Urgent problems at small x

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Regge theory provides an excellent fit to small-x structure-function data from $Q^2 = 0$ right up to the highest available values, but it also teaches us that conventional approaches to perturbative evolution are incorrect.

1. INTRODUCTION

During the 1960's, a lot was learnt about the analytic properties of scattering amplitudes. Much of this knowledge was incorporated in Regge theory, but it has been largely forgotten. However, Regge theory provides[1] the best available description of the structure function data at small x, right from $Q^2 = 0$ up to the very highest available values. It should not be regarded as a competitor for perturbative QCD; rather, it is complementary to it, and we need to learn how to make the two live together. In recent years, a belief has grown that the spectacular small-x behaviour seen at HERA may be associated with the collinear singularity of the DGLAP splitting function[2]. However, this belief conflicts with what we know about the analytic properties of the structure function[3].

2. REGGE FIT TO SMALL-x DATA

Regge theory should be valid at any value of Q^2, provided only that x is small enough. In its simplest form, it describes the structure function as a sum of fixed powers of x, multiplied by functions of Q^2:

$$F_2(x, Q^2) \sim \sum_i f_i(Q^2) x^{-\epsilon_i} \tag{1}$$

*This research is supported in part by the EU Programme "Training and Mobility of Researchers", Networks "Hadronic Physics with High Energy Electromagnetic Probes" (contract FMRX-CT96-0008) and "Quantum Chromodynamics and the Deep Structure of Elementary Particles" (contract FMRX-CT98-0194), and by PPARC

Regge theory tells us little about the coefficient functions $f_i(Q^2)$, beyond that they are analytic functions of Q^2. Also, we know from QED gauge invariance that at $Q^2 = 0$ they vanish at least linearly with Q^2. It may be that the assumption of simple powers of x is too simple, and it certainly must be corrected at some level, but it does fit the data extraordinarily well and so there is no reason to suppose that the correction is numerically significant at present x values. We find that three powers are sufficient: two are taken from our old fits[4] to hadronic total cross sections

$$\epsilon_1 = 0.08 \qquad \text{soft pomeron exchange} \tag{2}$$

$$\epsilon_2 = -0.45 \qquad f, a \text{ exchange} \tag{3}$$

The data require the remaining power to be

$$\epsilon_0 = 0.4 \tag{4}$$

with an error of about $\pm 10\%$. We call this the "hard pomeron".

We have made a fit[1] to the data at each available value of Q^2 to extract the values of the coefficient functions $f_i(Q^2)$. The data do not constrain the f, a-exchange coefficient function $f_2(Q^2)$ at all well, but the result for the hard-pomeron function $f_0(Q^2)$ and the soft-pomeron function $f_1(Q^2)$ are shown in figure 1.

Each vanishes at $Q^2 = 0$, as it has to. The hard-pomeron coefficient remains small until about $Q^2 = 10\ \mathrm{GeV}^2$, after which it rises approximately logarithmically. This is no surprise. What is surprising is that the soft-pomeron coefficient, after rising rapidly away from $Q^2 = 0$, reaches a peak at about $Q^2 = 10$ and then falls again.

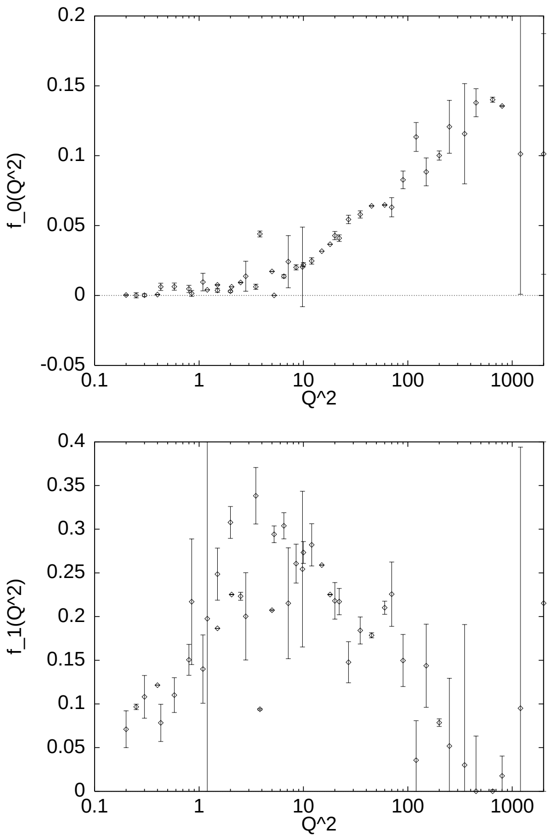

Figure 1. The coefficient functions f_0 and f_1 extracted from data at each Q^2; the error bars are from MINUIT

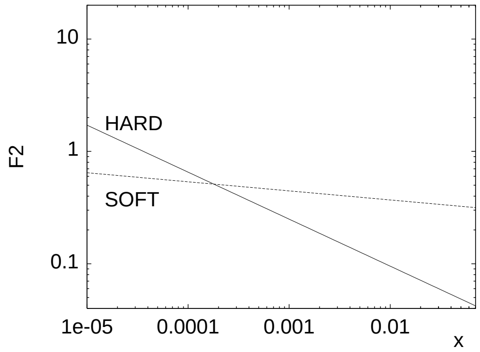

Figure 2. Hard and soft contributions to $F_2(x,Q^2)$ at $Q^2=5$

That is, soft-pomeron exchange is higher twist. For even quite large values of Q^2 this higher-twist component is a major part of the small-x structure function: see figure 2. This raises serious questions about all perturbative-QCD fits to structure functions.

The three-term form (1) gives an excellent fit to $F_2(x,Q^2)$ for $x<0.07$ and $0\le Q^2\le 2000$. With 8 free parameters, including ϵ_0, one can achieve a χ^2 per data point well below 1.0, so the exact values of the parameters are not completely determined by these data points. Figure 3 shows how such a fit compares with the largest-Q^2 data and the real-photon data. A combination of the hard and the soft pomerons also describes well[1] the data for the process $\gamma p\to\psi p$.

3. PERTURBATIVE EVOLUTION

If we Mellin transform with respect to x, the DGLAP equation[2] reads

$$\frac{\partial}{\partial \log Q^2}\mathbf{u}(N,Q^2)=\mathbf{P}(N,Q^2)\mathbf{u}(N,Q^2) \tag{5}$$

where $\mathbf{u}$ is a two-component object whose elements are the singlet quark distribution and the gluon distribution, while $\mathbf{P}$ is the splitting matrix. A power contribution

$$f(Q^2)x^{-\epsilon} \tag{6}$$

to $F_2(x,Q^2)$ corresponds to a pole

$$\frac{\mathbf{f}(Q^2)}{N-\epsilon} \tag{7}$$

in $\mathbf{u}(N,Q^2)$. Inserting such a pole into each side of (5) gives a differential equation for $f(Q^2)$. If we use the lowest-order approximation to the splitting matrix $\mathbf{P}$ the solution to this equation is that, for large Q^2, $f(Q^2)$ behaves[5] as a power of $\log Q^2$.

However, there is a serious problem with this lowest-order approximation. It gives $\mathbf{P}(N,Q^2)$ a pole at $N=0$, and it is this pole that largely determines the magnitude of the power of $\log Q^2$. Conventional perturbative-QCD fits to the data also rely on this pole to explain the rapid rise of F_2 at small x. But we know that in fact such a pole cannot be present: higher-order corrections must resum it away. We know this because at small Q^2

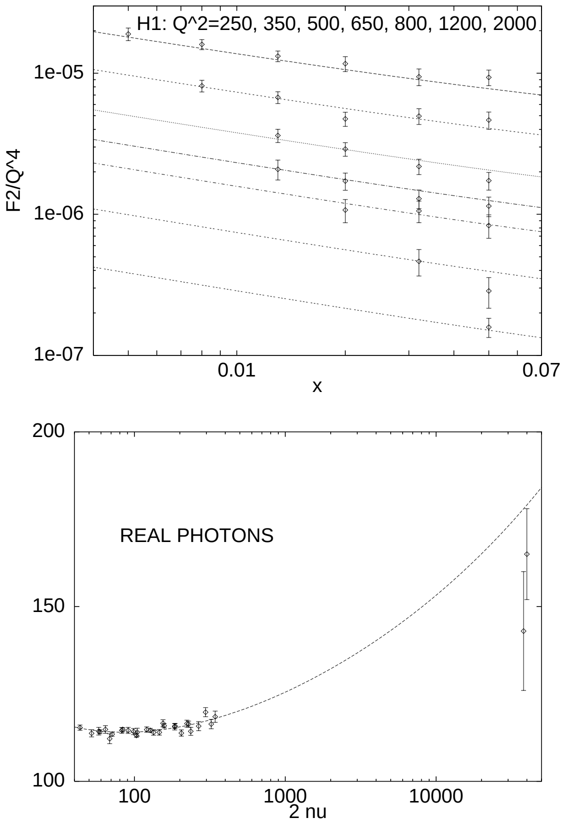

Figure 3. Regge fits to the largest-Q^2 data and the real-photon data

$\mathbf{u}(N,Q^2)$ does not have a singularity at $N=0$: rather, its singularities in the complex N-plane are the standard singularities of Regge theory — the soft pomeron, the mesons, and possibly also a hard pomeron. It is also supposed to be analytic in Q^2, so a singularity at $N=0$ cannot suddenly appear when we continue from small Q^2 up to beyond the values of Q^2 at which the DGLAP equation (5) begins to be valid. Thus $\mathbf{P}(N,Q^2)$ cannot have a pole at $N=0$, nor indeed at any other value of N.

At small N, the gg element of the splitting matrix is found by solving the equation[6]

$$\chi(P_{gg}(N,Q^2),Q^2)=N \tag{8}$$

where $\chi(\omega,Q^2)$ is the Lipatov characteristic function. In lowest order,

$$\pi\chi(\omega,Q^2)=3\alpha_S(Q^2)\,[2\psi(1)-\psi(\omega)-\psi(1-\omega)] \tag{9}$$

If one uses this approximation to $\chi(\omega,Q^2)$ one indeed finds that $P_{gg}(N,Q^2)$ is nonsingular at $N=0$, even though the terms of its expansion in powers of α_S are each singular at $N=0$. Compare the expansion of the function[7]

$$p(N,Q^2)=N-\sqrt{N^2-\alpha_S(Q^2)} \tag{10}$$

whose expansion is

$$p(N,Q^2)=\frac{\alpha_S(Q^2)}{2N}+\frac{\alpha_S^2(Q^2)}{8N^3}+\dots \tag{11}$$

but which is evidently finite at $N=0$. Near $N=0$ the expansion parameter $\alpha_S(Q^2)/N$ is so large that the expansion is illegal.

We know that $P_{gg}(N,Q^2)$ is finite at $N=0$, but we do not know how large it is, because the lowest-order approximation (9) to $\chi(\omega,Q^2)$ is apparently not a good one: the next-to-leading order correction is huge[8].

More work is needed!

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