Extraction of skewed parton distributions from experiment

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In this paper we will discuss algorithms for extracting skewed parton distributions (SPD's) from experiment as well as the relevant process and experimental observable suitable for the extraction procedure.

1. INTRODUCTION

The basic concept of SPD's [1–5] is best illustrated with the lowest order graph of deeply virtual Compton scattering (DVCS) in which a quark of momentum fraction x_1 leaves the proton and is returned to it with x_2. The two fractions not being equal is due to the fact that an on-shell photon is produced which necessitates a change in the + momentum in going from the virtual space-like photon with + momentum $-x_{bj}p_+$, to basically zero + momentum of the real γ. This sets $x_2 = x_1 - x$ and thus the skewedness parameter to x. Thus one has a nonzero momentum transfer onto the proton and the parton distributions (PDF's) which enter the process are no longer the regular PDF's since the matrix element of the quark (gluon) operator is now taken between states of unequal momentum rather than equal momentum.

2. APPROPRIATE PROCESS AND EXPERIMENTAL OBSERVABLE

The most desirable process for extracting SPD's is the one with the least theoretical uncertainty, the least singular Q^2 behavior and a proven factorization formula.

The process which fulfills all the above criteria is DVCS and the experimental observable which allows direct access to the SPD's is the azimuthal angle asymmetry A of the combined DVCS and Bethe-Heitler(BH) differential cross section. A is defined as [3]:

$$A = \frac{\int_{-\pi/2}^{\pi/2} d\phi \; d\sigma - \int_{\pi/2}^{3\pi/2} d\phi \; d\sigma}{\int_0^{2\pi} d\phi \; d\sigma}. \qquad (1)$$

The reason why this asymmetry is not zero is due to the interference term between BH and DVCS which is proportional to the real part of the DVCS amplitude. The factorized expression for the real part of the amplitude is [2]

$$Re\; T(x,Q^2) = \int_{-1+x}^{1} \frac{dy}{y} Re\; C_i(x/y,Q^2) \; f_i(y,x,Q^2). \qquad (2)$$

$Re\; C_i$ is the real part of the hard scattering coefficient (HSC) and f_i are the SPD's. At HERA one is mainly restricted to the small-x region where gluons dominate and thus i will be g. Thus Eq. (1) contains only measurable or directly computable quantities except Eq. (2) in the interference part. Therefore, one would now be able to extract the SPD's from on A, if one could deconvolute Eq. (2). However, for SPD's life is not as "simple" as in inclusive DIS where the deconvolution for F_2 is trivial, since the SPD's depend on two rather than one variable. Furthermore, the HSC depends on the same variables as the SPD's. These facts make the deconvolution of Eq. (2) impossible.

3. ALGORITHMS FOR EXTRACTING SPD'S

Rather than deconvoluting, one can expand the PDF's with respect to a complete set of orthogonal polynomials $P_j^{(\alpha_P)}(t)$. In this particular case we need the orthogonality of the polynomials to be on the interval $-1 \le t \le 1$ with $t = \frac{2y-x}{2-x}$ equivalent to $-1+x \le y \le 1$ as found in Eq. (2). One can then write the following expansion:

$$f^{q,g}(y,x,Q^2) = \frac{2}{2-x} \sum_{j=0}^{\infty} \frac{w(t|\alpha_P)}{n_j(\alpha_P)} P_j^{q,g}(t)$$

$$M_j^{q,g}(x,Q^2) \tag{3}$$

with $w(t|\alpha_P)$, $n_j(\alpha_P)$ and α_P being weight, normalization and a label determined by the choice of the orthogonal polynomial used. $M_j^{q,g}(x,Q^2)$ is given by:

$$M_j^{q,g}(x,Q^2) = \sum_{k=0}^{\infty} E_{jk}^{q,g}(x) f_k^{q,g}(x,Q^2), \tag{4}$$

where

$$f_j^{q,g}(x,Q^2) = \sum_{k=0}^{j} x^{j-k} B_{jk}^{q,g} \tilde{f}_k^{q,g}(x,Q^2). \tag{5}$$

$B_{jk}^{q,g}$ is an operator transformation matrix which fixes the NLO corrections to the eigenfunctions of the kernels and is thus just the identity matrix in LO. The upper limit in Eq. (4) is given by the constraint θ-functions present in the expansion coefficients, which are generally defined by

$$E_{jk}(\nu;\alpha_P|x) = \frac{\theta_{jk}}{(2x)^k} \frac{\Gamma(\nu)\Gamma(\nu+k)}{\Gamma(\frac{1}{2})\Gamma(k+\nu+\frac{1}{2})} \int_{-1}^{1} dt (1-t^2)^{k+\nu-\frac{1}{2}} \frac{d^k}{dt^k} P_j^{\alpha_P}\left(\frac{xt}{2-x}\right). \tag{6}$$

The moments of the SPD's evolve according to

$$\tilde{f}_j^{q,g}(x,Q^2) = \tilde{E}_j(Q^2,Q_0^2)\tilde{f}_j^{q,g}(x,Q_0^2) \tag{7}$$

where the evolution operator is a matrix of functions in the singlet case. Finally, the Gegenbauer moments of the SPD's at Q_0^2 are defined by

$$\begin{aligned}
\tilde{f}_j^q(x,Q_0^2) &= \int_{-1}^{1} dt \left(\frac{x}{2-x}\right)^j C_j^{3/2}\left(\frac{tx}{2-x}\right) f^q(t,x,Q_0^2)\ , \\
\tilde{f}_j^g(x,Q_0^2) &= \int_{-1}^{1} dt \left(\frac{x}{2-x}\right)^{j-1} C_{j-1}^{5/2}\left(\frac{tx}{2-x}\right) f^g(t,x,Q_0^2).
\end{aligned} \tag{8}$$

In LO order and at small x the above formalism simplifies. Owing to the conformal properties of the operators involved in the definition of the SPD's one finds the following expansion

$$\begin{aligned}
f^g(y_1,x,Q^2) &= \frac{2}{2-x} \sum_{j=0}^{\infty}\sum_{k=1}^{\infty} \frac{w(t|5/2)}{N_j(5/2)} E_{jk-1}^g(x) C_{j-1}^{5/2}(t) \tilde{f}_{k-1}^g(x,Q^2) \\
f^q(y_1,x,Q^2) &= \frac{2}{2-x} \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \frac{w(t|3/2)}{N_j(3/2)} E_{jk}^q(x) C_j^{3/2}(t) \tilde{f}_k^q(x,Q^2),
\end{aligned} \tag{9}$$

with $w(t|\nu) = (t(1-t))^{\nu-1/2}$ and the C_j^{ν}'s being Gegenbauer polynomials . The multiplicatively renormalizable moments evolve as above but with the explicit evolution operator:

$$\tilde{E}_j^{ik}(Q^2,Q_0^2) = T\exp\left(-\frac{1}{2}\int_{Q_0^2}^{Q^2} \frac{d\tau}{\tau}\gamma_j^{ik}(\alpha_s(\tau))\right) \tag{10}$$

where T orders the matrices of the regular LO anomalous dimensions (i,k = q,g) along the integration path.

Inserting Eq. (9) in Eq. (2) one obtains:

$$\begin{aligned}
Re\ T(x,Q^2) &= 2\sum_{j=0}^{\infty}\sum_{k=1}^{\infty} \tilde{E}_{k-1}(Q^2,Q_0^2) \\
&\tilde{f}_{k-1}^g(x,Q_0^2) E_{jk-1}^g(x) \int_{-1}^{1} \frac{dt}{2t+x} \frac{w(t|5/2)}{N_j(5/2)} \\
&Re\ C_g\left(\frac{1}{2}+\frac{t}{x},Q^2\right) C_{j-1}^{5/2}(t),
\end{aligned} \tag{11}$$

where we chose the factorization/renormalization scale to be equal to Q^2. As one can see the integral in the sum is now only over known functions and will yield, for fixed x, a function of j as will also do the expansion coefficients for fixed x. The evolution operator can also be evaluated and will yield for fixed Q^2 also just a function of j, which leaves the coefficients $\tilde{f}_{k-1}^g(x,Q_0^2)$ as the only unknowns. Since the lefthand side will be known from experiment for fixed x and Q^2, we are still in the unfortunate situation that a number is determined by the sum over j of an infinite number of coefficients. However measuring the real part at an infinite number of Q^2 for fixed x, one would have an infinite dimensional column vector on the lefthand side and on the right hand side one would have a square matrix times another column vector of coefficients of which the dimension is determined by the number of j. Since all the entries in the matrix are real and positive definite provided that there are no zero eigenvalues one can find

the inverse. Thus one can directly compute the moments of our initial parton distributions which are needed to reconstruct the skewed gluon distribution from Eq. (3).

The drawback of the above procedure is that this process has to be repeated anew for each x and that NLO evolution studies [4] indicate that one might need as much as $50-100$ polynomials to achieve enough accuracy at small-x. This, of course, would render this procedure useless in an experimental situation where a j of $5-10$ is possibly achievable!

A practical way out of the above mentioned predicament is by making a simple minded ansatz for the skewed gluon distribution in the different regions like $A_0 z^{-A_1}(1-z)^{A_2}$ for the DGLAP region where z is now just a dummy variable, plug this form in Eq. (2) and fit the coefficients to the data of the real part of the DVCS amplitude for fixed x and Q^2. One can repeat this procedure for different values of Q^2 and then interpolate between the different coefficients to obtain a functional form of the coefficients in Q^2, alternatively, after having extracted the values of the coefficients for different values of x at the same Q^2, use an evolution programm with the ansatz and the fitted coefficients as input and check whether one can reproduce the data for the real part at higher Q^2, thus checking the viability of the model ansatz.

To obtain an ansatz fullfilling the various constraints for SPD's (see Ji's and Radyushkin's references in [1]), one should start from the double distributions (DD) (see Redyushkin's references in [1].) which yield the skewed gluon distribution in the various regions

$$g(y,x) = \theta(y \geq x) \int_0^{\frac{1-y}{1-x}} dz G(y-xz,z) + \theta(y \leq x) \int_0^{\frac{y}{x}} dz G(y-xz,z). \quad (12)$$

Due to the fact that there are no anti-gluons, the above formula is enough to cover the whole region of interest $-1+x \leq y \leq 1$. What remains is to choose an appropriate model ansatz for G, for example,

$$G(z_1,z) = \frac{h(z_1,z)}{h(z_1)} f(z_1) \quad (13)$$

with $f(z_1)$ being taken from a diagonal parametrization with its coefficients now being left as variants in the skewed case and the normalization condition $h(z_1) = \int_0^{1-z_1} dz h(z_1,z)$ such that, in the diagonal limit, the DD just gives the diagonal distribution. The choice for $h(z_1,z)$ is a matter of taste but should be kept as simple as possible.

4. CONCLUSIONS AND OUTLOOK

In this note, we have showed that the extraction of skewed parton distributions from DVCS experiments is principally as well as practically possible and we have given explicit extraction algorithms. What remains to be done is to collect enough data for the algorithms to be applicable.

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