A model for parton distributions in hadrons

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The non-perturbative parton distributions in hadrons are derived from simple physical arguments resulting in an analytical expression for the valence parton distributions. The sea partons arise mainly from pions in hadronic fluctuations. The model gives new insights and a good description of structure function data.

Hard processes involving hadrons are calculated by folding perturbative QCD matrix elements with parton distributions describing the probability of finding a quark or a gluon in the hadron.

Perturbative QCD evolution describes the dependence of the parton distributions on the hard scale Q of the interaction. However, their dependence on the momentum fraction x at the lower limit for applying perturbative QCD, $Q_0 \approx 0.5-2$ GeV, are fitted to data using parameterisations, *e.g.* of the form [1–3]

$$f_i(x,Q_0) = N_i x^{a_i}(1-x)^{b_i}(1+c_i\sqrt{x}+d_i x) \quad (1)$$

The parameters in these functions have no direct physical meaning, making it difficult to interpret the results. To gain understanding of non-perturbative QCD we have developed a physical model [4] for the parton distributions at Q_0. This is here briefly described together with our latest developments.

The basic physical picture is that a probe with large resolution, compared to the hadron size, will see *free quarks and gluons* in quantum fluctuations of the hadron. The measuring time is short compared to the life-time of the fluctuation since the latter is determined by the confinement of quarks and gluons inside the hadron, as illustrated in Fig. 1. This makes it possible to describe the formation of the fluctuations independently of the measuring process.

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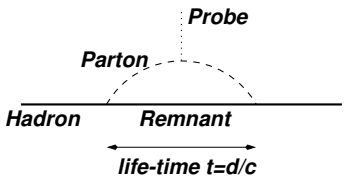

Figure 1. Fluctuation of a hadron into a parton and a remnant.

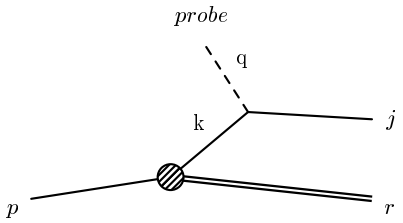

Figure 2. Kinematics for scattering on a parton in a hadron.

Our approach only intends to provide the four-momentum k of a single probed parton. All other information in the hadron wave function is neglected, treating the other partons collectively as a remnant with four-momentum r, see Fig. 2.

It is convenient to describe the process in the hadron rest frame where there is no preferred direction and hence spherical symmetry. The probability distribution for finding one parton is taken

as a Gaussian, which expressed in momentum space for a parton with four-momentum k and mass m_i is

$$f_i(k)dk = N(\sigma_i, m_i) e^{-\frac{(k_0 - m_i)^2 + k_1^2 + k_2^2 + k_3^2}{2\sigma_i^2}} dk, \quad (2)$$

where $\sigma = \frac{1}{d_h} \approx m_\pi$ is the inverse of the confinement length scale or hadron diameter.

The partonic structure is described using the light-cone momentum fraction $x = \frac{k_+}{p_+}$ which the parton has in the initial hadron. Since x is invariant under boosts in the z direction, the same will be true for the calculated parton distributions.

There are a number of constraints that must be fullfilled by the parton distributions. The normalisation for valence quarks is given by the sum rules

$$\int_0^1 f_i(x)dx = n_i, \quad (3)$$

and for the gluons by the momentum sum rule

$$\sum_i \int_0^1 x f_i(x)dx = 1. \quad (4)$$

There are also the kinematical constraints

$$m_i^2 \leq j^2 < W^2 \quad \text{and} \quad r^2 > \sum_i m_i^2 \quad (5)$$

given by the final partons being on-shell or time-like and the remnant having to include the remaining partons. These constraints also lead to $0 < x < 1$.

The parton model requires that W is well above the resonance region and that the resolution of the probe is much larger than the size of the hadron, *i.e.*

$$W \gg m_p \quad \text{and} \quad Q_0 \gg \sigma_i \quad (6)$$

The scale of the probe must also be large enough, $Q_0 \gg \Lambda_{QCD}$, for perturbative QCD to describe the evolution of the parton distributions from the starting scale Q_0.

In [4] we integrated Eq.(2) numerically to find the parton distributions since the kinematical constraints, Eqs.(5), are quite complicated in general. The problem is much simpler if the transverse momenta and the masses of the partons are neglected. It is then possible to derive [5] an analytical expression for the parton distributions,

$$f_i(x) = N'(\tilde{\sigma}_i) \exp\left(-\frac{x^2}{4\tilde{\sigma}_i^2}\right) \text{erf}\left(\frac{1-x}{2\tilde{\sigma}_i}\right), \quad (7)$$

where

$$\tilde{\sigma} = \frac{1}{d_h m_h} \approx \frac{m_\pi}{m_h}. \quad (8)$$

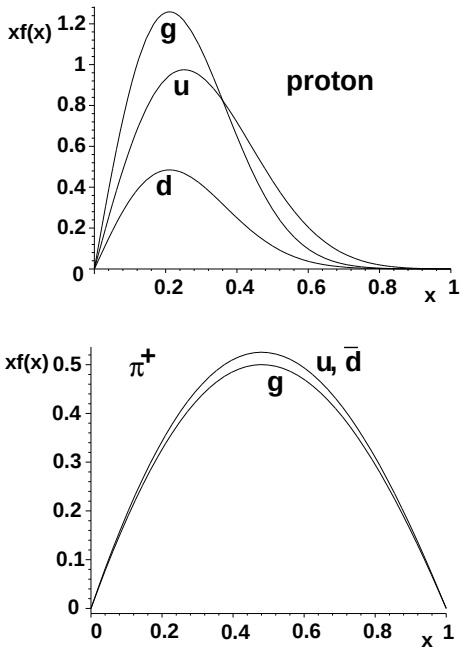

Figure 3. Valence distributions at $Q_0 \approx 0.85$ GeV in the proton and π^+.

The valence parton distributions for hadrons are here determined simply by the mass and size of the hadron! The resulting valence distributions for the proton ($\tilde{\sigma} \approx 0.15$) and the pion ($\tilde{\sigma} \approx 1$) are very reasonable as shown in Fig. 3. Note that the pion distributions are very similar to $xf(x) = 2x(1-x)$ and that one third of the pion momentum is carried by gluons.

The sea partons are described by hadronic fluctuations, *e.g.* for the proton $|p\pi^0\rangle + |n\pi^+\rangle + ...$, where the probe measures a valence parton in one of the two hadrons. The momentum distribution of pions in a hadronic fluctuation is assumed to

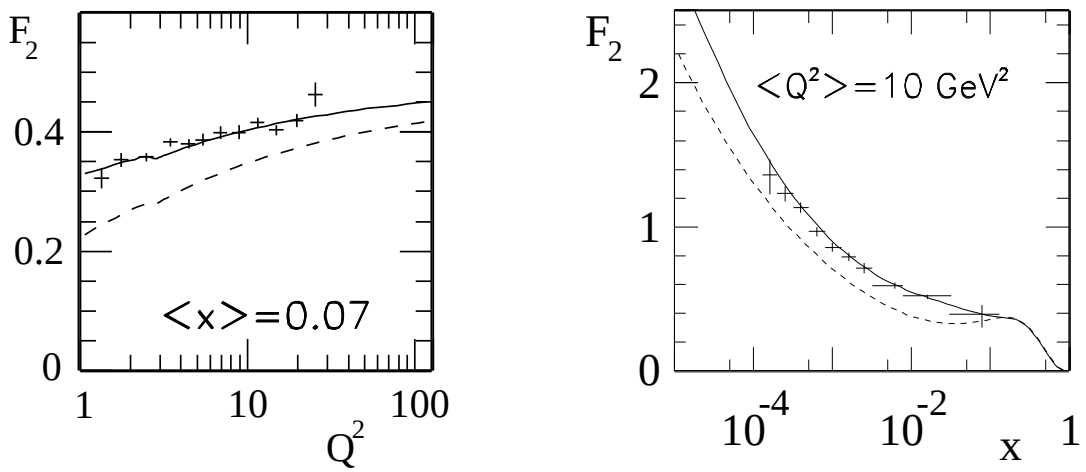

Figure 4. Data from NMC [6] and ZEUS [7] on the proton structure function $F_2(x, Q^2)$ compared to the model with valence partons only (dashed) and including sea partons (full).

follow from the same model as for the valence partons, with the differences that the mass can not be neglected and the width $\sigma_\pi \approx 50$ MeV is smaller, related to the longer range of pionic strong interactions.

Using these valence and sea parton x distributions at $Q_0 = 0.85$ GeV, next-to-leading order DGLAP evolution in the CTEQ program [1] was applied to obtain the parton distributions at larger Q. The proton structure function $F_2(x, Q^2)$ can then be calculated and compared with deep inelastic scattering data, as illustrated in Fig. 4 and detailed in [4]. The model does remarkably well, in view of its simplicity and few parameters (6). Of course, conventional parton density parameterisations give much better fits, probably mainly due to their many more parameters (≈ 20).

The main part of the proton structure is determined by the valence distributions, but the sea gives an important contribution at small x, as can also be seen from the comparison with the measured F_2.

Including all pion fluctuations will give a flavour asymmetric sea with $\bar{d} > \bar{u}$ as also observed experimentally [8], but the numerical details remain to be investigated.

The model predicts the valence parton distributions for all hadrons, but heavy quarks give more complicated analytical expressions [5]. Numerical results on strange and charmed mesons are shown in [4]. In addition, a study based on Monte Carlo has been made [9] to investigate intrinsic strange and charm quarks in the proton.

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