Structure function evolution at next-to-leading order and beyond

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Results are presented of two studies addressing the scaling violations of deep-inelastic structure functions. Factorization-scheme independent fits to all ep and μp data on F_2 are performed at next-to-leading order (NLO), yielding $\alpha_s(M_Z) = 0.114 \pm 0.002_{\rm exp}\,(+0.006-0.004)_{\rm th}$. In order to reduce the theoretical error dominated by the renormalization-scale dependence, the next-higher order (NNLO) needs to be included. For the flavour non-singlet sector, it is shown that available calculations provide sufficient information for this purpose at $x > 10^{-2}$.

1. INTRODUCTION

One of the important objectives of studying structure functions in deep-inelastic scattering (DIS) is a precise determination of the QCD scale parameter Λ (i.e., the strong coupling α_s) from their scaling violations. In this talk we briefly present results of two studies [1,2] aiming at an improved control and a reduction of the corresponding theoretical uncertainties.

2. FLAVOUR-SINGLET EVOLUTION IN NLO [1]

The evolution of structure functions is usually studied in terms of scale-dependent parton densities and coefficient functions. In this case the predictions of perturbative QCD are affected by two unphysical scales: the renormalization scale μ_r and the mass-factorization scale μ_f. While the former is unavoidable, the latter can be eliminated by recasting the evolution equations in terms of observables [3]. In the flavour-singlet sector, this procedure results in

$$\frac{d}{d\ln Q^2}\begin{pmatrix} F_2 \\ F_B \end{pmatrix} = \mathcal{P}\left(\alpha_s(\mu_r), \frac{\mu_r^2}{Q^2}\right) \otimes \begin{pmatrix} F_2 \\ F_B \end{pmatrix} \qquad (1)$$

with $F_B = dF_2/d\ln Q^2$ or $F_B = F_L$. The kernels $\mathcal{P}$ are combinations of splitting functions and coefficient functions which become prohibitively complicated in Bjorken-x space at NLO. Thus Eqs. (1) are most conveniently treated using modern complex Mellin-moment techniques [4].

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We have performed leading-twist NLO fits to the F_2^p data of SLAC, BCDMS, NMC, H1, and ZEUS. Statistical and systematic errors have been added quadratically, the normalization uncertainties have been taken into account separately. The singlet/non-singlet decomposition has been constrained by the F_2^n/F_2^p data of NMC. The initial shapes $F_{2,B}(x,Q_0^2)$ are expressed via standard parametrizations for parton densities at $\mu_f = Q_0$.

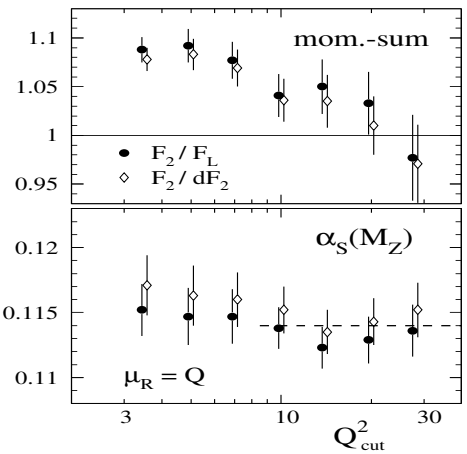

Figure 1. The dependence of the fit results for the energy-momentum sum and for $\alpha_s(M_Z)$ on the Q^2-cut imposed in addition to $W^2 > 10$ GeV^2.

In order to establish the kinematic region which can be safely used for fits of α_s in the leading-twist NLO framework, the lower Q^2-cut applied to the data has been varied between 3 and 30 GeV^2. When the normalized momentum sum of

the partons defining the $F_{2,B}$ initial distributions is left free, the fits with $Q^2_{\rm cut} < 10$ GeV^2 prefer values significantly different from unity, see Fig. 1. Also shown in this figure is the $Q^2_{\rm cut}$-dependence of the fitted values for $\alpha_s(M_Z)$, now imposing the momentum sum rule. The results for $Q^2_{\rm cut} \leq 7$ GeV^2 tend to lie above the $Q^2_{\rm cut} \geq 10$ GeV^2 average of $\alpha_s(M_Z) = 0.114$ (dashed line).

In Fig. 2 we display the renormalization scale dependence of the $\alpha_s(M_Z)$ central values for the safe choice $Q^2_{\rm cut} = 10$ GeV^2. The conventional, but somewhat ad hoc, prescription of estimating the theoretical error by the variation due to $0.25 \leq \mu_r^2 \,/\, Q^2 \leq 4$ results in

$$\alpha_s(M_Z) = 0.114 \pm 0.002\,{}_{\rm exp}\,{}^{+\,0.006}_{-\,0.004}\Big|_{\rm scale} \quad . \qquad (2)$$

Other theoretical uncertainties are considerable smaller and can be neglected at this point. The uncertainty due to possible higher-twist contributions, for instance, can be estimated at about 1% via the target-mass effects included in the fits.

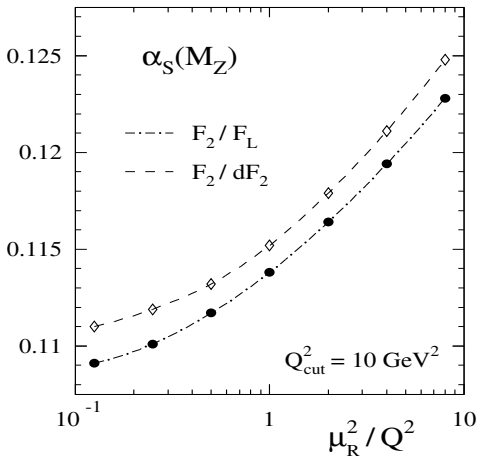

Figure 2. The dependence of the optimal values for $\alpha_s(M_Z)$ on the renormalization scale μ_r.

3. NON-SINGLET EVOLUTION IN NNLO [2]

The theoretical error in Eq. (2) clearly calls for NNLO analyses. The necessary contributions to the β-function [5] and the coefficient functions [6] are known. However, only partial results are available for the 3-loop terms $P^{(2)}(x)$ in the splitting-function expansion ($a_s \equiv \alpha_s/4\pi$)

$$P = a_s P^{(0)} + a_s^2 P^{(1)} + a_s^3 P^{(2)} + \ldots \quad . \qquad (3)$$

For the non-singlet part of F_2 considered here (NS$^+$), present information comprises the lowest five even-integer moments [7], the full N_f^2 piece [8], and the most singular small-x term [9].

We have performed a systematic study of the constraints imposed on $P_{\rm NS}^{(2)+}(x)$ by these results. Four approximations spanning the current uncertainty range are shown in Fig. 3, together with their convolutions with a typical input shape.

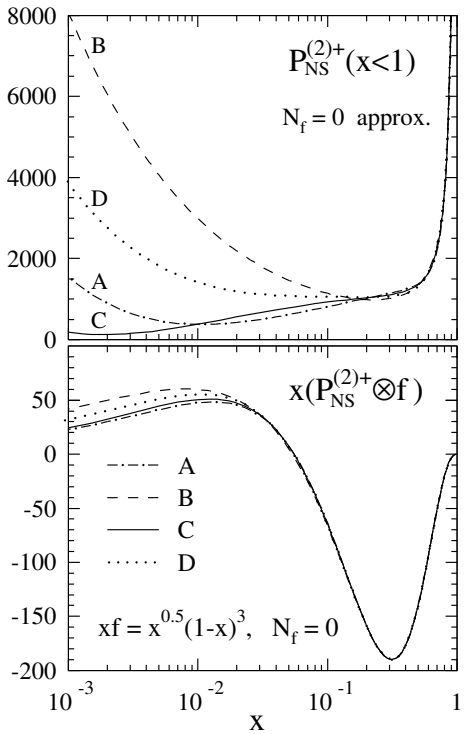

Figure 3. Representative approximate results for the flavour-number independent part of the 3-loop non-singlet$^+$ $\overline{\rm MS}$ splitting function.

$P_{\rm NS}^{(2)+}(x)$ is well determined at $x \geq 0.15$, with a total spread of about 15% at $x \simeq 0.3$. At (non-asymptotically) small x its behaviour is rather unconstrained despite the known leading $x \to 0$ contribution. As the splitting functions enter

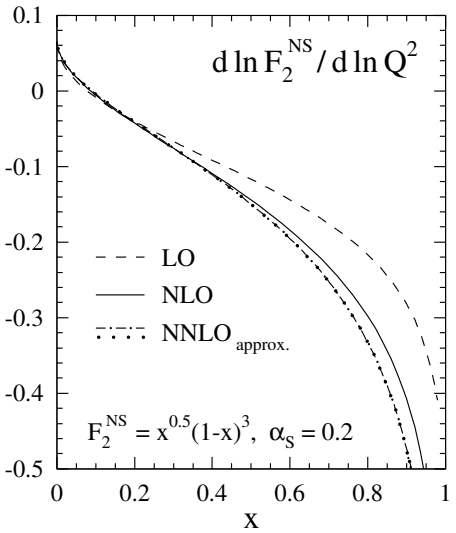

Figure 4. The first three steps in the expansion of the scaling violations of the non-singlet component of F_2 for typical input parameters.

scaling violations always via convolutions

$$(P \otimes f)(x) = \int_x^1 dy/y \; P(x/y)\, f(y) \qquad (4)$$

with smooth initial distributions $f(x)$, the residual uncertainties are much reduced for observables over the full x-range. In the present case they prove to be negligible at $x > 0.02$.

The net effect of the NNLO correction is finally illustrated in Fig. 4, where the scale-derivative of $F_2^{\rm NS}$ is shown for $\mu_r = Q$ and $N_f = 4$, using an α_s-value typical for the fixed-target region. The inclusion of this correction into fits is expected to lead to a slightly lower central value for α_s and a considerably reduced theoretical uncertainty.

4. SUMMARY AND OUTLOOK

We have analyzed present $ep/\mu p$ F_2-data in a factorization-scheme independent framework [1]. We find that $Q^2, W^2 > 10$ GeV^2 is a safe region for leading-twist NLO fits of α_s. Our central value is close to that of the standard pre-HERA analysis in [10], but lower than the recent result of [11] using a lower Q^2-cut of 2 GeV^2. The irreducible renormalization-scale uncertainty turns out to be larger than expected from [10].

We have derived approximate x-space expressions for the 3-loop non-singlet splitting functions $P_{\rm NS}^{(2)}$, including error estimates [2]. This approach is complementary to, but more flexible than, the integer-moment procedures pursued in [12,13]. The remaining uncertainties of $P_{\rm NS}^{(2)}$ are small for the evolution at $x > 10^{-2}$, thus allowing for detailed NNLO analyses in this region. An extension to the singlet case is in preparation.

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