Bootstrap of the Gluon Reggeization

V.S. Fadin [a] *

[a]Budker Institute for Nuclear Physics and Novosibirsk State University, 630090 Novosibirsk, Russia

Gluon reggeization is extremely important for the description of high energy process in perturbative QCD. I discuss “bootstrap” conditions for the reggeization, imposed by the s -channel unitarity.

The reggeization of elementary particles is one of the remarkable properties of QCD. Contrary to QED, where the electron does reggeize in perturbation theory, but the photon remains elementary, the gluons do reggeize in QCD [1, 2] as well as the quark [3]. The gluon reggeization plays the key role in the derivation of the BFKL equation [4], which is the most common basis for the description of small x processes.

The BFKL equation is a particular case (for forward scattering, i.e. $t = 0$ and vacuum quantum numbers in the $t-$channel) of the equation for the $t-$channel partial waves of the elastic amplitudes [4]. For brevity, we will use the term “BFKL equation” for the general case as well. In the approach based on the gluon reggeization the amplitude for the high energy process $A + B \to A' + B'$ at fixed momentum transfer $\sqrt{-t}$ is presented as the convolution

$$\mathcal{A}_{AB}^{A'B'} = s \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}\vec{q}_1^{\,\prime 2}} \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}\vec{q}_2^{\,\prime 2}} \frac{d\omega}{\sin(\pi\omega)}$$

$$\left[\left(\frac{-s}{s_0}\right)^{\omega} - \tau\left(\frac{s}{s_0}\right)^{\omega}\right] \sum_{\mathcal{R},\nu} \Phi_{A'A}^{(\mathcal{R},\nu)}\left(\vec{q}_1;\vec{q};s_0\right)$$

$$\times \frac{G_{\omega}^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2,\vec{q}\right)}{(2\pi)^{D-1}} \Phi_{B'B}^{(\mathcal{R},\nu)}\left(-\vec{q}_2;-\vec{q};s_0\right) , \qquad (1)$$

where the momenta are defined in Fig.1.

Let me stress, that that the intermediate zigzag lines in Fig.1 denote not gluons, but reggeized gluons which would only become gluons in the absence of interaction.

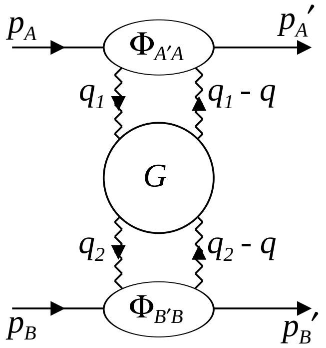

Fig.1

Figure 1. The scattering amplitude of the process $A + B \to A' + B'$.

The first two integrations in (1) are performed over $D-2$ dimensional vectors which are transverse to the momenta p_A and p_B of the initial particles. The space-time dimension, D, is taken to be $D = 4+2\varepsilon$ in order to regularize the infrared divergences. Everywhere in this paper $q_i' \equiv q_i - q$, where $q \simeq \vec{q}$ is the momentum transfer in the process $A+B \to A'+B'$. The sum in (1) is taken over irreducible representations $\mathcal{R}$ of the colour group, which are contained in a product of two adjoint representations, and over states ν from full sets of states for these representations. The third integration is performed along the imaginary axis over the Mellin variable ω. s_0 is some energy scale and τ is the signature equal to $+1(-1)$ for symmetric (antisymmetric) representations. $\Phi_{A'A}^{(\mathcal{R},\nu)}$ and $\Phi_{B'B}^{(\mathcal{R},\nu)}$ are the impact factors projected on the states $(\mathcal{R},\nu)$, which describe the $A \to A'$ and $B \to B'$ particle transitions in the particle-

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reggeon scattering processes shown by the upper and lower blobs of Fig. 1. $G_\omega^{(\mathcal{R})}$ are the Mellin transforms of the Green's functions for reggeon-reggeon scattering.

The Green functions obey the equation

$$\omega G_\omega^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2,\vec{q}\right)=\vec{q}_1^{\,2}\vec{q}_1^{\,\prime\,2}\delta^{(D-2)}\left(\vec{q}_1-\vec{q}_2\right)$$

$$+\int\frac{d^{D-2}q_r}{\vec{q}_r^{\,\prime\,2}\vec{q}_r^{\,2}}\mathcal{K}^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_r;\vec{q}\right)G_\omega^{(\mathcal{R})}\left(\vec{q}_r,\vec{q}_2;\vec{q}\right), \qquad (2)$$

where the kernel

$$\mathcal{K}^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right)=\left[\omega\left(-\vec{q}_1^{\,2}\right)+\omega\left(-\vec{q}_1^{\,\prime 2}\right)\right]$$

$$\times\delta^{(D-2)}\left(\vec{q}_1-\vec{q}_2\right)+\mathcal{K}_r^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right) \qquad (3)$$

is given as the sum of the "virtual" part, defined by the gluon trajectory $j(t)=1+\omega(t)$ and the "real" part $\mathcal{K}_r^{(\mathcal{R})}$, related to the real particle production in reggeon-reggeon collisions.

Due to the gluon reggeization the representation (1), as well as Eq. (2) for the Green functions and the form (3) of the kernel, are valid not only in the leading logarithmic approximation (LLA) [4], when the leading terms $(\alpha_S \ln s)^n$ are resummed, but also in the next-to-leading approximation (NLA) as well, when the $\alpha_S(\alpha_S \ln s)^n$ terms are also resummed. Both the kernel of the BFKL equation and the impact factors are unambiguously defined in terms of the gluon regge trajectory and the effective vertices for the reggeon-particle interactions. Recently, the calculation of the radiative corrections to the kernel of the BFKL equation for forward scattering was completed [5] and the explicit form of the kernel in the NLA was obtained [6]. A number of subsequent papers (see, for instance, [7]) was devoted to investigation of its consequences.

The representation (1) was obtained using the s-channel unitarity and assuming the gluon reggeization. But the reggeization was proven only in LLA. In the NLA till now it was checked only in the first three orders of perturbation theory in α_S. Evidently more powerful tests of the reggeization are desirable. One such test gives us the requirement of the self-consistency of the BFKL approach, or, in other words, the compatibility of the gluon reggeization with the s-channel unitarity. The representation (1) contains a mixture of various irreducible representations $\mathcal{R}$ of the colour group in the $t-$channel. The most interesting representations are the colour singlet (Pomeron channel) and the antisymmetric colour octet (gluon channel). The self-consistency requires that in the gluon channel the two interacting reggeized gluons reproduce the reggeized gluon itself. This "bootstrap" supports the idea of the reggeization in such a strong way that its fulfillment leaves practically no doubts that the idea is correct. In the LLA the fulfillment of the "bootstrap" was explicitly demonstrated in [4]. The "bootstrap" conditions in the NLA were derived in [8].

For the sake of brevity, the term "gluon reggeization" used by us means that in each order of perturbation theory amplitudes of the processes at large relative energies of participating particles and fixed (i.e. not increasing with s) momentum transfers have the regge form. For the amplitudes of the elastic scattering process $A+B\to A'+B'$ with gluon quantum numbers in the t-channel and negative signature it means:

$$(\mathcal{A}_8^-)_{AB}^{A'B'}=\Gamma_{A'A}^c\left[\left(\frac{s}{t}\right)^{j(t)}-\left(\frac{s}{-t}\right)^{j(t)}\right]\Gamma_{B'B}^c\,, \qquad (4)$$

where c is a color index and $\Gamma_{P'P}^c$ are the particle-particle-reggeon (PPR) vertices which do not depend on s. This form is assumed to be valid in the LLA as well as in the NLA.

Together with the elastic amplitude the derivation of the BFKL equation in LLA and NLA is based on the reggeized form of production amplitudes in the multi-regge kinematics.

The first bootstrap condition [8] is:

$$\frac{g^2Nt}{2\left(2\pi\right)^{D-1}}\int\frac{d^{D-2}q_1}{\vec{q}_1^{\,2}\vec{q}_1^{\,\prime\,2}}\int\frac{d^{D-2}q_2}{\vec{q}_2^{\,2}\vec{q}_2^{\,\prime\,2}}\mathcal{K}^{(8)}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right)$$

$$=\omega^{(1)}\left(t\right)\left(\omega^{(1)}\left(t\right)+\omega^{(2)}\left(t\right)\right)\ . \qquad (5)$$

The second condion reads:

$$\frac{ig\sqrt{N}t}{\left(2\pi\right)^{D-1}}\int\frac{d^{D-2}q_1}{\vec{q}_1^{\,2}\vec{q}_1^{\,\prime\,2}}\Phi_{A'A}^{(8,c)}\left(\vec{q}_1;\vec{q};s_0\right)$$

$$=\Gamma_{A'A}^c\omega^{(1)}\left(t\right)+\Gamma_{A'A}^{c(B)}$$

$$\times \frac{1}{2}\left[\omega^{(2)}(t)-\left(\omega^{(1)}(t)\right)^2 \ln\left(\frac{\vec{q}^{\,2}}{s_0}\right)\right] . \tag{6}$$

The first condition was checked [9] in the part concerning the quark contribution. In the one-loop approximation (LLA) the trajectory

$$\omega^{(1)}(t) = \frac{g^2 N t}{2\,(2\pi)^{D-1}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}\vec{q}_1^{\,\prime\,2}} \tag{7}$$

is purely gluonic. The quark contribution appears at the two-loop level only and has the form

$$\omega_Q^{(2)}(t) = g^2 t \int \frac{d^{(D-2)}q_1}{(2\pi)^{D-1}} \frac{[F(\vec{q})-2F(\vec{q}_1)]}{\vec{q}_1^{\,2}\vec{q}_1^{\,\prime\,2}},$$

$$F(\vec{q}) = \frac{2g^2 N n_f \Gamma\left(2-\frac{D}{2}\right)\Gamma^2\left(\frac{D}{2}\right)}{(4\pi)^{\frac{D}{2}}\Gamma(D)\,(\vec{q}^{\,2})^{(2-\frac{D}{2})}} . \tag{8}$$

The quark contribution to the "real" part of the kernel in the gluon channel is [9]

$$\mathcal{K}r^{(8)Q}(\vec{q}_1,\vec{q}_2;\vec{q}) = \frac{g^4 n_f N}{(2\pi)^{D-1}} \frac{\Gamma(1-\epsilon)}{\epsilon(4\pi)^{2+\epsilon}} \frac{\Gamma^2(2+\epsilon)]}{\Gamma(4+2\epsilon)}$$

$$\times \left\{ \frac{(\vec{k}^2)^\epsilon}{\vec{k}^2}(\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}+\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}) + \vec{q}^{\,2}\left((\vec{q}^{\,2})^\epsilon\right.\right.$$

$$\left. -(\vec{q}_1^{\,2})^\epsilon + (\vec{q}_2^{\,2})^\epsilon\right) - \frac{(\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}-\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2})}{\vec{k}^2} \times$$

$$\left.\left((\vec{q}_1^{\,2})^\epsilon - (\vec{q}_2^{\,2})^\epsilon\right) + (\vec{q}_1 \leftrightarrow \vec{q}_1^{\,\prime}, \vec{q}_2 \leftrightarrow \vec{q}_2^{\,\prime})\right\} . \tag{9}$$

The kernel $\mathcal{K}^{(8)}(\vec{q}_1,\vec{q}_2;\vec{q})$, according to (3), is expressed through the trajectory and the "real" part. The above expressions give

$$\int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}\vec{q}_2^{\,\prime\,2}} \mathcal{K}^{(8)Q}(\vec{q}_1,\vec{q}_2;\vec{q}) = \frac{g^4 n_f N}{(2\pi)^{D-1}} \frac{\Gamma(-\epsilon)}{(4\pi)^{2+\epsilon}}$$

$$\times \frac{\Gamma^2(2+\epsilon)}{\Gamma(4+2\epsilon)} \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}} \left\{ \frac{\vec{q}_1^{\,2}}{\vec{k}^2}\left((\vec{q}_2^{\,2})^\epsilon + (\vec{q}_2^{\,\prime\,2})^\epsilon\right.\right.$$

$$\left. -(\vec{q}_1^{\,2})^\epsilon - (\vec{q}_1^{\,\prime\,2})^\epsilon\right) - \frac{\vec{q}^{\,2}}{\vec{q}_2^{\,\prime\,2}}\left((\vec{q}^{\,2})^\epsilon - (\vec{q}_1^{\,2})^\epsilon - (\vec{q}_2^{\,2})^\epsilon\right)$$

$$\left. + \ (\vec{q}_1 \longleftrightarrow \vec{q}_1^{\,\prime})\right\} , \tag{10}$$

where $\vec{k} = \vec{q}_1 - \vec{q}_2$. Putting the r.h.s. expression into (5) and using (7),(8), it is easy to verify that the "bootstrap" equation (5) is satisfied.

The gluon part of the condition (5) is now under consideration. The second bootstrap condition (6) is discussed by M.I. Kotsky (these Proceedings).

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