2}{2}\partial_\alpha\partial^\beta\Big)\Big\} H_n^{(4)}\left(x^2|\Box\right) T_{\beta n}(x). \tag{8}$$

Of course, contraction with x^α leads to $\overset{\circ}{T}_n(x)$.
(3) *Skew tensorial harmonic polynomials*:

$$\overset{\circ}{T}_{[\alpha\beta]n}(x) = \Big\{\delta^\mu_{[\alpha}\delta^\nu_{\beta]} + \frac{1}{(n+1)n}\Big(x_{[\alpha}\delta^\mu_{\beta]}\partial^\nu(x\partial) - \frac{x^2}{2}\partial_{[\alpha}\delta^\mu_{\beta]}\partial^\nu\Big) - \frac{2}{(n+2)(n+1)n}x_{[\alpha}\partial_{\beta]}x^\mu\partial^\nu\Big\} \times H_n^{(4)}\left(x^2|\Box\right) T_{[\mu\nu]n}(x) \tag{9}$$

with the convention $T_{[\alpha\beta]} = (T_{\alpha\beta} - T_{\beta\alpha})/2$.

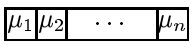
In principle also harmonic tensors like $\overset{\circ}{T}_{(\alpha\beta)n}(x)$, related to gluonic operators, and of higher orders may be constructed. Obviously, any *harmonic tensor function* may be represented as a series expansion into corresponding polynomials.

4. TWIST DECOMPOSITION OF QUARK OPERATORS

In order to select the different spin content of these tensorial harmonic polynomials we have to observe the complete symmetry related to $[m]$. Let us give some selected examples.
1. (Pseudo) scalar operators: First we consider the scalar quark operator

$$O(0,\kappa x) = \bar\psi(0)(\gamma x)U(0,\kappa x)\psi(\kappa x) \tag{10}$$
$$O_n(x) = x^\alpha x^{\mu_1}\ldots x^{\mu_n}\bar\psi(0)\gamma_{(\alpha}D_{\mu_1}\ldots D_{\mu_n)}\psi(0).$$

Only Young pattern (i) is relevant:



contains $\quad \tau = 2, 4, \ldots$.

Therefore, $O_n(x)$ gets traceless by applying, according to Eq. (6), the harmonic projection operator. Using the integral representation of Euler's Beta–function, $(n+1-k)!(k-1)!/(n+1)! = B(n+2-k,k) = \int_0^1 \mathrm{d}t\, t^{n+1-k}(1-t)^{k-1}$ one

resums these local operators $\overset{\circ}{O}_n(x)$ to obtain the traceless nonlocal scalar twist 2 operator

$$O^{\rm tw2}(0,\kappa x) \equiv \overset{\circ}{O}(0,\kappa x) = \sum_{n=0}^{\infty} \frac{\kappa^n}{n!} \overset{\circ}{O}_n(x) = \qquad (11)$$

$$O(0,\kappa x) + \sum_{k=1}^{\infty} \int_0^1 dt \frac{(1-t)^{k-1}(-x^2)^k \Box^k}{t^{k-1} 4^k k!(k-1)!} O(0,\kappa t x).$$

The sum contains the trace terms which are to be subtracted from $O(0,\kappa x)$; obviously, they vanish in the limit $x \to \tilde{x}$. Therefore, the scalar operator (10), if taken on LC, already has minimal twist 2. The analogous conclusion holds for the pseudo scalar operator obtained by replacing $\gamma_\mu \to \gamma_5\gamma_\mu$.
2. (Axial) vector operators: Now we consider the vector operators

$$O_\alpha(0,\kappa\tilde{x}) = \bar{\psi}(0)\gamma_\alpha U(0,\kappa\tilde{x})\psi(\kappa\tilde{x}) \qquad (12)$$
$$O_{\alpha n}(x) = x^{\mu_1}\ldots x^{\mu_n}\bar{\psi}(0)\gamma_\alpha D_{(\mu_1}\ldots D_{\mu_n)}\psi(0)$$

The relevant symmetry classes are determined by the Young patterns (i) and (ii):

$\alpha\,|\,\mu_1\,|\,\mu_2\,|\,\cdots\,|\,\mu_n$ contains $\tau = 2,4,\ldots,$

$\mu_1\,|\,\mu_2\,|\,\cdots\,|\,\mu_n$ / α contains $\tau = 3,4,\ldots,$

Proceeding analogous to the scalar case we obtain for the symmetry class (i)

$$O_\alpha^{\rm tw2}(0,\kappa x) = \partial_\alpha \int_0^1 {\rm d}\lambda\, \overset{\circ}{O}(0,\kappa\lambda x), \qquad (13)$$

and for the symmetry class (ii) (14)

$$O_\alpha^{\rm tw3}(0,\kappa x) = (g_\alpha^\mu (x\partial) - x^\mu\partial_\alpha)\int_0^1 {\rm d}\lambda \overset{\circ}{O}_\mu(0,\kappa\lambda x),$$

where $\overset{\circ}{O}_\alpha(0,\kappa x)$ has to be determined according to Eq. (8). Both expressions contain trace terms which are related to twist $\tau = 4$ and higher.
3. (Skew) tensor operators: Here we distinguish two cases, the true tensor case (A)

$$M_{[\alpha\beta]}(0,\kappa x) = \bar{\psi}(0)\sigma_{\alpha\beta}U(0,\kappa x)\psi(\kappa x), \qquad (15)$$
$$M_{[\alpha\beta]n}(x) = x^{\mu_1}\ldots x^{\mu_n}\bar{\psi}(0)\sigma_{\alpha\beta}D_{(\mu_1}\ldots D_{\mu_n)}\psi(0),$$

and case (B) with one additional contraction

$$M_\alpha(0,\kappa x) = \bar{\psi}(0)\sigma_{\alpha\beta}\tilde{x}^\beta U(0,\kappa x)\psi(\kappa x), \qquad (16)$$
$$M_{\alpha n}(x) = x^\beta M_{[\alpha\beta]n}(x).$$

The relevant symmetry classes are determined by the Young patterns (ii) and (iii):

$\beta\,|\,\mu_1\,|\,\mu_2\,|\,\cdots\,|\,\mu_n$ / α contains $\tau = 2,3,4\ldots,$

$\mu_1\,|\,\mu_2\,|\,\cdots\,|\,\mu_n$ / β / α contains $\tau = 3,4,\ldots.$

Obviously, in case (B) the Young pattern (iii) does not contribute. ¿From pattern (ii) we obtain

$$M_{[\alpha\beta]}^{\rm tw2}(0,\kappa x) = 2\partial_{[\beta}\int_0^1 {\rm d}\lambda\,\lambda\, \overset{\circ}{M}_{\alpha]}(0,\kappa\lambda x), \qquad (17)$$

$$M_\alpha^{\rm tw2}(0,\kappa x) = \overset{\circ}{M}_\alpha(0,\kappa\lambda x); \qquad (18)$$

the contributions from pattern (iii) begin with

$$M_{[\alpha\beta]}^{\rm tw3(iii)}(0,\kappa x) = \Big((x\partial)\delta^\nu_{[\beta} - 2x^\nu\partial_{[\beta}\Big) \times \int_0^1 {\rm d}\lambda\,\lambda\, \overset{\circ}{M}_{\alpha]\nu}(0,\kappa\lambda x), \qquad (19)$$

where $\overset{\circ}{M}_{\alpha\nu}(0,\kappa x)$ is determined by Eq. (9).

5. PPROJECTION ONTO THE LIGHT–CONE

In the preceding Section we stated only the leading contributions for the different symmetry classes, and did not carry out the various differentiations which are necessary to obtain the explicit expressions for the corresponding operators. Of course, these differentiations have to be performed before projecting onto the light–cone. As can be seen easily after that projection only contributions up to twist $\tau_{\rm max} = 4$ will survive; cf. Eq. (1). This shows that the nonlocal light–ray operators under consideration have a unique decomposition which is given by

$$\begin{aligned} O_\alpha &= O_\alpha^{\rm tw2} + O_\alpha^{\rm tw3} + (O_\alpha^{\rm tw4(i)} + O_\alpha^{\rm tw4(ii)}), \\ M_{[\alpha\beta]} &= M_{[\alpha\beta]}^{\rm tw2} + M_{[\alpha\beta]}^{\rm tw3(ii)} + M_{[\alpha\beta]}^{\rm tw4(ii)} \\ &\quad + M_{[\alpha\beta]}^{\rm tw3(iii)} + M_{[\alpha\beta]}^{\rm tw4(iii)}, \\ M_\alpha &= \tilde{x}^\beta M_{[\alpha\beta]} = M_\alpha^{\rm tw2} + M_\alpha^{\rm tw3(ii)}. \end{aligned}$$

for the vector and skew tensor cases. The resulting expressions for general values (κ_1,κ_2) are

$$O^{\rm tw2}(\kappa_1\tilde{x},\kappa_2\tilde{x}) = \bar{\psi}(\kappa_1\lambda\tilde{x})(\gamma\tilde{x})U(\kappa_1\lambda\tilde{x},\kappa_2\lambda\tilde{x})\psi(\kappa_2\lambda\tilde{x}) \equiv \tilde{x}^\mu O_\mu(\kappa_1\tilde{x},\kappa_2\tilde{x}) \tag{20}$$

$$O^{\rm tw2}_\alpha(\kappa_1\tilde{x},\kappa_2\tilde{x}) = \int_0^1 {\rm d}\lambda \Big(\partial_\alpha + \frac{1}{2}(\ln\lambda)\, x_\alpha \Box\Big) x^\mu O_\mu(\kappa_1\lambda x,\kappa_2\lambda x)\big|_{x=\bar{x}} \tag{21}$$

$$O^{\rm tw3}_\alpha(\kappa_1\tilde{x},\kappa_2\tilde{x}) = \int_0^1 {\rm d}\lambda \Big(\delta^\mu_\alpha (x\partial) - (x^\mu\partial_\alpha + x_\alpha\partial^\mu) - (\ln\lambda)\, x_\alpha \Box x^\mu\Big) O_\mu(\kappa_1\lambda x,\kappa_2\lambda x)\big|_{x=\bar{x}} \tag{22}$$

$$O^{\rm tw4}_\alpha(\kappa_1\tilde{x},\kappa_2\tilde{x}) = \int_0^1 {\rm d}\lambda \Big(x_\alpha\partial^\mu + \frac{1}{2}(\ln\lambda)\, x_\alpha \Box x^\mu\Big) O_\mu(\kappa_1\lambda x,\kappa_2\lambda x)\big|_{x=\bar{x}} \tag{23}$$

$$M^{\rm tw2}_{[\alpha\beta]}(\kappa_1\tilde{x},\kappa_2\tilde{x}) = 2\int_0^1 {\rm d}\lambda \Big\{ \lambda\, \partial_{[\beta}\delta^\mu_{\alpha]} - (1-\lambda)\big(2x_{[\alpha}\partial_{\beta]}\partial^\mu - x_{[\alpha}\delta^\mu_{\beta]}\Box\big)\Big\} M_\mu(\kappa_1\lambda x,\kappa_2\lambda x)\big|_{x=\bar{x}} \tag{24}$$

$$\begin{aligned} M^{\rm tw3}_{[\alpha\beta]}(\kappa_1\tilde{x},\kappa_2\tilde{x}) = \int_0^1 {\rm d}\lambda \Big\{ & \lambda\big((x\partial)\delta^\nu_{[\beta} - 2x^\nu\partial_{[\beta}\big)\delta^\mu_{\alpha]} - \frac{1-\lambda^2}{\lambda}\Big(x_{[\alpha}\partial_{\beta]}x^{[\mu}\partial^{\nu]} \\ & + x_{[\alpha}\big(\delta^{[\mu}_{\beta]}(x\partial) - x^{[\mu}\delta_{\beta]}\big)\partial^{\nu]} - x_{[\alpha}\delta^{[\mu}_{\beta]}x^{\nu]}\Box\Big)\Big\} M_{\mu\nu}(\kappa_1\lambda x,\kappa_2\lambda x)\big|_{x=\bar{x}} \end{aligned} \tag{25}$$

$$M^{\rm tw4}_{[\alpha\beta]}(\kappa_1\tilde{x},\kappa_2\tilde{x}) = \int_0^1 {\rm d}\lambda \frac{1-\lambda}{\lambda}\Big\{\big(x_{[\alpha}\delta^{[\mu}_{\beta]}x^{\nu]}\Box - 2x_{[\alpha}\big(\delta^{[\mu}_{\beta]}(x\partial) - x^{[\mu}\delta_{\beta]}\big)\partial^{\nu]}\Big\} M_{\mu\nu}(\kappa_1\lambda x,\kappa_2\lambda x)\big|_{x=\bar{x}} \tag{26}$$

$$M^{\rm tw2}_\alpha(\kappa_1\tilde{x},\kappa_2\tilde{x}) = M_\alpha(\kappa_1\lambda x,\kappa_2\lambda x) - \tilde{x}_\alpha\partial^\mu \int_0^1 {\rm d}\lambda\, \lambda\, M_\mu(\kappa_1\lambda x,\kappa_2\lambda x)\big|_{x=\bar{x}} \tag{27}$$

$$M^{\rm tw3}_\alpha(\kappa_1\tilde{x},\kappa_2\tilde{x}) = \tilde{x}_\alpha\Big(\frac{1}{2}x^\mu\Box - \partial^\mu(x\partial)\Big)\int_0^1 {\rm d}\lambda\, (\lambda\ln\lambda)\, M_\mu(\kappa_1\lambda x,\kappa_2\lambda x)\big|_{x=\bar{x}} \tag{28}$$

put together in the Table above. To obtain that results some formulas like $\int_0^1 d\lambda \int_0^1 dt f(\lambda t)/t = \int_0^1 d\lambda(1-\lambda)f(\lambda)/\lambda$ are used and various partial integrations have to be performed. The two different twist–4 contributions of the vector operator may be read off from Eqs. (21,22) as the terms multiplied by x_α; analogously for the twist–3 part $M^{{\rm tw3(ii)}}_{[\alpha\beta]}$ of the tensor operator Eq. (24).

In conlusion we remark that because of relations (13) and (17) the twist–2 vector and scalar operators and the skew tensor and vector operators, as well as their anomalous dimensions, are directly connected. This leads to nontrivial relations of their non–forward matrix elements, i.e. the *double distribution amplitudes* and their evolution kernels [8]. The extension to non–local conformal LC operators is straightforward. For a detailed presentation of our results see Ref. [7].

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