Initial conditions and evolution of off-diagonal distributions

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We briefly discuss the problem of specifying initial conditions for evolution of off-diagonal (skewed) parton distributions. We present numerical results to show that evolution rapidly washes out differences of input.

1. Introduction

Off-diagonal (or skewed) parton distributions provide important information about the non-perturbartive structure of the nucleon [1]. They can in principle be measured in such processes as deeply virtual Compton scattering, diffractive vector meson production or diffractive high-p_T jet production. Just as for the ordinary (diagonal) parton distributions, the QCD evolution equations [2] play an important role in the determination of skewed parton distributions. As usual input distributions are required. However, this is more complicated than in the diagonal case since the skewed parton distributions depend on additional variable – the asymmetry parameter $\xi \sim p - p'$. We use the symmetric formulation of Ji [1] in which the skewed parton distributions are given by functions $H(x,\xi)$ with support $-1 \leq (x,\xi) \leq 1$, see [3] for more details. Here we discuss various ways to specify the initial conditions and show how the differences in the input distributions disappear on evolution. In particular we illustrate the conclusion of [3] that the skewed distributions $H(x,\xi)$, at small x and ξ, are fixed by the conventional diagonal partons.

2. Constraints imposed on $H(x,\xi)$

The distributions $H(x,\xi)$ have to fulfil several conditions with respect to the variable ξ. First, the time-reversal invariance and hermiticity impose the condition [1]

$$H(x,\xi) \;=\; H(x,-\xi)\,. \tag{1}$$

Thus $H(x,\xi)$ is an even function of ξ. The second condition states that in the limit $\xi = 0$ we recover the ordinary diagonal parton distributions

$$H(x,0) \;=\; H^{diag}(x)\,. \tag{2}$$

The third condition is more complicated but has a simple origin, [1,4]. The N^{th} moment of $H(x,\xi)$ is a polynomial in ξ of the order N at most

$$\int_{-1}^{1} dx\, x^{N-1}\, H(x,\xi) \;=\; \sum_{i=0}^{[N/2]} A_{N,i}\, \xi^{2i}\,, \tag{3}$$

which also embodies condition (1). Finally we impose continuity of $H(x,\xi)$ at the border $x = \pm\xi$ between two different physical regions, see [3]. This ensures that the amplitude of a physical process, described by skewed distributions, is finite. All these conditions have to be fulfilled in the construction of the input for evolution. They are, of course, conserved during the evolution.

3. Initial conditions for evolution

There are two equivalent ways to evolve $H(x,\xi,\mu^2)$ up in the scale μ^2. The first method, which is most appropriate at small x,ξ, uses the evolution of the Gegenbauer moments of $H(x,\xi)$ and the Shuvaev transform [5,6] to find the final answer in the $x-$space [3]. In the second method the solutions are found after imposing initial conditions and numerically solving the evolution equations, directly in the $x-$space. Here we supplement the studies of [3] by presenting results from the second approach.

Condition (3) is difficult to fulfil in order to specify initial distributions at a certain scale μ_0^2. It may be facilitated by the use of the double distribution $\mathcal{F}(\tilde{x}, \tilde{y})$ [4]

$$H(x,\xi) \;=\; \int_{\mathcal{R}} d\tilde{x} d\tilde{y}\; \mathcal{F}(\tilde{x},\tilde{y})\; \delta(x - (\tilde{x} + \xi\tilde{y}))\;, \quad (4)$$

where $\mathcal{R}$ is the square $|\tilde{x}| + |\tilde{y}| \leq 1$ This prescription introduces a nontrivial mixing between x and ξ. Condition (1) is guaranteed if $\mathcal{F}(\tilde{x},\tilde{y}) = \mathcal{F}(\tilde{x},-\tilde{y})$. We still have freedom to add to (4) a function $\mathrm{sign}(\xi)\, D(x/\xi)$, antisymmetric in x/ξ, contained entirely in the ERBL-like region $|x| < \xi$ [7].

The only problem left is to build in the ordinary diagonal distributions in the prescription (4). To do this we take

$$\mathcal{F}(\tilde{x},\tilde{y}) \;=\; h(\tilde{y})\, H^{diag}(\tilde{x}) \;/ \int_{-1+|\tilde{x}|}^{1-|\tilde{x}|} dy'\, h(y')\;, \quad (5)$$

where different choices of h give, via (4), different initial conditions for $H(x,\xi)$. Three choices,

$$h(\tilde{y}) \;=\; \begin{cases} \delta(\tilde{y}) \\ (1-\tilde{y}^2\,)^{p+1} \qquad p = 0, 1\;, \\ \sin(\pi\, \tilde{y}^2) \end{cases} \quad (6)$$

are shown in Fig. 1, with $H^{diag}(x)$ given by GRV [8] at $\mu_0^2 = 0.26$ GeV^2. The first choice gives simply the diagonal input $H^{diag}(x)$, independent of ξ. The second, with $p = 0(1)$ for quarks (gluons), generates an input form similar to that obtained from the model of [3,6] in which the Gegenbauer moments of $H(x,\xi)$ are ξ-independent. This property is conserved by the evolution. The exact form of the double distribution in this case can be found in [9]. The last choice was selected so as to give an oscillatory input behaviour in the ERBL-like region.

4. Discussion

Fig. 1 shows the quark non-singlet, quark singlet and gluon skewed distributions for the three input models evolved up in μ^2 for $\xi = 0.03$. The results for larger values of ξ are qualitatively the same [10].

We see that the form of $h(\tilde{y})$ has the most influence in the ERBL region. However even in this region evolution soon washes out the differences. Already by $\mu^2 = 100$ GeV^2 the three curves are close to each other. In the DGLAP region, $|x| > \xi$, the curves are almost identical while in the ERBL region, $|x| < \xi$, they approach the asymptotic form for each particular skewed parton distribution. Such behaviour is especially important in view of the fact that one set of the curves is obtained from the pure diagonal input parton distributions. In this way we illustrate the result presented in [3] that to a good accuracy at small ξ, the skewed distributions $H(x,\xi;\mu^2)$ are completely known in terms of conventional partons. Thus, to summarize, the nonperturbative information contained in the diagonal input parton distributions and particular features of the evolution equations for skewed parton distributions are sufficient for their determination.

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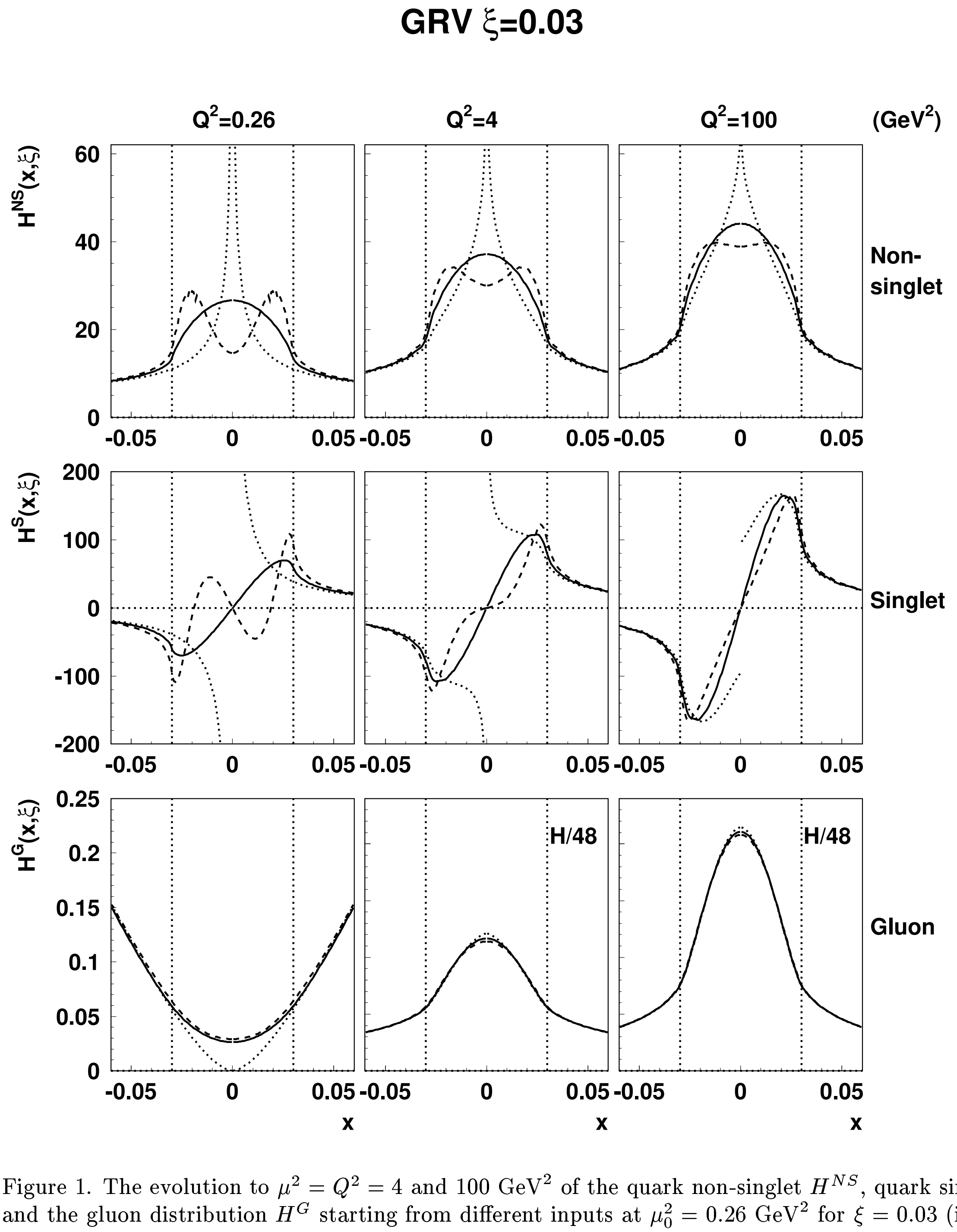

Figure 1. The evolution to $\mu^2 = Q^2 = 4$ and 100 GeV^2 of the quark non-singlet H^{NS}, quark singlet H^S and the gluon distribution H^G starting from different inputs at $\mu_0^2 = 0.26$ GeV^2 for $\xi = 0.03$ (indicated by the dotted vertical lines $x = \pm\xi$). The dotted curves correspond to evolution from diagonal input, the continuous curves to the model of [3] and the dashed curves to oscillatory input in the ERBL-like region.