Hamiltonian for Reggeon Interactions in QCD

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Next-to-leading corrections to the BFKL equation are discussed in the framework of the BLM renormalization scheme. The duality properties of the Hamiltonian for the Reggeon interactions in the multi-colour QCD are reviewed. The Odderon and other states constructed from three reggeized gluons are considered.

1. BLM APPROACH TO THE BFKL POMERON

Recently the next-to-leading corrections to the kernel of the BFKL equation ([1],[2]) were calculated; see Refs. [3] and references therein. In the $\overline{\text{MS}}$ renormalization scheme their contribution to the Pomeron trajectory is negative and big. It turns out [4], that the intercept of the BFKL pomeron is stable $\omega \sim 0.17$ in a rather wide interval $1\,GeV^2 < Q^2 < 100\,GeV^2$ of the photon virtualities in the non-abelian physical schemes (MOM and Υ) combined with the Brodsky-Lepage-Mackenzie (BLM) optimal scale setting [5].

Using this procedure, one can obtain in particular the cross-section $\sigma_{\gamma\gamma}$ for the high energy virtual photon scattering measured recently by the $L3$ group [6]. The comparison between our results and the experimental data is presented in Fig. 1 (for details see [7]).

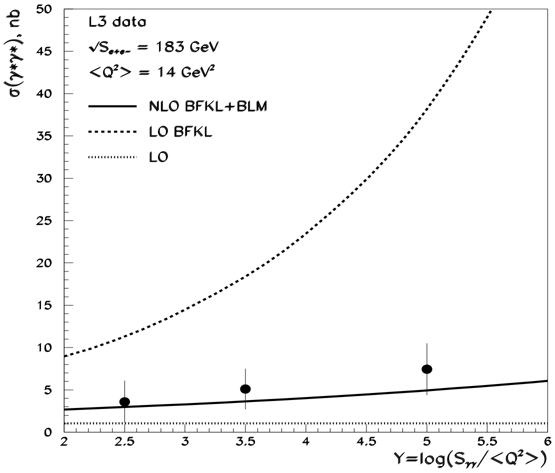

Figure 1. NLO BFKL improved by BLM vs. L3 data.

Note, that the impact factors for the virtual photons with $Q_1^2 = Q_2^2 = Q^2$ are known only in the Born approximation (see [2], [8]). Further, the Regge factor was taken by us in the form $(cs/Q^2)^j$ with $c = 1$. The true value of the multiplier c can be estimated only after calculating a two-loop contribution to the impact factors. Apart from the dependence from the renormalization scheme next-to-leading corrections to the BFKL equation contain an uncertainty related with the resummation of the terms responsible for the correlation between the particles with close rapidities. This effect leads to an increase of the BFKL intercept in comparison with its BLM value [7].

2. DUALITY OF THE REGGEON INTERACTIONS

Even after taking into account the next-to-leading corrections the total cross-section obtained from the BFKL equation violates the Froissart bound $\sigma_t < a \ln^2 s$. To restore the s-channel unitarity one should estimate the contribution of diagrams with an arbitrary number of

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reggeized gluons in the t-channel.

In the simplest case corresponding to the generalized leading logarithmic approximation the number n of the reggeized gluons is conserved when the rapidity grows. The high energy asymptotics $s^{1+\Delta_n}$ of the scattering amplitudes is related with the ground state energy E of the corresponding Schrödinger equation [9] as follows

$$\Delta_n = -\frac{g^2}{8\pi^2}E. \qquad (1)$$

In the multi-colour limit $N_c \to \infty$ the colour structure of the total Hamiltonian is simplified and in the impact parameter representation H has the property of the holomorphic separability [10]

$$H = \frac{1}{2}(h+h^*),\ h = \sum_{k=1}^{n} h^{k,k+1}\ ,\ [h,h^*]=0\,, \qquad (2)$$

where

$$h^{kl} = \ln(p_k p_l) + \frac{1}{p_k}\ \ln(\rho_{kl})\, p_k + \frac{1}{p_l}\ \ln(\rho_{kl})\, p_l + 2\gamma.$$

Here we used the complex coordinates $\rho_k = x_k + iy_k,\ p_k = i\partial/\partial_{\rho_k}$ and $\rho_{kl} = \rho_k - \rho_l,\ \gamma = -\psi(1)$.

The wave function of the compound state of n reggeized gluons in the multi-colour limit is holomorphically factorizable

$$\Psi = \sum_{r<s} C_{rs}\, \psi_r\, \widetilde{\psi_s}\,, \qquad (3)$$

where ψ_r and $\widetilde{\psi_s}$ are degenerate solutions of the corresponding Schrödinger equations in the holomorphic and anti-holomorphic subspaces:

$$h\psi = \epsilon\psi\,,\ h^*\widetilde{\psi} = \widetilde{\epsilon}\widetilde{\psi}\,,\ E = \frac{1}{2}(\epsilon+\widetilde{\epsilon})\,. \qquad (4)$$

The coefficients C_{rs} are chosen from the condition of the single-valuedness of Ψ in the two-dimensional impact parameter space.

The holomorphic Hamiltonian h has the integrals of motion $q_r, r = 1,2...n$ [10]

$$q_r = \sum_{i_1<i_2<...i_r} \rho_{i_1 i_2}\rho_{i_2 i_3}...\rho_{i_n i_1}\, p_{i_1} p_{i_2}...p_{i_k} \qquad (5)$$

with the properties

$$[q_r, q_s] = 0\,,\ [q_r, h] = 0\,. \qquad (6)$$

Moreover, it coincides with the local Hamiltonian of the integrable spin chain [11].

The integrability of the model is related with the duality symmetry of the Hamiltonian h and of the integrals of motion q_r under the canonical transformation

$$p_r \to \rho_{r,r+1} \to p_{r+1} \qquad (7)$$

combined with their transposition [12]. For the compound state of three reggeized gluons one can use for the wave function the conformal ansatz

$$\Psi_{m,\widetilde{m}} = \left(\frac{\rho_{23}}{\rho_{20}\rho_{30}}\right)^m \left(\frac{\rho_{23}^*}{\rho_{20}^*\rho_{30}^*}\right)^{\widetilde{m}} \Phi_{m,\widetilde{m}}(x,x^*)\,,$$

depending on the anharmonic ratio $x = \frac{\rho_{12}\rho_{30}}{\rho_{10}\rho_{32}}$. In this case the duality equation for $\Phi_m^{\widetilde{m}} = \Phi_{m,\widetilde{m}}(\overrightarrow{x})$ can be presented in the pseudo-differential form

$$|x(1-x)|^2\ (i\partial)^{1+m}(i\partial^*)^{1+\widetilde{m}}\ \Phi_m^{\widetilde{m}} = |\lambda|\ \Phi_{1-m}^{1-\widetilde{m}}\,,$$

where $m, \widetilde{m}$ are conformal weights and λ is the eigen value of the first non-trivial integral of motion [12].

3. THREE-REGGEON COMPOUND STATES

One can write the holomorphic Hamiltonian for the Odderon wave function $\Phi_m(P)$ in the P-representation ($P = x\partial$) in the normal form:

$$\frac{h}{2} = \ -\log(x) + 3\gamma +$$

$$\psi(1-P) + \psi(-P) + \psi(m-P) + \sum_{k=1}^{\infty} x^k\, f_k(P)\,,$$

where $f_k(P)$ is the meromorhic function known explicitly [12].

The Odderon function $\Phi_m(P)$ in the P-representation can be expanded in series over $1/\lambda$

$$\Phi_m(P) = \sum_{n=0}^{\infty} \lambda^{-n}\Phi_m^{(n)}(P)\,,\ \Phi_m^{(0)}(P) = 1\,, \qquad (8)$$

where the coefficients $\Phi_m^{(n)}(P)$ turn out to be polynomials of the order $4n$ satisfying recurrence relations due to the above duality equation [12].

Using these relations one can obtain the asymptotic expansion of the holomorphic energy ϵ as a function of λ:

$$\frac{\varepsilon}{2} = \log(\lambda) + 3\gamma -$$
$$\left(\frac{3}{448} + \frac{13}{120}(m-1/2)^2 - \frac{1}{12}(m-1/2)^4\right)\frac{1}{\lambda^2} + \ldots$$

This expansion can be used with some accuracy even for the lowest energy $\epsilon = 0.49$ corresponding to $\lambda = 0.20\,i$ [13].

There is a solution of the duality equation with $\lambda = 0$:

$$\varphi_{m,\widetilde{m}}(\overrightarrow{x}) = 1 + (-x)^m(-x^*)^{\widetilde{m}} + (x-1)^m(x^*-1)^{\widetilde{m}}.$$

This expression vanishes at the points $x = 0$, $x = 1$ and $x = \infty$ only when the conformal spin is odd: $m - \widetilde{m} = 2k + 1$. The corresponding wave function in this case is anti-symmetric under the transmutation of the gluon coordinates in the impact parameter space. Due to the Bose statistics for gluons the colour part of the wave function should be also anti-symmetric and therefore this state has the pomeron quantum number. Acting on this solution by the Hamiltonian written in the x representation [12] we obtain the energy

$$2E = \psi(1-m) + \psi(m) + \psi(1-\widetilde{m}) + \psi(\widetilde{m}) + 4\gamma\,.$$

The corresponding singularity of the t-channel partial wave is situated at $\omega = 0$ for $m = 1$, $\widetilde{m} = 0$. Because the impact factor should vanish at $\overrightarrow{k}_i = 0$, this singularity does not give any contribution to the scattering amplitude. However the other anti-symmetric states constructed from three reggeized gluons survive and they are responsible for the singular behaviour of the structure function $g_2(x)$ at $x \to 0$.

For the case of even values of the conformal spin the normalization integral for $\varphi_{m,\widetilde{m}}(x)$ is divergent. Nevertheless one can attempt to consider the trial function coinciding with this solution everywhere except the small regions around points $x = 0, x = 1$ and $x = \infty$, where it is zero. It turns out, that this trial function does not satisfy the sum rules which follow from the duality equation [12]. As a consequence of this circumstance the different definitions of the Hamiltonian give the different values for the energy. The problem of even values of n deserves a more accurate investigation.

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