Towards a Lattice Calculation of Δq and δq *

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Within the framework of lattice QCD a high statistics computation of the nucleon axial and tensor charges is given. Particular attention is paid to the chiral and continuum extrapolations.

1. INTRODUCTION

The axial and tensor charges of the nucleon are defined by ($s^2 = -m_N^2$):

$$\begin{aligned} \langle \vec{p},\vec{s}|\mathcal{A}^\mu|\vec{p},\vec{s}\rangle &= 2s^\mu \Delta q(\mu) \qquad (1)\\ \langle \vec{p},\vec{s}|\mathcal{T}^{\mu\nu}|\vec{p},\vec{s}\rangle &= \frac{2}{m_N}\left(s^\mu p^\nu - s^\nu p^\mu\right)\delta q(\mu) \end{aligned}$$

with $\mathcal{A}^\mu = \overline{q}\gamma^\mu\gamma_5 q$, $\mathcal{T}^{\mu\nu} = i\overline{q}\sigma^{\mu\nu}\gamma_5 q$. The chirality and charge conjugation of these operators are $(+,+)$ and $(-,-)$ respectively. The charges have a parton model interpretation:

$$\begin{aligned} \Delta q(\mu) &= \int_0^1 dx\,[\{q_\uparrow(x,\mu) - q_\downarrow(x,\mu)\}+ \\ &\qquad \{\overline{q}_\uparrow(x,\mu) - \overline{q}_\downarrow(x,\mu)\}] \\ \delta q(\mu) &= \int_0^1 dx\,[\{q_\perp(x,\mu) - q_\top(x,\mu)\}- \\ &\qquad \{\overline{q}_\perp(x,\mu) - \overline{q}_\top(x,\mu)\}] \qquad (2) \end{aligned}$$

where x is the fraction of nucleon momentum carried by parton quark density $q_\bullet(x,\mu)$ at scale μ, in scheme $\mathcal{S}$. Δq is related to the lowest moment of the g_1 structure function and can be measured in (polarised) DIS, while h_1 (for δq) having a $-$ chirality must be found in a reaction allowing the quarks to have a different chirality, such as (polarised) Drell-Yan.

Known for the charges is that $\Delta u - \Delta d = g_A \approx 1.26$, (from neutron decay) and in the heavy quark limit $\Delta u = \delta u = 4/3$, $\Delta d = \delta d = -1/3$. Indeed in the non-relativistic limit as fermions (ie here quarks) are eigenstates of γ_0 then we expect $\Delta q = \delta q$.

2. THE LATTICE APPROACH

The hypercubic discretisation of the Euclidean path integral for QCD with lattice spacing a and resulting Monte Carlo evaluation of the partition function allows, in principle, a fundamental test of QCD. That being said, the lattice programme is rather like an experiment: careful account must be taken of error estimations and extrapolations. There are three limits to consider:

1. The box size must be large enough so that finite size effects are small. Currently sizes of $\sim$ 2fm seem large enough (the N diameter is $\sim$ 0.8fm).

*Talk given by R. Horsley.

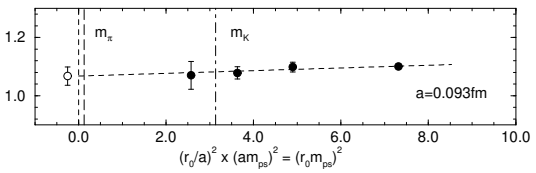

Figure 1. The χ-extrapolation for g_A.

2. The chiral limit, when the quark mass approaches zero. It is difficult to calculate quark propagators at quark masses much below the strange quark mass. For each a value, three (or four) heavier quark masses are used and a linear extrapolation is made to the chiral limit, see Fig. 1. (Within our precision, there is no difference to extrapolating to the u/d quark mass or to the chiral limit.)

3. The continuum limit, $a^2 \to 0$ (the leading order discretisation effects of the action can be so arranged to be $O(a^2)$, [1]). We have performed simulations at three a values corresponding approximately to (0.093, 0.068, 0.051) fm, [a^{-1} = (2.12, 2.90, 3.85)GeV] using scale $r_0 = 0.5$fm, [$r_0^{-1} = 395$MeV], [2].

Our lattice calculation of matrix elements is standard, see eg [3]. We only note here that we are working in the 'quenched approximation' when the fermion determinant in the partition function is set equal to one. The nucleon matrix elements consist of two diagrams: a quark line connected piece and a quark line disconnected piece ($qldis$). Only the quark line connected piece is calculated. Thus, strictly speaking, from eq. (2), we can only compute $\delta u - \delta d$ and $\Delta u - \Delta d$. However, due to the additional negative sign in eq. (2) for δq, we might also expect to be able to determine δu, δd separately. (Both the above described approximations allow considerable savings in computer time.)

3. RENORMALISATION

In general operators (or matrix elements) must be renormalised $\mathcal{O}_R^{\mathcal{S}}(\mu) = Z_{\mathcal{O}}^{\mathcal{S}}(\mu)\mathcal{O}_{bare}$ (in scheme $\mathcal{S}$, at scale μ) before they can be compared with experimental results. For the axial current ensuring $PCAC$ on the lattice gives $Z_{\mathcal{A}}$ and has been calculated in [4]. The tensor current is more complicated. It is first convenient to define a rgi–operator, which is both scheme and scale independent by

$$\mathcal{T}^{rgi} = \Delta Z_{\mathcal{T}}^{\mathcal{S}}(\mu)\mathcal{T}_R^{\mathcal{S}}(\mu) \qquad (3)$$

with

$$\left[\Delta Z_{\mathcal{T}}^{\mathcal{S}}(\mu)\right]^{-1} = \qquad (4)$$
$$\left[2b_0(g^{\mathcal{S}})^2\right]^{\frac{d_{\mathcal{T},0}}{2b_0}} \exp\left[\int_0^{g^{\mathcal{S}}} d\xi \left[\frac{\gamma_{\mathcal{T}}^{\mathcal{S}}(\xi)}{\beta^{\mathcal{S}}(\xi)} + \frac{d_{\mathcal{T},0}}{b_0\xi}\right]\right]$$

The factor $[\Delta Z_{\mathcal{T}}^{\overline{MS}}]^{-1}$ is plotted in Fig. 2. Given $\mathcal{T}^{rgi}$ then from Fig. 2 and eq. (3) we

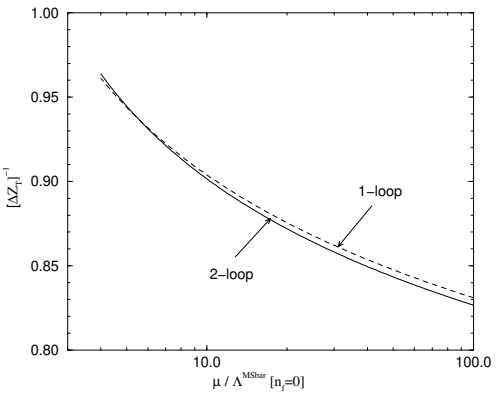

Figure 2. $[\Delta Z_{\mathcal{T}}^{\overline{MS}}]^{-1}$ using the one and two loop expansions of the beta and gamma functions, [5] in units of $\Lambda^{\overline{MS}}$ for $n_f = 0$ flavours. The relation to the scale r_0 is given in [6]. So, eg, at μ = 1, 2, 5 GeV, $[\Delta Z_{\mathcal{T}}^{\overline{MS}}]^{-1} = 0.959_7^7$, 0.911_4^5, 0.870_3^3 respectively.

can thus find $\mathcal{T}_R^{\overline{MS}}(\mu)$ at any desired scale μ. For $Z_{\mathcal{T}}^{\overline{MS}}$ only the result for one loop perturbation theory is known. A non-perturbative result would be desirable, at present we shall use 'tadpole improved' perturbation theory, which re-expands the perturbation series, [7], using a physical coupling constant and removes the lattice 'tadpole' diagrams. Our explicit procedure using $g^{\overline{MS}}$ is described in [8]. To check that this gives a plausible result, we calculate the ρ-tensor decay constant, which also requires the same renormalisation constant. This decay constant is defined by

$$\langle 0|\mathcal{T}^{\mu\nu}|\rho(\vec{p},\lambda)\rangle = i\left(e_\lambda^\mu p^\nu - e_\lambda^\nu p^\mu\right) f_\rho^\perp(\mu) \qquad (5)$$

This gives for $f_\rho^{\perp rgi}/m_\rho$ the results shown in Fig. 3. Ball and Braun, [9], using the sum rule

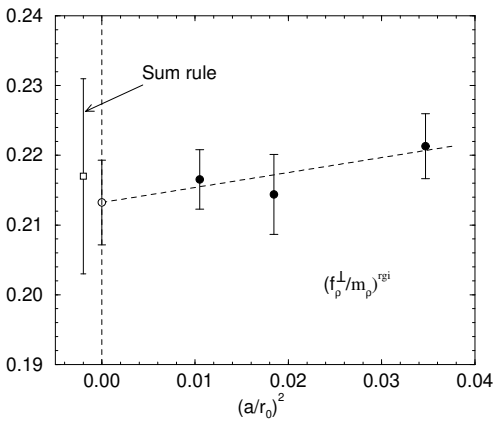

Figure 3. The $a^2 \to 0$ limit for $f_\rho^{\perp rgi}/m_\rho$.

approach find a value for $f_\rho^\perp(\mu = 1\text{GeV})$ of 160 ± 10MeV ($n_f = 3$). Using Fig. 2, this converts to $f_\rho^{\perp rgi}/m_\rho = 0.217(14)$. From Fig. 3 we see that there is good agreement between the lattice and sum rule methods.

4. RESULTS

We now present our results. In Fig. 4 we show $g_A = \Delta u - \Delta d$. There appears to be

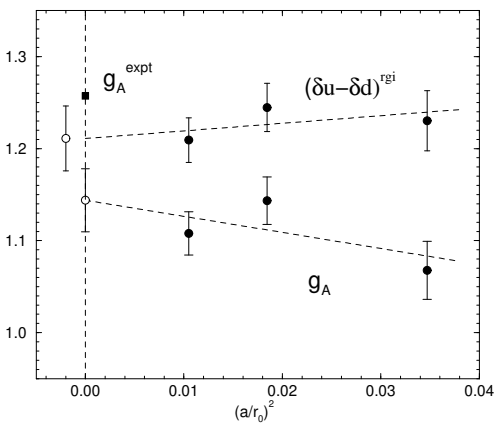

Figure 4. The $a^2 \to 0$ limit for g_A and $(\delta u - \delta d)^{rgi}$. Also shown is the experimental result for g_A.

an a^2 gradient (in distinction to the other results). It seems difficult to reach the phenomenological result. Perhaps this is due to a quenching effect. Also in this figure we show $(\delta u - \delta d)^{rgi}$. It would seem that we have a milder $O(a^2)$ gradient and also that $(\delta u - \delta d)^{rgi} \gtrsim g_A$. This might indicate that a non-relativistic description of spin structure for the 'quenched' nucleon is reasonable. Note, however, that we are well away from the non-relativistic limit of 5/3. To conclude, in the Table, we give our $n_f = 0$ continuum results:

$g_A = \Delta u - \Delta d$	1.14(3)
Δu	0.889(29) $+\Delta q_{qldis}$
Δd	-0.236(27) $+\Delta q_{qldis}$
$(f_\rho^\perp/m_\rho)^{rgi}$	0.213(6)
$(\delta u - \delta d)^{rgi}$	1.21(4)
$(\delta u)^{rgi}$	0.980(30)
$(\delta d)^{rgi}$	-0.234(17)

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