Hard Diffraction and QCD Multi-Pomeron Vertices

R. Peschanski[a]

[a]CEA, Service de Physique Théorique, CE-Saclay
F-91191 Gif-sur-Yvette Cedex, France

We discuss the phenomenological and theoretical implications of recent progresses in the evaluation of multi-Pomeron vertices in high-energy perturbative QCD.

1. MULTI-POMERON VERTICES IN THE QCD DIPOLE MODEL

The QCD dipole model [1] happens to be a quite useful representation of the QCD perturbative resummation at leading logarithms at high incident energy (or, equivalently, at small Bjorken x) known as the BFKL QCD Pomeron [2]. On a phenomenological ground, it provides an "s-channel" description of hard diffraction [3] which appears to be quite successful in describing the present data at HERA [4]. On a theoretical ground, the dipole model represents the $1/N_c$ limit of perturbative QCD, and leads to interesting simplifications in the calculation of triple [5] and multiple [6] Pomeron vertices. The present contribution gives an overview of these phenomenological and theoretical results.

2. THEORY

The main result of applying the QCD dipole model to the calculation of multi-Pomeron vertices can be briefly described as follows [6]: The $1 \to p$ Pomeron vertex can be obtained [7] from the calculation of the QCD dipole multiplicity density (i.e. the probability for finding p dipoles $\rho_{b_0}\rho_{b_1},...,\rho_{p_0}\rho_{p_1}$ in an initial one $\rho_{a_0}\rho_{a_1}$) coming from the solution of a integro-differential equation. The solution reads:

$$B_{1\to p} = \int \frac{d^2\rho_0...d^2\rho_p}{|\rho_{01}\ \rho_{12}...\rho_{p0}|^2} \times E^{h_0}(\rho_{0\alpha},\rho_{1\alpha})...E^{h_p}(\rho_{p\pi},\rho_{0\pi}), \quad (1)$$

with $\rho_{ij} = \rho_i - \rho_j$ (resp. $\bar\rho_{ij} = \bar\rho_i - \bar\rho_j$). The $\rho_\alpha...\rho_\pi$ are auxiliary variables which play the rôle of dipole c.o.m. coordinates. The $E^h(\rho_{i\delta},\rho_{j\delta}) = (-1)^n \left(\frac{\rho_{ij}}{\rho_{i\delta}\rho_{j\delta}}\right)^h \times \left(\frac{\bar\rho_{ij}}{\bar\rho_{i\delta}\bar\rho_{j\delta}}\right)^{\bar h}$, are [8] the $SL(2,\mathbb{C})$ eigenvectors labeled by the quantum numbers of the irreducible unitary representations, namely $h = i\nu + \frac{1-n}{2}$, $\tilde h = 1-\bar h = i\nu + \frac{1+n}{2}$, ($n \in \mathbb{Z}$, $\nu \in \mathbb{R}$). Interestingly enough, the expression of the functions $B_{1\to p}$ can be considered as correlation functions, namely

$$B_{1\to p} \equiv \langle 0|\Phi^{h_0}(\rho_\alpha)\Phi^{h_1}(\rho_\beta)...\Phi^{h_p}(\rho_\pi)|0\rangle \quad (2)$$

where the $\Phi^h(\rho)$ are suitably defined operators. An explicit construction following e.g. the BPZ construction [9] can be given [6], as briefly summarized now.

A substantial simplification occurs when we consider the Fourier transform of the Φ^h operator in momentum space:

$$\Phi^h(q) = \int d^2x\ e^{-iqx}\Phi^h(x)\ . \quad (3)$$

One may define the Hilbert space, vacuum state and operators in such a way that:

$$\left\langle \Phi^{h_0}(q_0)\dots\Phi^{h_p}(q_p)\right\rangle = \delta(q_0+\ldots+q_p)\left\langle \phi^{h_0}(q_0)\dots\phi^{h_p}(q_p)\right\rangle, \quad (4)$$

where the $\phi^h(q)$ act on the Hilbert space $f \in L^2(\mathbb{C})$ with the vacuum $|0\rangle = 1$ by

$$[\phi^h(q)f](k) = \mathcal{E}^h_{k,q}\cdot f(k-q) \quad (5)$$

and

$$\mathcal{E}^h_{k,q} \equiv \int d^2\rho d^2\rho'\ e^{iq\rho'+ik(\rho-\rho')}\ \frac{E^h(\rho,\rho')}{|\rho-\rho'|^2} \quad (6)$$

can be explicitly calculated in terms of hypergeometric functions [6]. The correlation functions

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$$\left\langle \phi^{h_0}(q_0)...\phi^{h_p}(q_p)\right\rangle = \int d^2k\ \mathcal{E}^{h_0}_{k,q_0} \times \mathcal{E}^{h_1}_{k-q_0,q_1}...\mathcal{E}^{h_p}_{k-q_0-...-q_{p-1},q_p}\ . \quad (7)$$

All dipole correlators and thus QCD Pomeron vertices are just expressed by a *single* integral over a product of $\mathcal{E}^h_{k,q}$ functions.

The expression (7) allows a rather simple and attractive representation of Pomeron vertices (see Fig.1) in terms of a one-loop integral in momentum space with vertices defined by the functions $\mathcal{E}^h_{k,q}$. Each Pomeron interacts in momentum space *via* a 3-vertex defined by a function $\mathcal{E}^h_{k,q}$ or $\bar{\mathcal{E}}^h_{k,q}$ depending whether it is created or annihilated. The momentum is conserved at each vertex.

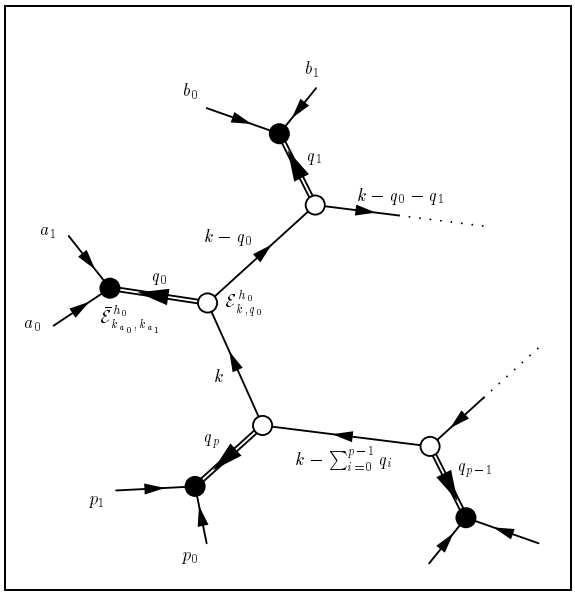

Figure 1. Graphical representation of the $1 \to p$ QCD Pomeron amplitude. White circles: internal $\mathcal{E}^h_{k,q}$ vertex functions entering the one-loop integral. Black circles: complex conjugates external vertices $\bar{\mathcal{E}}^h_{k,q}$ coupling the external gluons to the interacting BFKL Pomerons. Double lines: BFKL Pomerons.

3. PHENOMENOLOGY

In the dipole model approach, hard diffraction in γ^*-proton processes is determined [3] by the interaction between color dipole states describing the photon and the proton. Indeed, it is well-known that the photon can be analyzed in terms of $q\bar{q}$ configuration while it has been shown [10] that the small-x structure function of the proton can be described by a collection of primordial dipoles with subsequent perturbative QCD evolution. More specifically [3], the combination of the dipole description of perturbative QCD at high energy and the Good-Walker mechanism [11] leads to a unified description of the proton total and diffractive structure functions [12].

In the dipole approach, two components are shown to contribute to the diffractive structure function. First, a *quasi-elastic* component corresponds to the elastic interaction of two dipole configurations. It is expected to be dominant in the finite β region, i.e. for small relative masses of the diffractive system. Interestingly, it is related [13] to the solution of the conformal coupling of a $q\bar{q}$ state to the BFKL Pomeron.

Second, there is an *inelastic* component where the initial photon dipole configuration is diffractively dissociated in multi-dipole states by the target. This process is expected to be important at small β (large masses). In this case, the theoretical calculation is directly related [5] to the triple QCD Pomeron vertex, which is a particular case of the vertices considered in the previous section. Indeed, an explicit evaluation of the quantity $B_{1\to 2}$ (see equation (1)) has been performed using the dipole model and leads to a rather large value [5] of the triple QCD Pomeron coupling corresponding to the calculation of $B_{1\to 2}$ at the Pomeron saddle-point values $h_0 = h_1 = h_2 = \frac{1}{2}$.

Thus, in the QCD dipole model approach of hard diffraction, the conformal properties of the BFKL Pomerons through their couplings and vertices are relevant and could be tested by the phenomenological approach. The paper [4] contains a fit of the published diffractive data [14] with 7 free parameters. It also includes a phenomenological secondary Regge trajectory which is known to play a rôle in the limited domain of large mass and

small rapidity gap [14]. The fit is successful showing that this approach is a good candidate for a deeper understanding of hard diffraction. Note that non-leading logs perturbative corrections are phenomenologically taken into account via the parameterization of an effective BFKL singularity with intercept lower than the bare value. Also, effective vertices of the *primordial dipole* distribution in the non perturbative proton target are not theoretically known but verify certain constraints, as discussed in detail in [10]. Both these aspects deserve more study in the future.

4. CONCLUSIONS AND OUTLOOK

The dipole approach to QCD at high energy appears to be quite successful in analyzing deep inelastic scattering at small x and in particular hard diffraction processes. At the theoretical level, it allows an evaluation of the complicated QCD multi-Pomeron vertices among which the triple Pomeron coupling is of relevance in the phenomenological studies. Recently also the first calculation [15] of 1-loop Pomeron contributions have been performed in the same framework.

Among the problems to be addressed to in the near future, the intriguing relationship of the QCD multi-pomeron vertices with conformal field theories arises the question whether the already known [8] global conformal invariance of the BFKL kernel can be enlarged to some kind of Virasoro algebra. On the phenomenological ground, the application of the QCD dipole model to Tevatron results on diffraction and their comparison with HERA emerge as one of the most interesting questions.

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REFERENCES

1. A.H. Mueller, Nucl. Phys. **B415** (1994) 373, **B437** (1995) 107; A. H. Mueller, B. Patel, Nucl. Phys. **B425** (1994) 471.
2. L.N. Lipatov, Sov. J. Nucl. Phys. **23** (1976) 642; V.S. Fadin, E.A. Kuraev and L.N. Lipatov, Phys. Lett. **B60** (1975) 50; E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Sov. Phys. JETP **44** (1976) 45, **45** (1977) 199; I.I. Balitsky and L.N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822.
3. A. Bialas, R. Peschanski, Phys. Lett. **B378** (1996) 302, **B387** (1996) 405.
4. S. Munier, R. Peschanski and Ch. Royon, Nucl. Phys. **B534** (1998) 297.
5. A. Bialas, H. Navelet, R. Peschanski, Phys. Lett. **B427** (1998) 147, Phys. Rev. **D57** (1998) 6585.
6. R. Janik and R. Peschanski, hep-ph/9901426, to be published
7. R. Peschanski, Phys. Lett. **B409** (1997) 491.
8. L.N. Lipatov, Sov. Phys. JETP **63** (1986) 904.
9. A.A. Belavin, A.M. Polyakov, A.B. Zamolodchikov, Nucl. Phys. **B241** (1984) 333.
10. S. Munier, R. Peschanski, Nucl. Phys. **B524** (1998) 377.
11. M. L. Good, W. D. Walker, Phys. Rev. **120** (1960) 1857; see, for a related discussion of hard diffraction , R. Peschanski, hep-ph/9805325.
12. A. Bialas, R. Peschanski, Ch. Royon, Phys. Rev. **D57** (1998) 6899.
13. H. Navelet, R. Peschanski, Nucl. Phys. **B515** (1998) 269; A. Bialas, H. Navelet, R. Peschanski, hep-ph/9811345, to be published in Eur. Phys. J.
14. C. Adloff et al., H1 coll., Z.Phys. **C76** (1997) 613; J. Breitweg, ZEUS coll., Eur. Phys. J. **C1** (1998) 81; soon in Nucl. Phys.**B**.
15. H. Navelet, R. Peschanski, Phys. Rev. Lett. **82** (1999) 1370 and hep-ph/9810359.