Overview of variable flavour number schemes

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We discuss problems with the construction of variable flavour number schemes.

Charm quark electroproduction is used to extract the gluon density $g(x,\mu^2)$ of the proton in deep inelastic lepton-hadron scattering. This method assumes that the charm component in the proton wave function is negligible (i.e., only extrinsic production). The Born approximation to $F_{2,c}(x,Q^2,m_c^2)$ (the charm component of the total $F_2(x,Q^2)$), is then given by the virtual-photon-gluon-fusion process. Extrinsic charm production also receives contributions from virtual-photon-quark subprocesses in next-to-leading order (NLO), but the HERA experiments [1] [2] are carried out at small x where the light flavour densities are very small compared to the gluon density so these contributions are small.

Two descriptions of extrinsic charm production have been used. In the first one the charm quark is massive ($m_c \approx 1.4$ GeV$/c^2$) and the partonic cross sections or coefficient functions are described by NLO perturbation theory [3]. The input parton densities are defined in the three-light-flavor scheme. This is a completely consistent picture at low scales and, after incorporating a fragmentation function for the $c \to D$-meson transition (and a possible string-drag effect), fits the exclusive distributions measured in the H1 [4] and ZEUS [5] experiments rather well. The second approach treats the charm quark as massless so that it can be represented by a parton density $c(x,\mu^2)$, with the boundary condition $c(x,\mu^2) = 0$ for $\mu \le m_c$. The two approaches are intimately related as was shown in [6] where the large logarithms of the type $\ln^i(Q^2/m_c^2)$, which appear in the fixed order perturbation series when $Q^2 \gg m_c^2$, were incorporated into new four flavor parton densities (including the calculable $c(x,\mu^2)$). The result is that $F_{2,c}(x,Q^2,m_c^2)$, which in the first approach is written as convolutions of three light parton densities with heavy quark coefficient functions, becomes, after resummation, as convolutions of four light parton densities with light parton coefficient functions. It is clear that the first approach is better when the charm quark pair is produced near threshold ($\hat{s} = Q^2(1-x)/x \ge 4m_c^2$) where m_c cannot be neglected. On the other hand far away from threshold, where $Q^2 \gg m_c^2$, the large logarithms mentioned above dominate the structure function so that the second approach becomes more appropriate. Both are characterized by the number of active flavours involved in the description of the parton densities which are available for fixed flavour schemes in [7], [8] and [9]. As most of the experimental data occur in the intermediate kinematical region a third approach, called the variable flavour number scheme (VFNS), has been introduced to describe $F_{2,c}(x,Q^2,m^2)$. A VFNS prescription was given in lowest order pQCD in [10]. A proof of factorization to all orders was recently given in [11]. A different VFNS approach, also generalized to all orders, was given in [12]. Finally a third version of a VFNS scheme was presented in [13]. Previous reviews of these schemes are available in [14]. The differences between the three versions can be attributed to several ingredients entering their construction. In any VFNS approach one needs two sets of parton densities, namely three flavour densities (which are obtained from fits to experimental data) and four flavour densities which are calculated from the former. Both parametrizations should sat-

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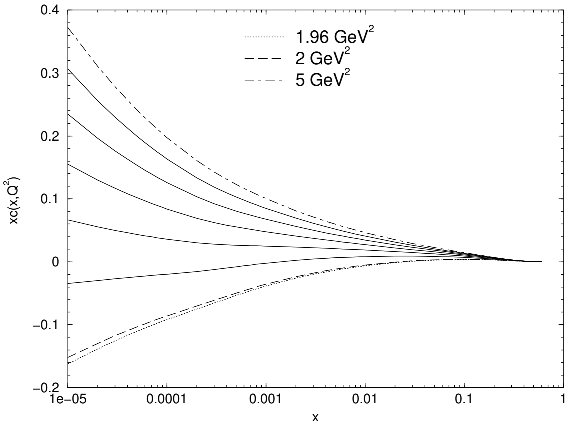

Figure 1. $c(x,Q^2)$ versus x for fixed Q^2.

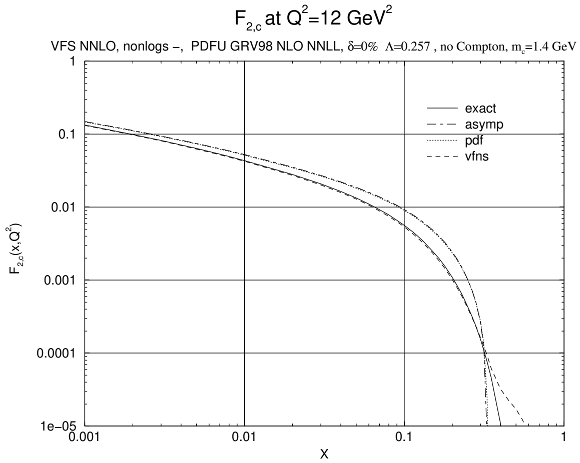

Figure 2. $F_{2,c}(x,Q^2)$ versus x for fixed Q^2.

isfy the matching relations across flavour thresholds derived in [6]. The recent CTEQ5HQ [8] and MRST98 [9] parton densities incorporate VFNS schemes [10] and [13] respectively. However the reason they obtain different charm densities is due to their choice of different data to fit their gluon densities.

The first problem in any VFNS scheme is how to impose the matching condition on $c(x,\mu^2)$, which should vanish at a low scale because it is not present in the three flavour scheme. In lowest order the relevant operator matrix element is proportional to $\ln\mu^2/m_c^2$, which is zero at $\mu^2 = m_c^2$ so one can start the charm density at this point and subtract a term from the $g(x,\mu^2)$ to maintain the momentum sum rule for four flavours when $\mu^2 > m_c^2$. In [13] a more complicated matching condition involving the derivatives of the structure functions is imposed at this scale. However this matching condition spoils the relation between the coefficient functions and the partonic reactions.

In higher order pQCD the matching conditions across flavour thresholds involve non-logarithmic terms [6]. This implies that there is no scale μ^2 below which one can set $c(x,\mu^2) = 0$ for all x. If one continues to start the charm density at $\mu^2 = m_c^2$ then there is an x dependent boundary condition at that point. (An alternative procedure would be to remove the non-logarithmic terms from $c(x,\mu^2)$ and absorb them into the massless quark coefficient functions at the cost of defining a different mass factorization scheme for the latter. This approach was used in [12]). We have recently constructed in [15] a new four flavor density set satisfying the two-loop matching conditions where $c(x,\mu^2) \neq 0$ at $\mu = m_c$.

Next one must examine the mass factorization procedure to be carried out before the large logarithms in Q^2/m_c^2 can be resummed. The problem is that the mass factorization cannot be carried out on the level of $F_{2,c}(x,Q^2,m_c^2)$ alone, because one also needs contributions from the light parton components of the structure functions. The latter contain the heavy charm quark loop contributions to the gluon self energies, which appear in the virtual corrections to the light parton coefficient functions. These corrections have to be combined with contributions from gluon splitting into heavy $c\bar{c}$ pairs, which belong to the charm components (not the light quark components) of the structure functions. This problem was solved in [12]. Note that there is never any difficulty in taking $m_c \to 0$ in the totally inclusive quantity $F_2(x,Q^2)$. However, in the massless charm approach one has to be careful to define the charm contribution $F_{2,c}(x,Q^2,m_c^2)$ in an infra-red safe way. This is discussed in detail in [15].

In Fig.1 we plot the $xc(x,\mu^2 = Q^2)$ from [15] for Q^2 =1.96,2,2.5,3,3.5,4,4.5 and 5 (GeV/c)^2. To obtain this density we started the QCD evolution of the three flavour parton densities at the low scale used in [7] and using the NLO splitting functions evolved them up to scale $\mu^2 = m_c^2 = 1.96$ GeV^2. At this value we added discontinuities in all the parton densities from the matching conditions. One can see that $c(x,\mu^2)$ starts from negative values at small x so it cannot increase as fast as in the LO case where it started from zero for all x or as fast as the NLO case provided $c(x,\mu^2)$ is presented in the $\overline{\mathrm{MS}}$ scheme. The charm density resembles that from the MRST98 (central gluon) set [9], which does not increase as fast as that in CTEQ5HQ set [8].

One consequence of the different charm density is that the gluon evolution in μ^2 is now steeper than before so that the four flavour momentum sum rule is satisfied for $\mu^2 > m_c^2$. We have evolved all the new four flavor densities from $\mu^2 = m_c^2$ to larger scales again using the NLO splitting functions. This analysis assumes that the unknown NNLO splitting functions can be neglected.

The values of $F_{2,c}(x,Q^2,m_c^2)$ in VFNS schemes depend on the magnitudes of the four flavour densities. The figure 2 shows the contributions to the BMSN version of a VFNS scheme [12] at a scale $Q^2 = 12$ (GeV$/c)^2$, where $F_2^{\rm vfns}(4) = F_2^{\rm exact}(3) + F_2^{\rm asypm}(3) - F_2^{\rm pdf}(4)$. We have used the scale where $\mu^2 = m_c^2$ for $Q^2 < m_c^2$ and $\mu^2 = m_c^2 + Q^2(1 - m_c^2/Q^2)^2/2$ for $Q^2 > m_c^2$.

It is interesting to compare this plot with the corresponding one in [12], where we did not implement the non-logarithmic terms in the parton densities but put them in the coefficient functions. The $F_{2,c}(x,Q^2,m_c^2)$ agree very well with each other at small x demonstrating the invariance of $F_{2,c}(x,Q^2,m_c^2)$ under the change in the mass factorization scheme. At large x (near threshold) there are small differences due to the inclusion of even higher order terms in α_s. As one decreases Q^2 there is a dramatic fall-off in both $F_{2,c}^{\rm asymp}(3)$ and $F_{2,c}^{\rm pdf}(4)$, but the difference between them stays very small. Further plots for both $F_{2,c}(x,Q^2,m_c^2)$ and $F_{2,L}(x,Q^2,m_c^2)$ are presented in [15] together with a more detailed discussion of the threshold region.

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