Skewed distributions fixed by diagonal partons at small x, ξ and $\gamma^* p \to Vp$ at HERA

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We show that the skewed parton distributions are completely determined at small x and ξ by the conventional diagonal partons. We study the application to diffractive vector meson production at HERA.

1. INTRODUCTION

Data are becoming available for processes which are described by off-diagonal (or skewed) parton distributions. A relevant example is diffractive vector meson production at HERA, $\gamma^* p \to Vp$ with $V = \rho, J/\psi$ or Υ, where at high $\gamma^* p$ c.m. energy, W, the cross section is dominated by the two-gluon exchange diagram

$$\frac{d\sigma}{dt}(\gamma^* p \to Vp)\bigg|_{t=0} = \ldots [x_2 g(x_1, x_2; \mu^2)]^2 \quad (1)$$

where g is the off-diagonal ($x_1 \neq x_2$) gluon distribution with

$$\begin{aligned} x_1 &= (Q^2 + M_{q\bar{q}}^2)/W^2, \\ x_2 &= (M_{q\bar{q}}^2 - M_V^2)/W^2 \ll x_1, \end{aligned} \quad (2)$$

see ref. [1]. $M_{q\bar{q}}$ is the mass of the $q\bar{q}$ system produced by a photon of virtuality Q^2. The relevant scale is $\mu^2 = z(1-z)Q^2 + k_T^2 + m_q^2$ where $z, 1-z$ and $\pm \boldsymbol{k}_T$ specify the momenta of the q and $\bar{q}$. The quadratic dependence on g in (1) shows that these data may offer a sensitive constraint on the gluon. Indeed our aim is to show that the off-diagonal distributions are fixed by the conventional diagonal partons, so that the data can, in principle, be included in a global parton analysis.

2. JI'S 'SYMMETRIZED' DISTRIBUTIONS

We shall use the "off-forward" distributions $H(x, \xi) \equiv H(x, \xi, t, \mu^2)$ with support $-1 \leq x \leq 1$ introduced by Ji [2], with the minor difference that the gluon $H_g = xH_g^{\rm Ji}$ [3]. They depend on the momentum fractions

$$x_{1,2} = x \pm \xi \quad (3)$$

carried by the emitted and absorbed partons at each scale μ^2 and on the momentum transfer variable $t = (p - p')^2$. The variables t and ξ do not change as we evolve the distributions up in the scale μ^2. In the limit $\xi \to 0$ they reduce to the conventional parton distributions

$$\begin{aligned} H_q(x,0) &= \begin{cases} q(x) & \text{for} \quad x > 0 \\ -\bar{q}(-x) & \text{for} \quad x < 0, \end{cases} \\ H_g(x,0) &= xg(x), \end{aligned} \quad (4)$$

and satisfy DGLAP evolution. In the limit $\xi \to 1$ they obey ERBL evolution. If we consider H_q at arbitrary values of ξ, then for $x > \xi$ and $x < -\xi$ we have DGLAP-like evolution for quarks and antiquarks respectively, while for $-\xi < x < \xi$ we have ERBL-like evolution for the emitted $q\bar{q}$ pair.

On account of the $x_1 \leftrightarrow x_2$ symmetry the distributions H_q, H_g are symmetric in ξ. We also have symmetry relations in terms of the x variable

$$\begin{aligned} H_q^{NS}(x,\xi) &= H_q^{NS}(-x,\xi), \\ H_q^{S}(x,\xi) &= -H_q^{S}(-x,\xi), \\ H_g(x,\xi) &= H_g(-x,\xi). \end{aligned}$$

where the superscripts S and NS denote singlet and non-singlet quarks respectively.

3. $H(x,\xi)$ IN TERMS OF CONFORMAL MOMENTS

The conformal moments[1] of the off-diagonal distributions,

$$\boxed{O_N(\xi,\mu^2) \;=\; \int_{-1}^{1} dx R_N(x_1,x_2) H(x,\xi),} \qquad (5)$$

are not mixed by evolution

$$O_N(\xi,\mu^2) \;=\; O_N(\xi,\mu_0^2)\left(\frac{\mu^2}{\mu_0^2}\right)^{\gamma_N}, \qquad (6)$$

where γ_N are the same anomalous dimensions as for diagonal partons. The R_N are known polynomials of degree N

$$R_N \;=\; \sum_{k=0}^{N} \left(\begin{array}{c} N \\ k \end{array}\right) \left(\begin{array}{c} N+2p \\ k+p \end{array}\right) x_1^k x_2^{N-k} \qquad (7)$$

with $p = 1, 2$ for quarks and gluons respectively. The O_N reduce to the usual moments in the limit $\xi \to 0$. For example for quarks

$$O_N \to M_N \;=\; \int_0^1 x^N q(x) dx, \qquad (8)$$

up to a normalizing factor $R_N(1,1)$.

The crucial step is to find the inverse relation to (5). That is to reconstruct $H(x,\xi)$ from a knowledge of the conformal moments. The result, due to Shuvaev [7], is

$$\boxed{H(x,\xi) \;=\; \int_{-1}^{1} dx' K(x,\xi;x') f(x')} \qquad (9)$$

where the kernel K is a known integral [7,8] and f is the Mellin transform

$$f(x') \;=\; \int \frac{dN}{2\pi i} (x')^{-N} O_N(\xi)/R_N(1,1). \qquad (10)$$

f reduces to the diagonal distribution for $\xi^2 \ll 1$. This follows since [2]

$$\begin{aligned} O_N(\xi) \;&=\; \sum_{k=0}^{[(N+1)/2]} O_{Nk}\xi^{2k} \\ &\simeq\; O_{N0} \;=\; O_N(0) = M_N R_N(1,1) \end{aligned} \qquad (11)$$

for small ξ^2. So the off-diagonal distribution H is completely determined in terms of the diagonal distribution f via (9).

4. A GOOD SMALL x,ξ APPROXIMATION

We can simplify (9) further if we assume that the diagonal partons have the form

$$xq(x) \;=\; N_q x^{-\lambda_q}, \quad xg(x) \;=\; N_g x^{-\lambda_g} \qquad (12)$$

for very small x. Then the x' integration can be performed analytically and

$$H_i(x,\xi) \;=\; \xi^{-\lambda_i - p} F_i\left(\frac{x}{\xi}\right) \qquad (13)$$

with $p = 1, 0$ for $i = q, g$ respectively. A full set of results for the off-diagonal/diagonal ratios,

$$R_i(x,\xi) \;=\; H_i(x,\xi)/H_i(x+\xi,0), \qquad (14)$$

can be found in [8]. There, the ratios $R_q^{NS,S}$ and R_g are plotted as functions of x/ξ for different values of λ_i. The scale dependence of the off-diagonal distributions, $H_i(x,\xi)$ of (13), and hence of the R_i, is hidden in the μ^2 dependence of the λ_i. Both λ_g and λ_q increase with increasing μ^2.

5. APPLICATION TO $\gamma^* p \to Vp$

The value of the ratio for the gluon distribution at $x = \xi$ is relevant for diffractive vector meson production, $\gamma^* p \to Vp$, at high energies, see (2). This ratio is given by[2]

$$R_g(x=\xi) \;=\; \frac{2^{2\lambda_g+3}}{\sqrt{\pi}} \frac{\Gamma\left(\lambda_g + \frac{5}{2}\right)}{\Gamma(\lambda_g+4)}. \qquad (15)$$

The cross section formula (1) may then be expressed in terms of the conventional diagonal gluon distribution g,

$$\left.\frac{d\sigma}{dt}(\gamma^* p \to Vp)\right|_{t=0} \;=\; \ldots \left[R_g x_1 g(x_1,\mu^2)\right]^2, \qquad (16)$$

where all the off-diagonal effects are contained in the known (enhancement) factor R_g^2. Of course

[1] Conformal moments were introduced in [4] for $\xi = 1$, and in [5] for $\xi \neq 1$; see also [6].

[2] This answer checks with the values of the ratio obtained by direct evolution of the off-diagonal and diagonal gluons in [1].

to calculate the cross section properly we must use the unintegrated gluon distribution and integrate over the transverse momenta of the exchanged gluons and of the q and $\bar{q}$ forming the vector meson.

To obtain the scale dependence of R_g, we first obtain the μ^2 dependence of λ_g of (12) from the behaviour of the gluon found in the global parton analyses. For example, the MRST partons [9] have $\lambda_g = 0.205$ and 0.38 at $\mu^2 = 4$ and 100 GeV^2 respectively. The appropriate scale for the diffractive process $\gamma^*(Q^2)p \to V(q\bar{q})p$ is $\mu^2 \simeq m_q^2 + Q^2/4$. In this way, for diffractive J/ψ and Υ photoproduction at HERA we find that the off-diagonal enhancement, R_g^2, is $(1.15)^2$ and $(1.32)^2$ respectively. However, for Υ photoproduction, x is not sufficiently small (~ 0.01) and we have to improve the assumption made in (12). If we take $xg \sim x^{-\lambda_g}(1-x)^6$ and perform the x' integration in (9) numerically, then we find an enhancement of $(1.41)^2$ for Υ photoproduction [10]. Moreover for ρ electroproduction it is found [11] that the enhancement due to off-diagonal effects of the $\gamma^* p \to \rho p$ cross section $d\sigma/dQ^2$, at the largest Q^2 of the HERA data, is more than a factor 2, which is just the enhancement needed to ensure a perturbative QCD description of the data.

6. DISCUSSION

The main conclusion is embodied in eqs. (9)–(11). That is the skewed distribution $H(x,\xi)$, at any scale, is fully determined at small x, ξ by knowledge of the diagonal parton distribution, at the same scale.

To be sure of this result we have checked that the analytic continuation of the conformal moments O_N in N is allowed [8]. A second consideration is that, from a formal point of view, we may add to the off-diagonal distribution any function which exists only in the ERBL-like region, $|x| < \xi$. In [8] we show such a contribution is negligible $O(\xi^2)$ at small ξ. So far our distributions allow the calculation of the imaginary part of the amplitude for the process. At small x and ξ it turns out that the real part may be calculated easily using a dispersion relation in the c.m. energy squared, W^2, and that the amplitude

$$A \;=\; i\mathrm{Im}A\,\frac{1+e^{-i\pi\lambda}}{1+\cos\pi\lambda}, \qquad (17)$$

where $A \propto (W^2)^\lambda$. Finally we note that our result remains valid at NLO, since there is no conformal mixing for $\xi^2 \ll 1$.

We conclude that, at small x, ξ, the skewed distributions $H(x,\xi;\mu^2)$ are completely known in terms of conventional partons. Thus data for processes which are described by such distributions can, in principle, be included in a conventional global analysis to better constrain the low x behaviour of the partons.

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REFERENCES

1. A.D. Martin and M.G. Ryskin, Phys. Rev. **D57** (1998) 6692.
2. X. Ji, Phys. Rev. Lett. **78** (1997) 610; Phys. Rev. **D55** (1997) 7114; J.Phys. **G24** (1998) 1181.
3. K. Golec-Biernat and A.D. Martin, Phys. Rev. **D59** (1999) 014029.
4. A.V. Efremov and A.V. Radyushkin, Phys. Lett. **B94** (1980) 245;
   M. Chase, Nucl.Phys. **B174** (1980) 109.
5. Th. Ohrndorf, Nucl.Phys. **B198** (1982) 26.
6. A.P. Bukhvostov, G.V. Frolov, L.N. Lipatov and E.A. Kuraev, Nucl.Phys. **B258** (1985) 601.
7. A. Shuvaev, `hep-ph/9902318`.
8. A. Shuvaev, K. Golec-Biernat, A.D. Martin and M.G. Ryskin, `hep-ph/9902410`, Phys. Rev. (in press).
9. A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, Eur.Phys.J. **C4** (1998) 463.
10. A.D. Martin, M.G. Ryskin and T. Teubner, Phys.Lett. **B454** (1999) 339.
11. A.D. Martin, M.G. Ryskin and T. Teubner, in preparation.