BFKL and CCFM final states

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I give a brief presentation of recent results on the equivalence of BFKL and CCFM small-x final states, and discuss their implications for phenomenology.

1. INTRODUCTION

There are two basic approaches to the study of final states at small x. The BFKL equation [1] is derived in the 'Multi-Regge' limit, i.e. assuming large rapidity intervals between successive emissions. This guarantees the leading logarithms $(\alpha_s \ln x)^n$ (LL) for sufficiently inclusive quantities (such as the total cross section, forward-jet rates) but not necessarily for more exclusive final properties, such as multiplicities or correlations.

A different approach, the CCFM equation [2], goes beyond the 'Multi-Regge' approximation, and explicitly treats coherence (angular ordering) and soft emissions (the $1/(1-z)$ part of the splitting function, where z is the fraction of energy remaining after a parton splitting). This guarantees the leading logarithms for any small-x observable, regardless of how exclusive it is.

The main practical disadvantage of the CCFM equation compared to the BFKL equation is that it is much more difficult to solve, both numerically and analytically. It is therefore of some importance to understand precisely in which situations the BFKL equation will give the correct answer.

A few years ago it was shown by Marchesini that for quantities such as multiplicities, the two equations differed at the level of double logarithms (DL) of x, $(\alpha_s \ln^2 x)^n$ [3]. More recently Forshaw and Sabio Vera [4] introduced a resolvability parameter μ_R and showed (at fixed order, subsequently extended to all orders by Webber [5]) that n-resolved-particle (n-jet) rates are the same in BFKL and CCFM at leading DL order (all terms $(\alpha_s \ln^2 x)^m (\alpha_s \ln x \ln Q)^n$). Since multiplicities are just a weighted sum of n-particle rates, but with $\mu_R = 0$, one is led to ask how these two, apparently contradictory, results are related.

A second issue is that the above results were obtained without any consideration of the soft emissions. What effect do they have?

These questions were discussed in [6] and the basic ideas are presented in the next section. Essentially, the inclusion of soft emissions leads to all BFKL and CCFM predictions being identical at LL level. The phenomenological implications of this result are discussed in sections 3 and 4.

2. THEORETICAL PROPERTIES OF FINAL STATES

The fundamental property used in [6] for the study of final-state properties, is that in both the BFKL and the CCFM equations it is possible to separate emissions which change the exchanged transverse momentum k (i.e. have the largest transverse momentum of all emissions so far — k-changing emissions) from those which don't (k-conserving). The k-changing emissions are responsible for determining the cross section, and can be shown, quite simply, to have the same structure in BFKL and CCFM.

It is the k-conserving emissions which are organised differently between BFKL and CCFM, so I will consider just those. For BFKL, Fig. 1 shows as the shaded region the distribution of k-conserving emissions in x (longitudinal momen-

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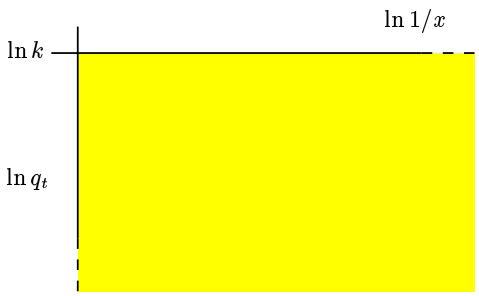

Figure 1. Distribution of BFKL emissions

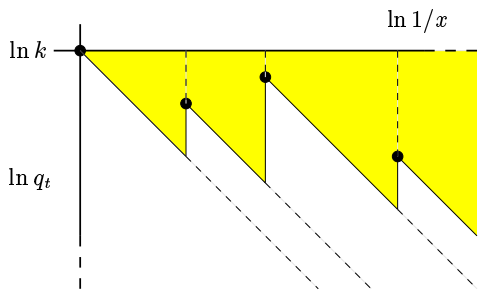

Figure 2. Distribution of CCFM emissions: discs are hard emissions, the shaded areas are the regions accessible to subsequent hard emissions. Soft emissions fill up the white space.

tum fraction of the emitted gluon) and q_t (transverse momentum of the gluon) space. Emissions are independent with mean density

$$\left\langle \frac{dn}{d\ln q_t \, d\ln 1/x} \right\rangle = 2\bar{\alpha}_s \,, \tag{1}$$

where $\bar{\alpha}_s = \alpha_s N_C/\pi$.

The CCFM equation has the extra constraint that emissions be ordered in rapidity. This is illustrated in Fig. 2 — the diagonal lines are constant rapidity ($\ln q_t/x$), and the requirement of a given emission having a larger rapidity than the previous one (being to the right of the diagonal line from the previous one) eliminates the small-q_t emissions. Mathematically this translates into a mean density of emissions of

$$\left\langle \frac{dn}{d\ln q_t \, d\ln 1/x} \right\rangle \simeq 2\bar{\alpha}_s e^{-\bar{\alpha}_s \ln^2 q_t/k} \,, \tag{2}$$

which differs from the BFKL result by a subleading factor (containing no logarithms of x). So for finite q_t one obtains the result that BFKL and CCFM emission rates are the same at LL, in accord with the results of Forshaw and Sabio Vera, and Webber [4,5]. But integrating over all q_t and x to get the total multiplicity gives

$$\langle n \rangle \simeq \sqrt{\pi \bar{\alpha}_s \ln^2 x} \,, \tag{3}$$

which is a double logarithm of x just as found by Marchesini in [3]. The message is that formally subleading transverse DLs $\alpha_s \ln^2 q_t$ play a fundamental role and can be thrown away only in specific circumstances.

So far in the CCFM equation we have considered results including just hard emissions, those from the $1/z$ part of the splitting function. The inclusion of the soft emissions, those from the $1/(1-z)$ part of the splitting function, changes the results radically, filling in the regions between the dashed lines of Fig. 2 in such a way that the combination of soft and hard emissions turns out to correspond to independent emissions with the density given by (1), identical to the result from the BFKL equation. This equivalence between BFKL and CCFM predictions holds at LL accuracy (all terms $(\alpha_s \ln x)^n$) [6]. There are actually still some differences at subleading transverse DL level $\alpha_s \ln^2 q_t$ but they are confined to the end of the chain of gluons and do not resum in such a way as to affect the LL results.

3. IMPLICATIONS FOR PHENOMENOLOGY

The above result on the LL equivalence of BFKL and CCFM final states is a formal statement. It has relevance for analytical calculations of the LL properties of final states, e.g. [7]. But the BFKL and CCFM equations have fundamentally different physical origins, and this is reflected in differences at subleading order: the BFKL equation (formulated as an evolution in x, for DIS) has a value of z (energy fraction remaining after a parton splitting) which is determined essentially by an arbitrary collinear cutoff μ, present for regularisation purposes:

$$\ln \frac{1}{z} \sim \frac{1}{2\bar{\alpha}_s \ln k/\mu} \,. \tag{4}$$

In the formal limit of small $\bar{\alpha}_s$, typical z values are small and there are no problems. But with $\mu/k \to 0$, $\langle z \rangle$ becomes arbitrarily close to 1. Since rapidities go as $\ln zq_t/(1-z)$, the rapidities of the emitted gluons (even the hardest, i.e. jets) depend, at subleading level, on the collinear cutoff.[2] In contrast, because of the explicit treatment of coherence and the separation of soft emissions, the CCFM equation never shows such pathological behaviour (z is well-defined), and so is a much better candidate for detailed phenomenology, for example in the form of a Monte Carlo program such as SMALLX [9] (for an application to HERA data see [10]).

But the CCFM equation is not entirely free of problems: there are subleading ambiguities in its implementation which can have large effects on its predictions [11]. Additionally the CCFM equation lacks an important symmetry: it works well evolving from a low transverse scale to a high one (DIS), but not in the opposite direction. The symmetry issue is actually resolved in the Linked Dipole Chain approach (LDC) [12], which like the CCFM equation has a separation of hard and soft emissions. But the LDC does not reproduce the BFKL cross section at LL level. It is not currently clear whether this might be related to its problems in describing the data.

4. OUTLOOK

The present phenomenological situation is that none of the approaches contains all of the physics that might be considered mandatory. Though the formal LL equivalence of BFKL and CCFM final states is of limited immediate relevance for phenomenology, it is an important step in our general understanding of small-x final states: together with information from the NLL corrections [13,14] it gives us a picture of the features required in future phenomenological approaches.

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[2] If one instead formulates the BFKL equation as an evolution in rapidity, as done for Monte Carlo implementations for the Tevatron [8] the problem disappears, but other serious difficulties arise for DIS, such as structure functions containing spurious transverse DLs.

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