Higher-twist corrections to nucleon structure functions from lattice QCD *

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A genuinely non-perturbative evaluation of higher-twist contributions to the structure functions of the nucleon, with all mixing effects and renormalon ambiguities taken care of, is presented. Higher-twist corrections turn out to be significant at moderate values of q^2.

1. INTRODUCTION

The calculation of power corrections to the deep-inelastic structure functions of the nucleon is a long-standing problem. Recent phenomenological studies [1] have shown that they account for more than 10% of the lower moments of F_2 at $q^2 = 5$ GeV^2. For the theoretical understanding of the structure functions, in particular their evolution as a function of q^2, it is thus important to gain quantitative control of these effects.

The theoretical basis for the calculation is the operator product expansion (OPE):

$$W_{\mu\nu} \equiv \langle p|J_\mu(q)J_\nu(-q)|p\rangle = \sum_{m,n} C^m_{\mu\nu,\mu_1\cdots\mu_n}(\mu^{-1})\,\langle p|\mathcal{O}^m_{\mu_1\cdots\mu_n}(\mu^{-1})|p\rangle, \quad (1)$$

where m distinguishes different operators with the same Lorentz symmetries. The Wilson coefficients C are universal and depend only on q and the renormalization point μ, while all dependence on the target momenta is contained in the operator matrix elements. The product of Wilson coefficients and matrix elements does not depend on μ, nor on the renormalization scheme.

Besides the familiar operators of twist two, such as

$$\mathcal{O}_{\mu_1\cdots\mu_n} = \bar\psi\gamma_{\{\mu_1}D_{\mu_2}\cdots D_{\mu_n\}}\psi - \text{traces},$$

the OPE (1) receives contributions from twist-four and higher operators, which give rise to power corrections, and which so far have been neglected in the theoretical analysis. A typical twist-four operator is

$$\mathcal{O}_{\mu_1\cdots\mu_n} = \bar\psi\gamma_{\mu_1}D_{\mu_2}\cdots D_\lambda D_\lambda\cdots D_{\mu_n}\psi.$$

In addition to the calculation of matrix elements of higher-twist operators, the evaluation of power corrections requires a non-perturbative computation of the Wilson coefficients. This is necessary because of renormalon ambiguities and mixing effects, which tie Wilson coefficients and higher-twist matrix elements together [2].

In [3] we have developed a method that allows a lattice calculation of the Wilson coefficients. With this method we can compute the r.h.s. of (1) entirely on the lattice:

$$\sum_{m,n} C^m_{\mu\nu,\mu_1\cdots\mu_n}(a)\,\langle p|\mathcal{O}^m_{\mu_1\cdots\mu_n}(a)|p\rangle,$$

$a = \mu^{-1}$ being the lattice cut-off, without having to introduce an intermediate (renormalization) scale parameter. This approach automatically takes care of all mixing effects, and it avoids renormalon ambiguities altogether.

*Talk given by G. Schierholz.

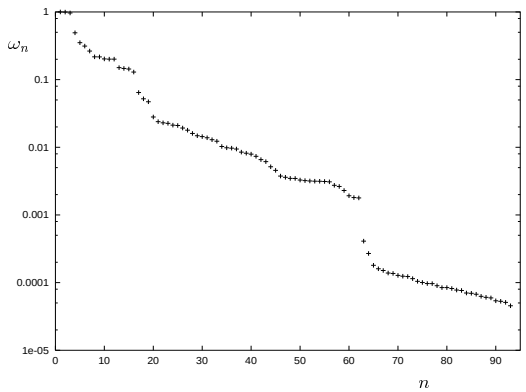

Figure 1. The eigenvalues ω_n against n.

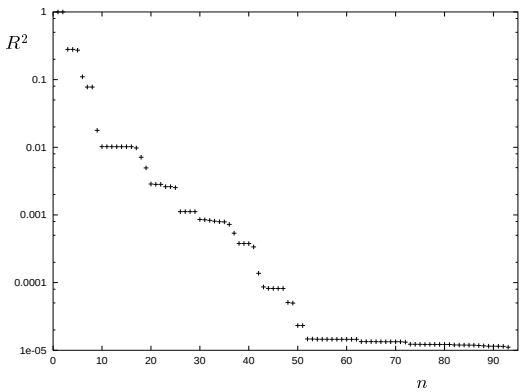

Figure 2. The residual error R^2 against n.

In this talk we shall present first results of an all-lattice calculation of higher-twist corrections to the structure function of the nucleon.

2. WILSON COEFFICIENTS

Let us concentrate on the Wilson coefficients first. The basic idea [3] is to compute $W_{\mu\nu}$, and the operator matrix elements in the OPE going with it, for off-shell quark states in Landau gauge, and to extract the Wilson coefficients from this information. Because the quarks are off-shell (and euclidean), we are far from the Bjorken limit, so that higher-twist operators are not suppressed.

We consider a system of quark states with momenta p_m, $m = 1, \cdots, M$. The photon momentum q is held fixed. We drop the Lorentz indices, and we label the operators and the associated Wilson coefficients by $n = 1, \cdots, N$. The matrix elements of the operators between states of momentum p_m are denoted by $O_n^{p_m}$, and the corresponding current matrix elements are called W^{p_m}. Note that both, $O_n^{p_m}$ and W^{p_m} are 4×4 matrices. The problem is then to solve the $N \times (M \times 16)$ system of equations

$$\begin{pmatrix} O_1^{p_1} & \cdots & O_N^{p_1} \\ \vdots & & \vdots \\ O_1^{p_M} & \cdots & O_N^{p_M} \end{pmatrix} \begin{pmatrix} C_1 \\ \vdots \\ C_N \end{pmatrix} = \begin{pmatrix} W^{p_1} \\ \vdots \\ W^{p_M} \end{pmatrix} \quad (2)$$

for the Wilson coefficients C_n. In the following we take into account all quark-bilinear operators with up to three covariant derivatives.

Generally, not all operators are independent, so that the system is overdetermined. Let us write (2) as

$$O\,C = W. \quad (3)$$

To solve (3), we apply a singular value decomposition, which is the standard method for solving overdetermined sets of equations [4]. Writing

$$O = U\,\omega\,V^T,$$

where U (V) is a column-orthonormal $(16 \times M) \times N$ ($N \times N$) matrix, and $\omega = \mathrm{diag}(\omega_n)$ with positive real eigenvalues ω_n arranged in descending order, the solution to (3) is

$$C = V\,\mathrm{diag}(1/\omega_n)\,U^T\,W.$$

In a calculation with infinite precision one would find $\omega_n = 0$ for any n exceeding the number of independent operators, and in these instances one would replace $1/\omega_n$ by zero. In a calculation like ours we can at most hope for a sharp drop of the ω's down to the level of the noise.

In total there are 1360 operators to be considered, which by symmetry reduce to 93. We have considered 70 different momenta, thus resulting in a 93×1120 system of equations. The calculations are done on a 32^4 lattice at $\beta = 6$ using Wilson fermions. In Fig. 1 we show a typical picture of the eigenvalues ω_n. We see a sharp drop at $n \approx 63$. In Fig. 2 we show the residual error $R^2 = |W - OC|^2/|W|^2$ as a function of n with $1/\omega_{n+1}, \cdots, 1/\omega_N$ set to zero. We see that the

error does not decrease significantly anymore for $n \gtrsim 63$, indicating that we have 'hit the noise', and that we may truncate the system at this value of n, what we will do.

3. STRUCTURE FUNCTION RESULTS

The next step is to compute the nucleon matrix elements of the operators entering the OPE (1). This, combined with the Wilson coefficients evaluated in the last section, then gives us the desired hadronic tensor $W_{\mu\nu}$ of the nucleon. For this calculation it is sufficient to consider a few different nucleon momenta only.

From $W_{\mu\nu}$ we can derive moments of the nucleon structure functions. Here we shall restrict ourselves to the lowest non-trivial moment of the unpolarized structure function. Projection onto spin two states gives [5]

$$\int \mathrm{d}\theta \sin^2\theta\, C_2^1(\cos\theta) W_{\lambda\lambda} \propto \int \mathrm{d}x\, (2xF_1 - F_L/2), \quad (4)$$

where $C_2^1(\cos\theta)$ is a Gegenbauer polynomial, and $\cos\theta = pq/|p||q|$. Similar results are obtained for other combinations of μ, ν.

The nucleon matrix elements are computed on a smaller, $16^3 32$ lattice (at the quoted value of β), and the combined results are extrapolated to the chiral limit. The calculations are done in the quenched approximation, so that our numbers refer to non-singlet structure functions.

We denote the r.h.s. of (4) by $\mathcal{M}_2(q^2)$. In the language of the parton model this corresponds to the moment $\langle x \rangle$. We write

$$\mathcal{M}_2(q^2) = m_2 + c_2/q^2,$$

separating the power correction from the leading, logarithmically varying contribution. This makes sense, because we can identify those higher-twist contributions which are logarithmically behaved, and those which are power behaved.

We present our results in Fig. 3. All our numbers should be regarded as preliminary. The first observation we make is that power corrections are large. They are positive, and at $q^2 = 5\,\mathrm{GeV}^2$ they amount to $\approx 30\%$ of the total contribution. The second, and equally important observation is that there is strong mixing between operators of twist two and four. This effect reduces the leading contribution (m_2) substantially, if compared with the result of our previous calculation [6] based on perturbative Wilson coefficients and renormalization constants. We now find much better agreement of this quantity with the phenomenological valence quark distribution functions.

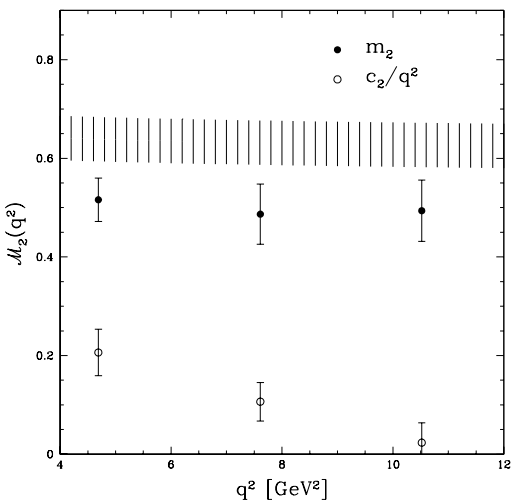

Figure 3. $\mathcal{M}_2(q^2)$ for the proton as a function of q^2. The hatched strip shows our old result [6] (where the width of the strip indicates the error) without higher-twist effects included.

REFERENCES

1. S. Liuti, Virginia preprint INPP-UVA-98-06 (1998) (`hep-ph/9809248`).
2. G. Martinelli and C. T. Sachrajda, Nucl. Phys. **B478** (1996) 660, and references therein.
3. S. Capitani, M. Göckeler, R. Horsley, H. Oelrich, P. Rakow, D. Petters and G. Schierholz, DESY preprint DESY 98-147 (1998) (`hep-lat/9809171`).
4. W. H. Press, S. A. Teukolsky, W. T. Vetterling and B. P. Flannery, *Numerical Recipes* (Cambridge University Press, 1986).
5. O. Nachtmann, Nucl. Phys. **B63** (1973) 237.
6. M. Göckeler, R. Horsley, E.-M. Ilgenfritz, H. Perlt, P. Rakow, G. Schierholz and A. Schiller, Phys. Rev. **D53** (1996) 2317.