Elements of a field theory of unitarity corrections

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I explain the motivation for investigating the high energy limit of QCD and give a brief presentation of recent progress in the understanding of unitarity corrections in this kinematic regime. Special emphasis is given to the conformal field theory structure discovered in these unitarity corrections.

1. INTRODUCTION

To the present day Regge theory [1] provides the most successful description of hadron–hadron scattering at high energy s and small momentum transfer t (see for example [2]). Regge theory is based on analyticity and unitarity of the S-matrix. The behaviour of hadronic scattering amplitudes is then encoded in the positions of Regge poles and cuts in the plane of complex angular momentum ω. Gribov's reggeon calculus [3] provides a consistent field theory of interacting reggeons.

On the other hand, QCD is firmly established as the correct microscopic theory of the strong interaction. It should therefore be possible to derive Regge theory from QCD and thus to understand Regge behaviour in terms of quarks and gluons. This is one of the most challenging problems in the physics of strong interactions and has not been resolved so far.

The difficulty is caused by the fact that the Regge limit is characterized by high parton densities. Moreover, many hadronic scattering processes in this kinematics are dominated by small momentum scales. Both facts indicate that non–perturbative effects dominate. Fortunately, however, there are a few scattering processes that can be treated perturbatively even at very high energy, namely those involving the scattering of small color dipoles. One such process is the scattering of highly virtual photons, in which the virtuality provides a hard scale. In the following we will have this process in mind. Our hope is, of course, that we can extract more general features of QCD in the Regge limit from our results.

The first step towards a QCD–based description of the high energy limit was done when the BFKL Pomeron was derived [4]. It describes a t-channel exchange with vacuum quantum numbers and resums the leading logarithms of the energy s which can compensate the smallness of the strong coupling constant. We expect the (perturbative) BFKL Pomeron to be applicable in processes that are governed by a single hard momentum scale. However, the BFKL Pomeron results in a power-like growth $\sigma \sim s^{0.5}$ of the cross–section at high energy. This will eventually lead to a violation of the Froissart bound — a consequence of unitarity stating that the growth of hadronic cross–sections can at most be logarithmic. In this sense the leading logarithmic approximation, i. e. the BFKL Pomeron, is inconsistent and has to be extended in order to restore unitarity.

2. BFKL POMERON AND GENERALIZED LEADING LOGARITHMIC APPROXIMATION

The BFKL Pomeron can be understood as the exchange of a bound state of two reggeized gluons in the t-channel. It is well–known that the restoration of unitarity requires the inclusion exchanges with more than two reggeized gluons in the t-channel, leading to the generalized leading

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logarithmic approximation. Therefore we are interested in amplitudes describing the production of n (reggeized) gluon in the t-channel, which can be related to the total cross section. These amplitudes $D_n^{a_1\cdots a_n}(\mathbf{k}_1,\ldots,\mathbf{k}_n)$ depend on the transverse momenta $\mathbf{k}_i$ of the gluons and carry n color labels in the adjoint representation of SU(N_c). In order to have unitarity in all subchannels the number of gluons in the t-channel should not be fixed. Therefore the amplitudes D_n obey a tower of coupled integral equations [5], the first of which ($n=2$) is identical to the BKFL equation. In the Regge limit the dynamics is effectively reduced to two dimensions, and consequently these equations are, like the BFKL equation, integral equations in two–dimensional tranverse momentum space. The complex angular momentum ω acts as an energy–like variable in the equations. Its conjugate variable, i. e. rapidity, can thus be interpreted as a time–like variable in this context.

The system of coupled integral equations is very complicated and a complete analytic solution seems currently well out of reach. Nevertheless, it turns out to be possible to extract very valuable information about the structure of the amplitudes.

3. REGGEIZATION AND FIELD THEORY STRUCTURE

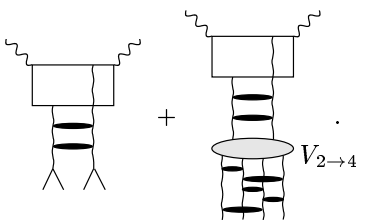
A very important property of QCD is the reggeization of the gluon in the Regge limit [6]. This phenomenon manifests itself in the emergence of a pole solution of the BFKL equation in the color octet channel. The particle corresponding to this pole carries the quantum numbers of a gluon: in a certain sense, the gluon appears to be a bound state of two gluons. This indicates that now the appropriate degrees of freedom are reggeized gluons — collective excitations of the gauge field — rather than elementary gluons.

A generalization of this phenomenon is found in the higher n-gluon amplitudes. The equation for the three–gluon amplitude can be solved [7],

$$D_3^{a_1a_2a_3}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = C_3 f_{a_1a_2a_3}[D_2(\mathbf{k}_1+\mathbf{k}_2,\mathbf{k}_3) - D_2(\mathbf{k}_1+\mathbf{k}_3,\mathbf{k}_2) + D_2(\mathbf{k}_1,\mathbf{k}_2+\mathbf{k}_3)]\,. \quad (1)$$

The three–gluon amplitude is thus a superposition of two–gluon amplitudes D_2. In each of the three terms two gluons (and in higher amplitudes even more gluons) arrange themselves in such a way as to form a 'more composite' reggeized gluon. (In this process the color part of the amplitude is crucial.) As a consequence, an actual three–gluon state in the t-channel does not occur.[2]

The four–gluon amplitude can be shown to have the following structure [7]:

$D_4 =$  (2)

Here the first (reggeizing) part is again a superposition of two–gluon amplitudes, coupled to the photons through a quark box diagram. In the second term the t-channel evolution starts with a two–gluon state that is coupled to a full four–gluon state via a new effective 2-to-4 transition vertex $V_{2\to4}$. The emerging structure is that of a quantum field theory in which states with different numbers of gluons are coupled to each other. The obvious question is whether this structure found here in the four–gluon amplitude persists in higher n-gluon amplitudes.

The necessary analysis of the five– and six–gluon amplitudes has recently been performed [8,9]. The five–gluon amplitude reggeizes completely, i. e. it is a superposition of two– and four–gluon amplitudes. The corresponding mechanism is of more general nature and we expect each amplitude with an odd number of gluons to be a superposition of lower amplitudes with even numbers of gluons.

A part of the six–gluon amplitude reggeizes again and can be written as a superposition of two– and four–gluon amplitudes. In addition, we have discovered a new piece in the field theory which has very interesting symmetry properties. However, further analysis is required in order to clarify whether it should be interpreted as a two–

[2] This does not affect the existence of the Odderon, since in our case the three–gluon system has even C-parity.

to–six transition vertex or as a superposition of new two–to–four vertices. In either case a new element is added to the field theory structure.

But the most important property of the six–gluon amplitude is that it fits nicely into the picture of a field theory structure of unitarity corrections. This strengthens our hope that the whole set of unitarity corrections can be formulated as an effective field theory in 2+1 dimensions, with rapidity acting as the time–like variable.

A closer analysis shows that the process of reggeization is crucial for the very emergence of the field theory structure. It is therefore desirable to further improve our understanding of this phenomenon. Recent progress in this direction includes the formulation of Ward–type identities relating amplitudes with different numbers of gluons [8].

As a further result the knowledge of the six–gluon amplitude allows us to establish the existence of a Pomeron–Odderon–Odderon vertex in perturbative QCD.

4. CONFORMAL INVARIANCE

One of the most interesting properties of the n-gluon amplitudes is their conformal invariance in impact parameter space. After a Fourier transformation from two–dimensional transverse momentum space to impact parameter space, $\mathbf{k}_i \to \vec{\rho}_i$, we can introduce complex coordinates $\rho = \rho_x + i\rho_y$. The amplitudes D_n are then invariant under Möbius transformations of the coordinates ρ_i,

$$\rho \to \rho' = \frac{a\rho + b}{c\rho + d}\,; \quad ad - bc = 1\,. \tag{3}$$

This result was found in [10] for the BFKL Pomeron, and in [11] for the 2-to-4 transition vertex $V_{2\to 4}$. The explicit expression for the new piece in the six–gluon amplitude has now been shown to exhibit this conformal invariance as well [8,9]. We therefore have reason to hope that the whole effective field theory will be conformally invariant in the two dimensions of impact parameter space.

5. SUMMARY AND OUTLOOK

The high energy limit of QCD requires the calculation of unitarity corrections to the perturbative Pomeron in the generalized leading logarithmic approximation. The amplitudes with upto six gluons in the t-channel have been calculated explicitly. There is strong evidence that a beautiful mathematical structure is hidden in these unitarity corrections. Specifically, we expect that the unitarity corrections can be cast into the form of a two–dimensional conformal field theory in impact parameter space with rapidity as a real parameter. This opens the fascinating possibility of applying the powerful methods of conformal field theory and — once the effective theory is identified — to derive the general features of high energy QCD, now bypassing the laborious explicit calculation of higher n-gluon amplitudes. An even more ambitious aim will eventually be the study of NLO corrections in this field theory.

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