Impact factors of charged QCD particles and second bootstrap condition for the gluon reggeization

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Results of NLLA calculation of the charged QCD particles non-forward impact factors are reported. Using these results the second bootstrap condition for the gluon reggeization is checked and its validity is confirmed.

The BFKL equation [1] is widely discussed now since it may shed the light at such an important question as the theoretical description of deep inelastic scattering at small Bjorken x. It gets especial importance due to present experimental results on proton's structure functions in this kinematical region from HERA (see, for example, the proceedings of this Workshop). Recently the next-to-leading (NLLA) corrections to this equation became available. These NLLA corrections are very large [2], so that we still have not a reliable theoretical predictions here. In such a situation one should look carefully on the assumptions which are made for gluon reggeization.

The exact meaning of the reggeization was explained in Ref. [3]. One of the important properties included into this term is the reggeized form of the elastic scattering amplitude (Fig.1) with octet colour state and negative signature in the t-channel in regge kinematical region $s \to \infty$, t - fixed (doesn't grow with s)

$$\mathcal{A}_8^{(-)} = \Gamma^c_{A'A} \left[\left(\frac{-s}{-t} \right)^{j(t)} - \left(\frac{s}{-t} \right)^{j(t)} \right] \Gamma^c_{B'B},$$
$$j(t) = 1 + \omega(t). \tag{1}$$

Here $\Gamma^c_{A'A}$ denotes the particle-reggeon effective interaction vertices and $\omega(t)$ is the reggeized gluon trajectory.

While in the leading approximation gluon reggeization was rigorously proven [4], in NLLA it only can be considered as an assumption, which is checked in the lowest order of perturbation theory, so that demonstrating its selfconsistency. The elastic scattering amplitude $\mathcal{A}_R$ (Fig.1) with the representation R of the colour group $SU(N)$ in the t-channel can be expressed through its t-channel partial wave f_R as follows:

$$(f_R(\omega, \vec{q}))^{A'B'}_{AB} = \int_{s_0}^{\infty} \frac{ds}{s^2} \left(\frac{s}{s_0} \right)^{-\omega} \mathrm{Im}_s \left(\mathcal{A}_R \right)^{A'B'}_{AB},$$
$$(\mathcal{A}_R)^{A'B'}_{AB} = \frac{s}{2\pi} \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{\sin(\pi\omega)}$$
$$\times \left[\left(\frac{-s}{s_0} \right)^{\omega} - \tau \left(\frac{s}{s_0} \right)^{\omega} \right] (f_R(\omega, \vec{q}))^{A'B'}_{AB}, \tag{2}$$

where τ is the signature, $\vec{q}$ - transverse momentum transfer ($t \approx -\vec{q}^{\,2}$) and an artificial scale s_0 is introduced to define the partial wave [5]. In the framework of the BFKL approach this partial wave has the following form [5]

$$(f_R(\omega, \vec{q}))^{A'B'}_{AB} = \frac{1}{(2\pi)^{D-2}} \int \frac{d^{D-2} q_1}{\vec{q}_1^{\,2} (\vec{q}_1 - \vec{q})^2}$$

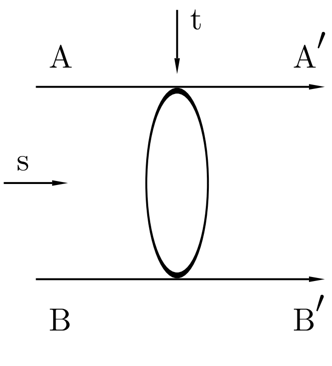

Figure 1. The elastic scattering amplitude.

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$$\times \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2-\vec{q})^2}\Phi^{(R,\nu)}_{A'A}(\vec{q}_1,\vec{q},s_0)$$
$$\times G^{(R)}_{\omega}(\vec{q}_1,\vec{q}_2,\vec{q})\Phi^{(R,\nu)}_{B'B}(-\vec{q}_2,-\vec{q},s_0), \qquad (3)$$

where $D=4+2\epsilon$ is space-time dimension, with ϵ being non-zero to remove the divergence. $G^{(R)}_{\omega}$ is the solution of the non-forward BFKL equation [3]

$$\omega G^{(R)}_{\omega}(\vec{q}_1,\vec{q}_2,\vec{q}) = \vec{q}_1^{\,2}(\vec{q}_1-\vec{q})^2\delta^{(D-2)}(\vec{q}_1-\vec{q}_2)$$
$$+\int \frac{d^{D-2}k}{\vec{k}^{\,2}(\vec{k}-\vec{q})^2}\mathcal{K}^{(R)}(\vec{q}_1,\vec{k},\vec{q})G^{(R)}_{\omega}(\vec{k},\vec{q}_2,\vec{q}), \qquad (4)$$

and Φ denotes non-forward impact factors of the scattered particles. To numerate colour states in the representation R the index ν was used in Eq.(3).

Such a reppresentation of BFKL approach 2, 3) for the elastic amplitude with $R=8$ and negative signature must evidently reproduce the assumed Regge form (1) with NLLA accuracy. This self-consistency requirement leads to the bootstrap conditions for the gluon reggeization [3]. We have considered [6] the second of these conditions which is the relation between the NLLA correction to the octet non-forward impact factor $\Phi^{(8,c)(1)}_{A'A}$, one and two loop contributions to the reggeized gluon trajectory $\omega^{(1)}$ and $\omega^{(2)}$ and Born and one loop particle-reggeon vertices $\Gamma^{c(B)}_{A'A}$ and $\Gamma^{c(1)}_{A'A}$.

$$ig\frac{\sqrt{N}t}{(2\pi)^{D-1}}\int \frac{d^{D-2}k}{\vec{k}^{\,2}(\vec{k}-\vec{q})^2}\Phi^{(8,c)(1)}_{A'A}(\vec{k},\vec{q},s_0) =$$
$$\Gamma^{c(1)}_{A'A}\omega^{(1)}(t)+\frac{1}{2}\Gamma^{c(B)}_{A'A}\Bigg[\omega^{(2)}(t)+$$
$$\left(\omega^{(1)}(t)\right)^2\ln\left(\frac{s_0}{-t}\right)\Bigg], \quad g^2=4\pi\alpha_s. \qquad (5)$$

This condition means that interaction of two t-channel reggeized gluons in the octet colour state with particle A must reproduce the usual particle-reggeon interaction. To check this one should calculate the NLLA impact factor according to the definition [3]

$$\Phi^{(R,\nu)}_{A'A}(\vec{q}_1,\vec{q},s_0) = \langle cc'|\hat{\mathcal{P}}_R|\nu\rangle \Phi^{cc'}_{AA'}(\vec{q}_1,\vec{q},s_0),$$
$$\Phi^{cc'}_{AA'}(\vec{q}_1,\vec{q},s_0) = \left(\frac{s_0}{\vec{q}_1^{\,2}}\right)^{\omega(q_1^2)/2}\left(\frac{s_0}{\vec{q}_1^{\,\prime 2}}\right)^{\omega(q_1'^2)/2}$$

$$\times\sum_{\{f\}}\int \theta(s_\Lambda - s_{AR})\frac{ds_{AR}d\rho_f}{2\pi}\Gamma^{c}_{\{f\}A}(q_1)$$
$$\times\left(\Gamma^{c'}_{\{f\}A'}(q_1')\right)^* - \frac{1}{2}\int \frac{d^{D-2}k}{\vec{k}^{\,2}(\vec{k}-\vec{q})^2}\Phi^{c_1c_1'(B)}_{AA'}(\vec{k},\vec{q})$$
$$\times\mathcal{K}^{c_1'c'(B)}_{rc_1c}(\vec{k},\vec{q}_1,\vec{q})\ln\left(\frac{s_\Lambda^2}{s_0(\vec{k}-\vec{q}_1)^2}\right). \qquad (6)$$

Here $\hat{\mathcal{P}}_R$ is projector on the representation R, q_1 ($\vec{q}_1$, $q_1^2\approx -\vec{q}_1^{\,2}$) and q_1' ($\vec{q}_1^{\,\prime}$, $q_1'^2\approx -\vec{q}_1^{\,\prime 2}$) are Reggeon momenta of the transverse component. The integration is performed over the standard phase space volume of the system $\{f\}$ produced

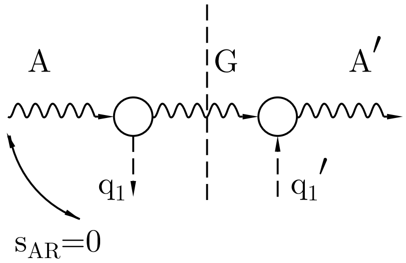

Figure 2. One gluon contribution to the gluon impact factor.

in particle-Reggeon collisions as well as over the particle-Reggeon invariant mass s_{AR}. $\Gamma^{c}_{\{f\}A}$ and $\Gamma^{c'}_{\{f\}A'}$ are the effective production amplitudes. $\mathcal{K}^{c_1'c'(B)}_{rc_1c}$ in Eq.(6) is the real particle production contribution to the non-forward BORN BFKL kernel, but without projecting it on a definite colour state in the t-channel. The sum is carried out over all systems $\{f\}$, which can be produced in the NLLA. In the case of the gluon impact factor, for example, there are three such contributions: the one-gluon (Fig.2), quark-antiquark (Fig.3) and two-gluon one, shown on Fig.4 together with the counterterm from Eq.(6). This counterterm and an artificial parameter s_Λ are necessary to remove the impact factor contribution of multi-regge kinematics (production with large invariant mass), which is accounted by leading term in the BFKL kernel. One-gluon and

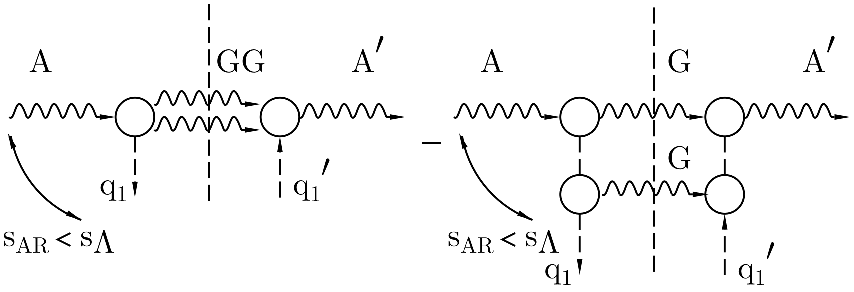

Figure 4. Two gluon contribution to the gluon impact factor.

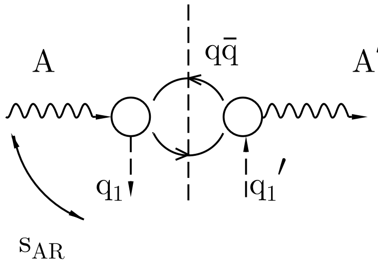

Figure 3. Quark-antiquark contribution to the gluon impact factor.

quark-antiquark pairs contributions do not contain the divergency from large gluon-Reggeon invariant mass integration region in Eq.(6), but the two-gluon contribution, taken separately, does. Therefore this contribution has a logarithmic s_Λ-dependence when $s_\Lambda \to \infty$. The counterterm cancels this artificial dependence (see Fig.4) of the impact factor due to factorization properties of two gluon in gluon-Reggeon collision amplitudes at large gluon-Reggeon invariant mass. Analogous cancellation appears also in the case of quark impact factor, where we have only two intermediate states in the NLLA: one-quark and one quark-gluon state.

Let us note, that from the representation (6) we can obtain all possible impact factors with arbitrary colour states in the t-channel. We have carried out such calculations for both gluon and quark non-forward impact factors for the case of QCD with massless quarks [6], performing the integrations in Eq.(6) in the form of the ϵ-expansion. For the octet impact factors we have obtained also general integral representations for arbitrary ϵ and quark masses. These expressions are necessary to check the bootstrap condition (5), which must be satisfied at arbitrary values of these parameters. We have found that the second bootstrap condition is satisfied for quark as well as for gluon impact factors for both helicity conserving and non-conserving cases, so that the NLLA gluon Reggeization is confirmed. For more details on the first bootstrap condition see V.S. Fadin's talk in these proceedings.

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