# Hard exclusive production of two pions

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The leading twist parametrization of dipion mass distribution in hard exclusive reactions is proposed. The parameters of this parametrization are related to quark distributions (usual and skewed) in the pion and to distribution amplitudes of mesons ($\pi$, $\rho$, etc.). We show that measurements of the shape of dipion mass distribution in hard exclusive reactions can give important information about partonic structure of the pion. The expression for the amplitude of the reaction $\gamma^*\gamma \to \pi\pi$ near the threshold in terms of singlet quark distribution in the pion is presented.

## 1. DIPION MASS DISTRIBUTION IN $\gamma^* + p \to \pi^+\pi^- + p$

Recently it became possible to measure with good precision two pion hard exclusive production in the deeply virtual photon fragmentation region in the range of dipion masses $0.4 \leq m_{\pi\pi} \leq 1.5$ GeV (see talks at this conference [1] and refs. [2,3]).

Owing to QCD factorization theorem for hard exclusive reaction [4] the dependence of the amplitude of the reactions

$$\gamma^* + T \to \pi\pi + T' \qquad (1)$$

on the dipion mass $m_{\pi\pi}$ factorizes at leading order into the universal (independent of the target) factor:

$$\int_0^1 \frac{dz}{z}\, \Phi^I(z,\zeta,m_{\pi\pi};Q^2)\ . \qquad (2)$$

Here $\Phi^I(z,\zeta,m_{\pi\pi};Q^2)$ is two-pion light cone distribution amplitude ($2\pi$DA), which depends on $z$–longitudinal momentum carried by the quark, $\zeta$ characterizing the distribution of longitudinal momentum between the two pions, and the invariant mass of produced pions $m_{\pi\pi}$, the superscript $I$ stands for isospin of produced pions ($I = 0, 1$). The dependence on the virtuality of the incident photon $Q^2$ is governed by the evolution equation. The $2\pi$DA's were introduced recently in [5] in the context of the QCD description of the process $\gamma^*\gamma \to 2\pi$, its detailed properties were studied in [6], we refer to these papers for details.

For the process (1) at small $x_{Bj}$ (see for *e.g.* recent measurements [2,3]) the production of two pions in the isoscalar channel is strongly suppressed relative to the isovector channel, because the former is mediated by $C$-parity odd exchange. At asymptotically large $Q^2$, QCD predicts the following simple form for the isovector $2\pi$DA [7,6]:

$$\Phi_{as}^{I=1}(z,\zeta,m_{\pi\pi}) = 6z(1-z)(2\zeta-1)F_\pi(m_{\pi\pi})\ , \qquad (3)$$

where $F_\pi(m_{\pi\pi})$ is the pion e.m. form factor measured with high precision in low energy experiments [8]. From eqs. (2,3) we conclude that at asymptotically large $Q^2$, QCD predicts unambiguously the shape of the dipion mass distribution:

$$\frac{dN(m_{\pi\pi})}{dm_{\pi\pi}^2} \propto \left(1 - \frac{4m_\pi^2}{m_{\pi\pi}^2}\right)^{3/2} |F_\pi(m_{\pi\pi})|^2\ . \qquad (4)$$

At non-asymptotic $Q^2$ the $2\pi$DA deviates from its asymptotic form (3). This deviation can be described by a few parameters which can be related to quark distributions (skewed and usual) in the pion and to distribution amplitudes of mesons ($\pi\rho$, etc. ), for details see [6]. In this case we propose the following parametrization of dipion mass distribution:

$$\frac{dN(m_{\pi\pi})}{dm_{\pi\pi}^2} \quad \propto \quad \left(1 - \frac{4m_\pi^2}{m_{\pi\pi}^2}\right)^{3/2} |F_\pi(m_{\pi\pi})|^2$$

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$$\times \quad \left(1 + D_1(m_{\pi\pi}, Q^2)\right)^2 \tag{5}$$
$$+ \quad \frac{3}{7}\left(1 - \frac{4m_\pi^2}{m_{\pi\pi}^2}\right)^2 |D_2(m_{\pi\pi}, Q^2)|^2,$$

where the functions $D_{1,2}(m_{\pi\pi}, Q^2)$ describe the deviation of the dipion mass distribution from its asymptotic form (4) and can be parametrized in the form:

$$\begin{aligned} D_1(m_{\pi\pi}, Q^2) &= A_1(Q^2)e^{b_1 m_{\pi\pi}^2} \\ &- \frac{6m_\pi^2}{m_{\pi\pi}^2} A_2(Q^2)e^{b_2 m_{\pi\pi}^2} \\ D_2(m_{\pi\pi}, Q^2) &= A_2(Q^2)e^{b_3 m_{\pi\pi}^2} . \end{aligned} \tag{6}$$

The dependence of $A_{1,2}(Q^2)$ on $Q^2$ is governed by the QCD evolution and in leading order, is given by:

$$A_{1,2}(Q^2) = A_{1,2}(\mu_0)\left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu_0)}\right)^{50/(99-6n_f)} . \tag{7}$$

With increasing of $Q^2$ the parameters $A_{1,2}(Q^2)$ go logarithmically to zero and one reproduces the asymptotic formula (4).

The parameters $b_i$ are $Q^2$ independent but, in principle, $m_{\pi\pi}$ dependent. The latter dependence is fixed by $\pi\pi$ scattering phase shifts, see [6].

In derivation of eq. (6) we neglect the production of pions in $G$-waves and higher because they are suppressed by powers of $1/\log(Q^2)$.

Interestingly that the parameters $A_{1,2}(\mu_0)$ and $b_i$ can be related to important parameters of quark distributions in the pion and meson distribution amplitudes [6]. Using soft pion theorems for $2\pi$DA's [6] one can express the second Gegenbauer moment of pion distribution amplitude to $A_{1,2}(\mu_0)$:

$$a_2^{(\pi)}(\mu_0) = A_1(\mu_0) + A_2(\mu_0) . \tag{8}$$

If one additionally uses the dispersion relation derived in [6] one gets the expression for the second Gegenbauer moment of $\rho$-meson distribution amplitude:

$$a_2^{(\rho)}(\mu_0) \approx A_1(\mu_0)\exp(b_1 m_\rho^2) . \tag{9}$$

The crossing relations [6] (see also [9,10]) allow to relate the parameter $A_2(\mu_0)$ to the third Mellin moment of quark distribution in the pion at normalization point $\mu_0$

$$\int_0^1 dx\; x^2(u^{\pi^+}(x) - \bar{u}^{\pi^+}(x)) = \frac{6}{7}\,A_2(\mu_0) . \tag{10}$$

In analysis of experiments on two pion diffractive production off nucleon (see e.g. [2,3]) the dipion mass distribution is usually fitted by, for example, Söding parametrization, which takes into account rescattering of produced pions on final nucleon. Let us note however that in the case of hard ($Q^2 \to \infty$) diffractive production the final state interaction of pions with residual nucleon is suppressed by powers of $1/Q^2$ relative to the leading twist amplitude. Also the parameters of Söding (or Ross-Stodolsky) parametrization are not related to any fundamental parameter of QCD. Here we proposed alternative leading-twist parametrization (6) describing the so-called "skewing" of two pion spectrum. The advantage of our parametrization is that its parameters can be related to fundamental quantities in QCD: quark distributions in the pion and meson distribution amplitudes.

## 2. REACTION $\gamma^*\gamma \to \pi\pi$ NEAR THRESHOLD

Another example of hard exclusive reaction where the $2\pi$DA enters is the reaction $\gamma^*\gamma \to \pi\pi$ with $Q^2$ of the photon large and the invariant mass of produced pions is small. In refs. [5,11] it was demonstrated that this processes is amenable to QCD description. The amplitude in the leading order can be expressed in terms of isoscalar $2\pi$DA [5]:

$$M_{\mu\nu} = \frac{5}{9}\, g_{\mu\nu}^{\perp} \int_0^1 \frac{dz}{z}\, \Phi^{I=0}(z, \zeta, m_{\pi\pi}; Q^2) . \tag{11}$$

Using the crossing relations and soft pion theorems we derive the expression for the amplitude $M_{\mu\nu}$ close to the threshold $m_{\pi\pi} = 2m_\pi$ in terms of singlet quark distribution in the pion. To this end we make one additional approximation: that production of two pions in the state with angular momentum $l$ is dominated by the operators with conformal spin $l-1$ and $l+1$. Higher the $Q^2$ the more this approximation is justified, see [12] for details. The final expression for the integral over $z$ entering eq. (11) has the form:

$$\int_0^1 \frac{dz}{z}\,\Phi^{I=0}(z,\zeta,m_{\pi\pi}\approx 2m_\pi) =$$
$$\int_0^1 \frac{Q(x)}{x}\Bigg(\frac{1-x^2}{\sqrt{(1+x)^2-4\zeta x}}$$
$$+\frac{1-x^2}{\sqrt{(1-x)^2+4\zeta x}}-2\Bigg)\,,$$

where the function $Q(x)$ is related to the singlet quark distribution in the pion:

$$Q(x) = 2q_\pi(x) - x\int_x^1 \frac{dy}{y^2}q_\pi(y)\,,$$

with

$$q_\pi(x) = u^{\pi^+}(x) + \bar{u}^{\pi^+}(x).$$

Note that the normalization points of $q_\pi(x)$ and $\Phi^{I=0}(z,\zeta,m_{\pi\pi})$ are the same.

## 3. CONCLUSIONS

We saw on two examples of hard exclusive reactions with production of two pions with relatively low invariant mass that the measurements of the shape of dipion mass distributions in these reactions can provide us with important information on partonic structure of the pion and two-pion resonances.

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