Nucleon Structure Functions from Constituent Quark

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We have used a constituent quarks model to describe the nucleon structure function, $F_2(x,Q^2)$, for a wide range of $x=[10^{-6},1]$ and $Q^2=[0.5,5000]GeV^2$. We have found that although F_2 rises as x decreases, but there exists some $x_0 \leq 10^{-4}-10^{-5}$, below which the rise of F_2 subsides drastically and hence, exhibits an almost flat behavior, compatible with the latest results from HERA, at least for low Q^2.

We describe the nucleon as a bound state of three constituent quarks which themselves have their own internal structures. This structure is generated perturbatively by QCD dynamics and it is evaluated in the next-to-leading order (NLO) in QCD. The nucleon structure function, F_2, then is related to the structure of the constituent quark by the convolution theorem. Hence, we can write:

$$F_2^p(x,Q^2)=\sum_c \int_x^1 dy G_{\frac{c}{p}}(y) f^c(\frac{x}{y},Q^2) \tag{1}$$

where the summation runs over constituent quarks, c, of the proton. $f^c(\frac{x}{y},Q^2)$ is the structure function of the constituent quark $c=U,D$. $G_{\frac{c}{p}}(y)$ is the probability of finding a constituent quark in the proton with momentum fraction y. These functions are easy to evaluate and are given in [1]:

$$G_{U/p}=7.98y^{0.65}(1-y)^2, \tag{2}$$

$$G_{D/p}=6.01y^{0.35}(1-y)^{2.3}. \tag{3}$$

Notice that these functions are independent of Q^2 and the nature of the probe used. $G_{c/p}$ summarizes the bound state problem and consequently leaves $f^c(\frac{x}{y},Q^2)$ in Eq. (1) free of the bound state complications. At high enough Q^2 it is the structure of the constituent quarks which are probed, but at sufficiently low Q^2 they behave as valance quarks. At large Q^2 we can write the structure of a U-type constituent quark as:

$$F_2^U=\frac{4}{9}(G_{\frac{u}{U}}+G_{\frac{\bar{u}}{U}})+\frac{1}{9}(G_{\frac{d}{U}}+G_{\frac{\bar{d}}{U}}+G_{\frac{s}{U}}+G_{\frac{\bar{s}}{U}})+... \tag{4}$$

To determine the parton distributions, we denote their moments by $M_x(n,s)$, where the subscript x stands for valence quarks u_v and d_v, as well as for the sea quarks, antiquarks and the gluon. The moments are functions of the evolution parameter, s.

$$s=ln\frac{ln\frac{Q^2}{\Lambda^2}}{ln\frac{Q_0^2}{\Lambda^2}} \tag{5}$$

where Λ and Q_0 are the scale parameters to be determined from the experiment.

$$M_u(n,s)_v=2U(n)\mathcal{M}^{NS}(n,s) \tag{6}$$

$$M_d(n,s)_v=D(n)\mathcal{M}^{NS}(n,s) \tag{7}$$

$$M_{sea}(n,s)= \tag{8}$$

$$\frac{1}{2f}\left[2U(n)+D(n)\right]\left[\mathcal{M}^S(n,s)-\mathcal{M}^{NS}(n,s)\right]$$

$$M_g=\left[2U(n)+D(n)\right]M_{gQ}(n,s) \tag{9}$$

where $U(n)$ and $D(n)$ are the moments of the inclusive constituent quark distributions of Eq. (2) and Eq. (3) $\mathcal{M}^S(n,s)$ and $\mathcal{M}^{NS}(n,s)$ are the moments of the singlet and non-singlet constituent quark structure functions and $M_{gQ}(n,s)$

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is the quark-to-gluon evolution function. These moments are calculated in the NLO. Here we give the parametric form of them:

$$xq_v(x) = a(1-x)^b x^c \tag{10}$$

$$xq_{sea(gluon)}(x) = \sum_i^3 a_i(1-x)^{b_i} \tag{11}$$

The parameters a, b, c, a_i, b_i, are further considered to be functions of s; for the valence sector:

$$a = a_0 + a_1 s + a_2 s^2 \tag{12}$$

similarly for b, and c. For the non-valence sector we have

$$a_i = \alpha_i + \beta_i exp(s/\gamma_i) \tag{13}$$

with similar form for b_i. In Fig. 1 we present the results for F_2^p at various x and Q^2.

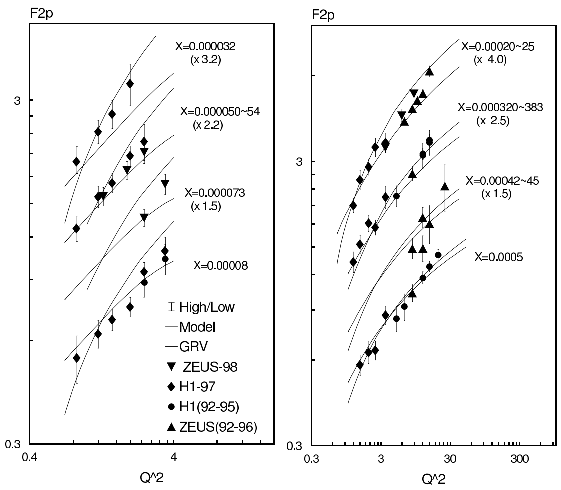

Figure 1. Proton structure function, F_2^p, as a function of Q^2 for various x values. Data are from HERA.

We stress that we have not made a complete fit to the data, however, in order to determine the parameters of the model we have used a single set of data at $Q^2 = 12GeV^2$. The rest of the wealth of the data are used just to check the validity of the model. As it is evident from the figure, the model agrees rather well with the experimental data down to 10^{-6} in x and $Q^2 = [0.5 - 5000]$. In these calculations we have also incorporation the flavor Symmetry Breaking of the sea using the conclusion of Ref. [1], which is also based on the same model as described here, but applied to low $p_\perp$-physics. Our result for the Gottfried sum rule with four flavors is:

$$S_G = \int_0^1 \frac{dx}{x}[F_2^p - F_2^n] = 0.27. \tag{14}$$

Exactly the same result is obtained from the latest MRST parameterization [2]. In a similar manner; we have calculated the gluon distribution and compared it against the data of Ref. [3]. Fig. 2 shows our gluon distribution at $Q^2 = 20GeV^2$.

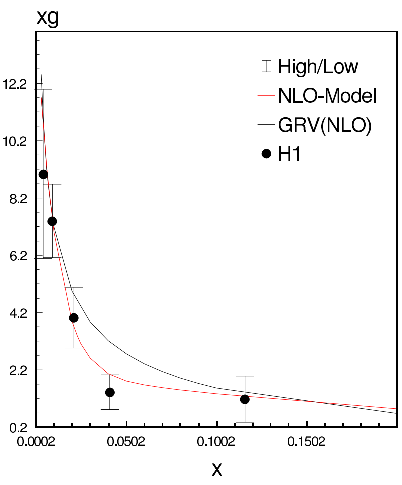

Figure 2. The gluon distribution, xg, as a function of x at $Q^2 \approx 20GeV^2$. The dashed line is our result and the solid line is that of GRV-94(NLO).

It is interesting that our results for $\frac{dF_2}{dln\,Q^2}$, when plotted in $x - Q^2$ plane shows that at low Q^2 and small x it ceases to increase and begins to fall, a feature that dose not follow from GLAP evolution equation and presently is an open question. In Fig. 3 we present the behavior of $\frac{dF_2}{dln\,Q^2}$.

In our model we find that as x decreases beyond $x \leq x_0 \approx 10^{-5}$, F_2^p flattens. This behavior is

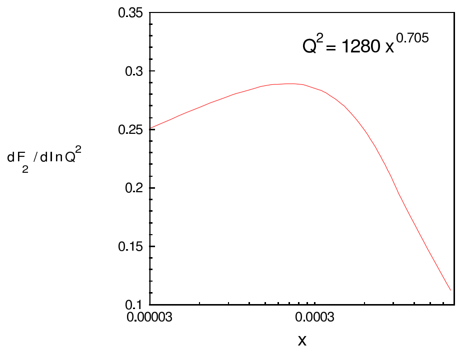

Figure 3. $\frac{dF_2^p}{dlnQ^2}$ in $x - Q^2$ plane.

a consequence of the model and is not assumed *a priori*. Such a behavior is now evident from HERA data and is presented in Fig. 4. x_0 is Q^2 dependent and as Q^2 increases it gets pushed yet to the lower values of x or gets washed out.

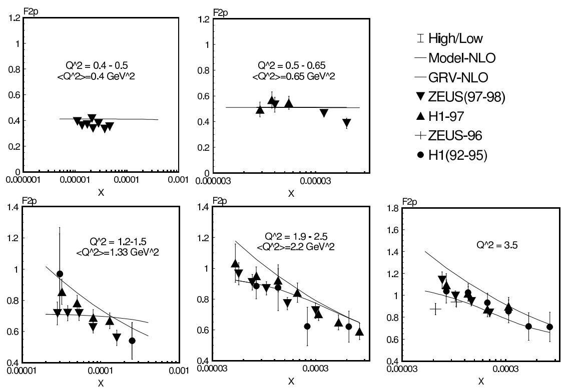

Figure 4. Variation of F_2^p as a function of x for $x < 10^{-4}$.

In Fig. 5 we also present the parton distributions as a function of x for some Q^2 values.

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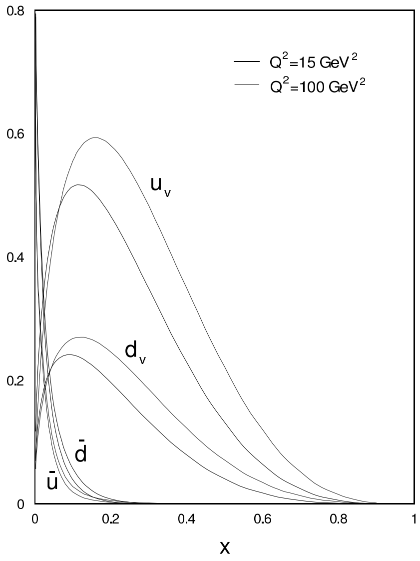

Figure 5. Parton distributions as a function of x.

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