Renormalization group improved BFKL equation *

Marcello Ciafaloni [a]

[a]Dipartimento di Fisica dell'Università, Firenze
and INFN, Sezione di Firenze, Italy

I report on the recent proposal of a generalized small-x equation which, in addition to exact leading and next-to-leading BFKL kernels, incorporates renormalization group constraints in the relevant collinear limits.

The calculation of next-to-leading $\log x$ corrections to the BFKL equation was completed last year [1,2] after several years of theoretical effort. The results, for both anomalous dimension and hard Pomeron, show however signs of instability due to both the size and the (negative) sign of corrections, possibly leading to problems with positivity also [3].

If we write the eigenvalue equation corresponding to the BFKL solution in the form [2]

$$\begin{aligned} \omega &= \bar{\alpha}_s(t)\left[\chi_0(\gamma) + \bar{\alpha}_s(\mu^2)\chi_1(\gamma) + \ldots\right], \\ t &= \log\frac{k^2}{\lambda^2}, \end{aligned} \qquad (1)$$

where $\omega = N-1$ is the moment index and γ is an anomalous dimension variable, the NL eigenvalue function has the shape of Fig 1, which completely overthrows the LL picture, even for coupling values as low as .04.

The basic reason for the instability above lies in the γ-singularity structure of χ_1 (cubic poles) which are of collinear origin, and keep track of the choice of the scaling variable, whether it is kk_0/s, or k^2/s, or k_0^2/s, in a two-scale hard process. An additional reason lies in the renormalization scale (μ) dependence of Eq (1), related to the method of solution.

In a recent proposal [4,6], both problems are overcome at once by a proper use of R.G. constraints on both kernel and solution. On one hand, the requirement of single-log scaling violations for both $k \gg k_0$ with Bjorken variable k^2/s and for the symmetrical limit, imply an ω-dependent shift of the γ-singularities in the kernel which resums the double logarithmic ones mentioned before.

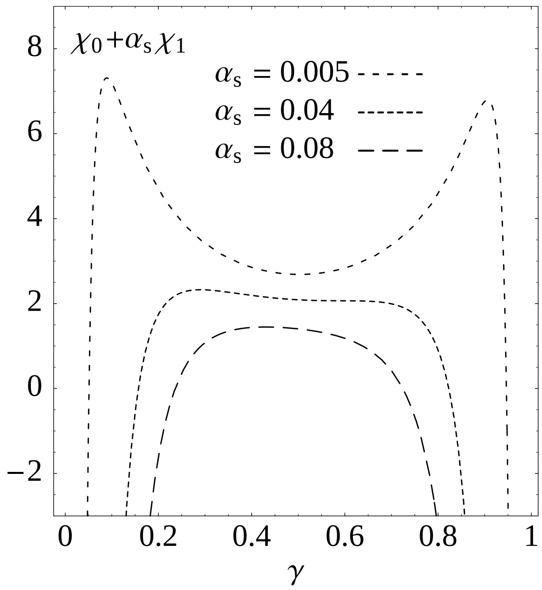

Figure 1. The BFKL eigenvalue function at NL accuracy, for $\alpha_s = 0.005$, 0.04 and 0.08.

On the other hand, a novel method of solution called ω-expansion [5] replaces α_s with ω as perturbative parameter of the subleading hierarchy, and allows a R.G. invariant formulation of the solution. More precisely, for large t the gluon

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Green's function takes the factorized form

$$G_\omega(\mathbf{k},\mathbf{k}_0) = \mathcal{F}_\omega(\mathbf{k})\tilde{\mathcal{F}}_\omega(\mathbf{k}_0),\ t - t_0 \gg 1, \quad (2)$$

where

$$\begin{aligned} \dot{g}_\omega(t) &\sim \mathbf{k}^2\mathcal{F}_\omega(\mathbf{k}) \\ &= \int \frac{d\gamma}{2\pi i} \exp\left[\gamma t - \frac{1}{b\omega}X(\gamma,\omega)\right] \end{aligned} \quad (3)$$

is the t-dependent unintegrated gluon density. The phase function X is given in terms of the effective eigenvalue function

$$\frac{\partial}{\partial\gamma}X(\gamma,\omega) = \chi(\gamma,\omega) = \chi_0^\omega(\gamma) + \omega\frac{\chi_1^\omega(\gamma)}{\chi_0^\omega(\gamma)} + \ldots, \quad (4)$$

which now has a fully stable ω-dependence (Fig.2).

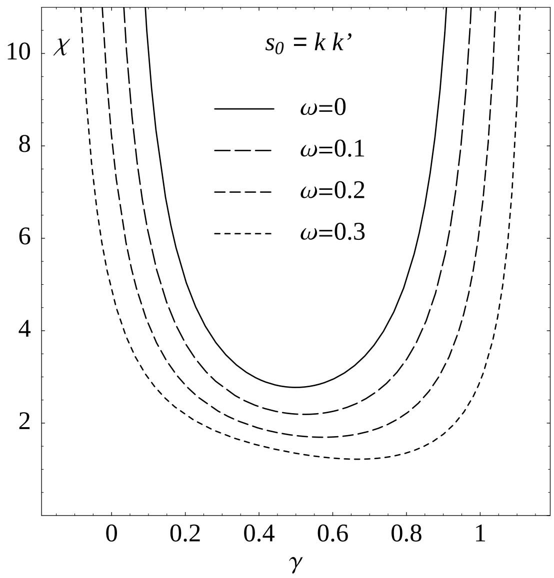

Figure 2. The effective eigenvalue function $\chi(\gamma,\omega)$ for various ω values.

The improved kernel eigenvalue functions χ_0^ω and χ_1^ω are constructed from the exact L+NL kernels, by incorporating the ω-shift requirement. The neglected terms in Eq.4 yield a small error, corresponding to a coupling change $\delta\alpha_s/\alpha_s = O(\alpha_s\omega)$, subleading in both $\frac{\alpha_s}{\omega}$ and α_s expansions of the small-x hierarchy.

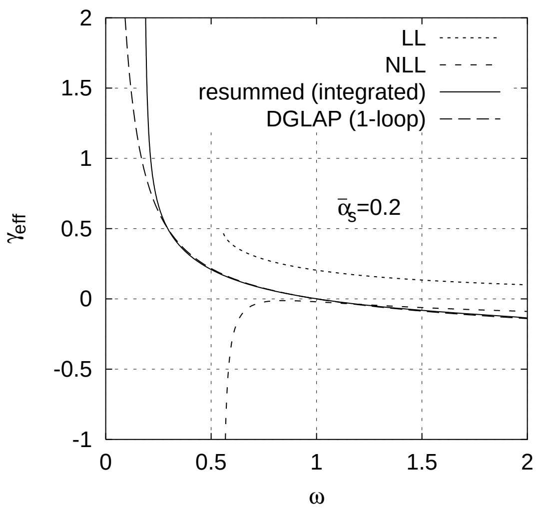

Figure 3. The resummed gluon anomalous dimension compared to various approximations.

The solution for the effective anomalous dimension $\gamma_{eff} = \dot{g}_\omega(t)/g_\omega(t)$ is shown in Fig.3, compared to L and NL approximations. The resummed result is remarkably similar to the fixed order value until very close to the singularity point $\omega_c(t)$, which lies below the saddle point breakdown value $\omega_s(t)$ used in previous NL estimates of the hard Pomeron. The latter signals the failure of the large-t saddle point $b\omega t = \chi(\bar{\gamma},\omega)$ to yield a reliable anomalous dimension $\bar{\gamma}$, due to infinite γ-fluctuations. The former is the position of the true t-dependent ω singularity, and is systematically lower [Fig.4]. No instabilities and very little renormalization scheme dependence are found.

The critical exponents $\omega_c(t)$ and $\omega_s(t)$ are actually both needed for a full understanding of the Green's function (2), whose coefficient $\tilde{\mathcal{F}}_\omega(\mathbf{k_0})$

carries the t-independent, leading Pomeron singularity, which is really nonperturbative. While a precise estimate of the latter requires extrapolating the small-x equation in the strong-coupling region $k^2 \simeq \Lambda^2$, one can argue [6] that ω_c and ω_s provide lower and upper bounds on ω_P, and thus a first rough estimate of the Pomeron intercept.

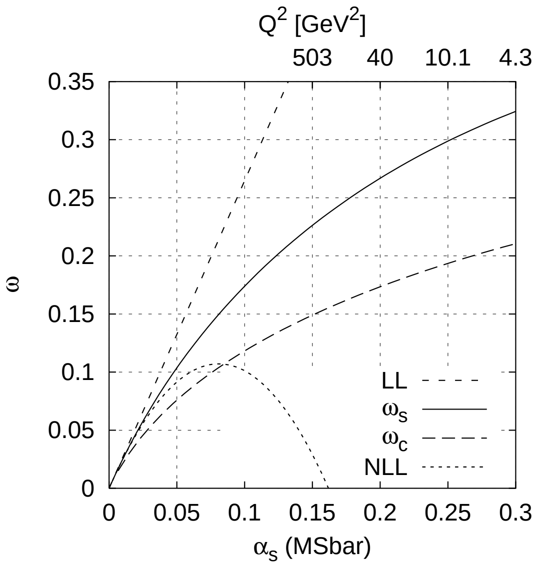

Figure 4. The resummed critical exponents $\omega_c(t)$ and $\omega_s(t)$, compared to L and NL estimates of the hard Pomeron.

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