NLL BFKL and NNLO

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The BFKL resummation at LL and NLL accuracy is briefly reviewed, with particular emphasis on the connection between the NLL corrections to the BFKL equation and exact NNLO calculations.

1. LL BFKL

In the limit of center-of-mass energy much greater than the momentum transfer, $\hat{s} \gg |\hat{t}|$, any scattering process is dominated by gluon exchange in the cross channel. Building upon this fact, the BFKL theory [1] models strong-interaction processes with two large and disparate scales, by resumming the radiative corrections to parton-parton scattering. This is achieved to leading logarithmic (LL) accuracy, in $\ln(\hat{s}/|\hat{t}|)$, through the BFKL equation, i.e. a two-dimensional integral equation which describes the evolution in transverse momentum space and moment space of the gluon propagator exchanged in the cross channel,

$$\omega\, f_\omega(k_a,k_b) \;= \qquad (1)$$
$$\frac{1}{2}\,\delta^2(k_a-k_b) \;+\; \frac{\bar{\alpha}_s}{\pi}\int \frac{d^2k_\perp}{k_\perp^2}\,K(k_a,k_b,k)\,,$$

with $\bar{\alpha}_s = N_c\alpha_s/\pi$, $N_c = 3$ the number of colors, k_a and k_b the transverse momenta of the gluons at the ends of the propagator, and with kernel K,

$$K(k_a,k_b,k) = \qquad (2)$$
$$f_\omega(k_a+k,k_b) - \frac{k_{a\perp}^2}{k_\perp^2+(k_a+k)_\perp^2}\,f_\omega(k_a,k_b)\,,$$

where the first term accounts for the emission of a gluon of momentum k and the second for the virtual radiative corrections, which *reggeize* the gluon. Eq. (1) has been derived in the *multi-Regge kinematics*, which presumes that the produced gluons are strongly ordered in rapidity and have comparable transverse momenta.

The solution of eq. (1), transformed from moment space to y space, and averaged over the azimuthal angle between k_a and k_b, is

$$f(k_a,k_b,y) \;= \int\frac{d\omega}{2\pi i}\,e^{\omega y}\,f_\omega(k_a,k_b) \qquad (3)$$
$$= \frac{1}{k_{a\perp}^2}\int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty}\frac{d\gamma}{2\pi i}\,e^{\omega(\gamma)y}\,\left(\frac{k_{a\perp}^2}{k_{b\perp}^2}\right)^\gamma\,,$$

with $\omega(\gamma) = \bar{\alpha}_s\chi(\gamma)$ the leading eigenvalue of the BFKL equation, determined through the implicit equation

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)\,. \qquad (4)$$

In eq. (3) the evolution parameter y of the propagator is $y = \ln(\hat{s}/\tau^2)$. The precise definition of the *reggeization* scale τ is immaterial to LL accuracy; the only requirement is that it is much smaller than any of the s-type invariants, in order to guarantee that $y \gg 1$. The maximum of the leading eigenvalue, $\omega(1/2) = 4\ln 2\bar{\alpha}_s$, yields the known power-like growth of f in energy [1].

What does the BFKL theory have to do with reality ? There is no evidence, as yet, of the necessity of a BFKL resummation either in the scaling violations to the evolution of the F_2 structure function in DIS (for a summary of the theoretical status, see ref. [2]), or in dijet production at large rapidity intervals [3]. The most promising BFKL footprint, as of now, seems to be forward jet production in DIS, where the data [4] show a better agreement with the BFKL resummation [5] than with a NLO calculation [6] (for a summary of dijet and forward jet production, see ref. [7]).

In a phenomenological analysis, the BFKL resummation is plagued by several deficiencies; the most relevant is that energy and longitudinal momentum are not conserved, and since the momentum fractions of the incoming partons are reconstructed from the kinematic variables of the outgoing partons, the BFKL prediction for a production rate may be affected by large numerical errors [8]. However, energy-momentum conservation at each gluon emission in the BFKL ladder can be achieved through a Monte Carlo implementation [9] of the BFKL equation (1).

Besides, because of the strong rapidity ordering between the gluons emitted along the ladder, any two-parton invariant mass is large. Thus there are no collinear divergences, no QCD coherence and no soft emissions in the BFKL ladder. Accordingly jets are determined only to leading order and have no non-trivial structure. Other resummations in the high-energy limit, like the CCFM equation [10] which has QCD coherence and soft emissions, seem thus better suited to describe more exclusive quantities, like multi-jet rates [11]. However, it has been shown that, provided the jets are resolved, i.e. their transverse energy is larger than a given resolution scale, the BFKL and the CCFM multi-jet rates coincide to LL accuracy [12].

2. NLL BFKL and NNLO

In addition to the problems mentioned above, the BFKL equation is determined at a fixed value of α_s (as a consequence, the solution (3) is scale invariant). All these problems can be partly alleviated by computing the next-to-leading logarithmic (NLL) corrections to the BFKL equation (1). In order to do that, the real [13] and the one-loop [14–16] corrections to the gluon emission in the kernel (2) had to be computed, while the reggeization term in eq. (2) needed to be determined to NLL accuracy [17].

From the standpoint of a fixed-order calculation, the NLL corrections [13–17] present features of both NLO and NNLO calculations. Namely, they contain only the one-loop running of the coupling; on the other hand, in order to extract the NLL reggeization term, an approximate evaluation of two-loop parton-parton scattering amplitudes had to be performed [17]. In addition, the one-loop corrections to the gluon emission in the kernel (2) had to be evaluated to higher order in the dimensional regularization parameter ϵ, in order to generate correctly all the singular and finite contributions to the interference term between the one-loop amplitude and its tree-level counterpart [15,16]. This turns out to be a general feature in the construction of the infrared and collinear phase space of an exact NNLO calculation [18], and can be tackled in a partly model-independent way by using one-loop eikonal and splitting functions evaluated to higher order in ϵ [19].

Building upon the NLL corrections [13–17] to the terms in the kernel (2), the BFKL equation was evaluated to NLL accuracy [20,21]. Applying the NLL kernel to the LL eigenfunctions, $(k_\perp^2)^\gamma$, the solution has still the form of eq. (3), with leading eigenvalue,

$$\begin{aligned} \omega(\gamma) &= \bar{\alpha}_s(\mu)\left[1 - b_0\bar{\alpha}_s(\mu)\ln(k_{a\perp}^2/\mu^2)\right]\chi_0(\gamma) \\ &+ \bar{\alpha}_s^2(\mu)\chi_1(\gamma) \end{aligned} \tag{5}$$

where $b_0 = 11/12 - n_f/(6N_c)$ is proportional the one-loop coefficient of the β function, with n_f active flavors, and μ is the $\overline{MS}$ renormalization scale. $\chi_0(\gamma)$ is given in eq. (3), and $\chi_1(\gamma)$ in ref. [20]. In eq. (5), the running coupling term, which breaks the scale invariance, has been singled out.

Both the running-coupling and the scale-invariant terms in eq. (5) present problems that could undermine the whole resummation program (for a summary of its status see ref. [22]). Firstly, the NLL corrections at $\gamma = 1/2$ are negative and large [20] (however, eq. (5) no longer has a maximum at $\gamma = 1/2$ [23]). Secondly, double transverse logarithms of the type $\ln^2(k_{a\perp}^2/k_{b\perp}^2)$, which are not included in the NLL resummation, can give a large contribution and need to be resummed [24]. Double transverse logarithms appear because the NLL resummation is sensitive to the choice of reggeization scale τ; e.g. the choices $\tau^2 = k_{a\perp}k_{b\perp}$, $k_{a\perp}^2$ or $k_{b\perp}^2$, which are all equivalent at LL, introduce double transverse logarithms one with respect to the others at NLL. An alternative, but related, approach is to introduce a cut-off Δ

as the lower limit of integration over the rapidity of the gluons emitted along the ladder [25,26]. This has the advantage of being similar in spirit to the dependence of a fixed-order calculation on the factorization scale, namely in a NLL resummation the dependence on the rapidity scale Δ is moved on to the NNLL terms [26], just like in a NLO exact calculation the dependence on the factorization scale is moved on to the NNLO terms.

Finally, we remark that so far the activity has mostly been concentrated on the NLL corrections to the Green's function for a gluon exchanged in the cross channel. However, in a scattering amplitude this is convoluted with process-dependent impact factors, which must be determined to the required accuracy. In a NLL production rate, the impact factors must be computed at NLO. For dijet production at large rapidity intervals, they are given in ref. [27].

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