The NNLO QCD analysis of the CCFR data for xF_3 : is there still the room for the twist-4 terms ?

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The results of the more detailed NNLO QCD analysis of the CCFR data for xF_3 SF are presented. The factorization scale uncertainties are analyzed. The NNLO results for $\alpha_s(M_Z)$ and twist-4 contributions are obtained. Despite the fact that the amplitude of the x-shape of the twist-4 factor is consequently decreasing at the NLO and NNLO, our new QCD analysis seems to reveal the remaining twist-4 structure at the NNLO level. The definite N^{3}LO uncertainties are fixed using the [0/2] Padé resummation technique.

It is known that the CCFR collaboration provided not long ago rather precise experimental data for xF_3 SF of νN DIS and extracted the value of $\alpha_s(M_Z)$ using the NLO DGLAP analysis[1]. In its process the twist-4 contributions were taken into account using the infrared renormalon (IRR) model of Ref.[2] with its parameter, fixed as 1/2 of the originally proposed one.

In the series of the subsequent papers[3]-[5] we concentrated on the attempts to fit the CCFR data at the NNLO level with the help of the Jacobi polynomial-Mellin moments version of the DGLAP method [6]- [8], based on the following equation:

$$xF_3^{N_{max}} = x^\alpha (1-x)^\beta \sum_{n=0}^{N_{max}} \Theta_n^{\alpha,\beta}(x) \times \qquad (1)$$
$$\sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) M_{j+2}(Q^2) + \frac{h(x)}{Q^2}$$

where $h(x)$ is the twist-4 contribution.

We used the results of calculations of the NNLO corrections to the coefficient functions[9] and the analytical expressions for the NNLO corrections to the anomalous dimensions of the non-singlet moments with $n = 2, 4, 6, 8, 10$ [10], supplemented with the given in Ref.[3] $n = 3, 5, 7, 9$ similar numbers, obtained using the smooth interpolation procedure of Ref.[11].

Using the fits with free Jacobi polynomial parameters α, β, we found that their values $\alpha \approx 0.7$, $\beta \approx 3$ correspond to the minimum in the plane (α, β). The form of $h(x)$ was fixed (1) through the IRR model of Ref.[2] with its coefficient $A_2^{'}$ considered as the free parameter and (2) as the function, modeled by free parameters $h_i = h(x_i)$, where x_i are the points of the experimental data bining. The QCD evolution of the moments has the following form

$$\frac{M_n(Q^2)}{M_n(Q_0^2)} = \left(\frac{A_s(Q^2)}{A_s(Q_0^2)}\right)^{\frac{\gamma_{NS}^{(0)}}{2\beta_0}} \frac{AD(n,Q^2)C^{(n)}(Q^2)}{AD(n,Q_0^2)C^{(n)}(Q_0^2)} \qquad (2)$$

where $A_s(Q^2) = \alpha_s(Q^2)/(4\pi)$ is the $\overline{MS}$-scheme expansion parameter, $AD(n,Q^2) = 1 + p(n)A_s(Q^2) + q(n)A_s(Q^2)^2 + ...$ comes from the

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Table 1
The Q_0^2 dependence of $\Lambda^{(4)}_{\overline{MS}}$ [MeV]. LO^* means that in the LO-fits NLO α_s is used; NLO^* ($NNLO^*$) indicates that in the NLO ($NNLO$) fits $NNLO$ (N^3LO) α_s is used. [0/2] marks the results of the N^3LO expanded [0/2] Padé fits with α_s defined at the N^3LO.

$Q_0^2(GeV^2)$	5	8	10	20	50	100
LO	266 ±35	266±35	265±34	264±35	264±36	263±36
LO^*	382 ±38	380±41	380±40	379±46	378±43	377±42
NLO	341 ±30	340±40	340±35	339±36	337±34	337±37
NLO^*	322 ±29	321±33	321±33	320±34	319±36	318±36
$NNLO$	293 ±30	312±33	318±33	326±35	326±36	325±36
$NNLO^*$	284 ±28	303±31	308±32	316±33	316±33	315±34
[0/2]	293 ±29	323±32	330±35	335±37	326±36	319±35

expansion of anomalous dimension, $C^{(n)}(Q^2) = 1 + C^{(1)}(n)A_s(Q^2) + C^{(2)}(n)A_s(Q^2)^2 + ...$ is the coefficient function of $M_n(Q^2)$ and $M_n(Q_0^2)$ is parametrized at the factorization scale Q_0^2. In the case of $f = 4$ numbers of flavours the numerical values of $p(n)$, $q(n)$, $C^{(1)}(n)$ and $C^{(2)}(n)$ are given in Ref.[5]. We will use the expansion of α_s through the powers of $1/ln(Q^2/\Lambda^2_{\overline{MS}})$ in the LO, NLO, NNLO and N^{3}LO, which contains the 4-loop term of the QCD β-function [12].

Here we complete previous analysis of the CCFR data of Refs.[4,5], performed in the case of $Q_0^2 = 5\ GeV^2$, by varying Q_0 in the wide region. The fits were done for the CCFR data, cutten at $Q^2 > 5\ GeV^2$, without twist-4 effects, but with target mass corrections included. The results for $\Lambda^{(4)}_{\overline{MS}}$ are given in Table 1 for different Q_0^2. The LO and NLO results are stable to the choice of Q_0^2. The results of the LO^* fits are higher than the LO ones, and from this level taking into account of other perturbative QCD effects are decreasing the values of $\Lambda^{(4)}_{\overline{MS}}$. The NNLO results are sensitive to the variation of Q_0. The values of $\Lambda^{(4)}_{\overline{MS}}$ become stable for $Q^2 \geq 20\ GeV^2$ only. The same effect is manifesting itself for the results of the N^3LO fits. This effects can be related to the peculiar behavior of the NNLO perturbative QCD expansion of M_2. Using the numerical values of $p(2)$, $q(2)$, $C^{(1)}(2)$ and $C^{(2)}(2)$, given in Ref.[5], we obtain

$$AD(2,Q^2)C^{(2)}(Q^2) = 1 - 0.132A_s(Q^2) \quad (3)$$
$$-46.155A_s(Q^2)^2 + ...$$

Thus, it is safer to start the evolution from $Q_0^2 = 20\ GeV^2$, where the numerical value of the A_s^2 contribution in Eq.(3) is smaller.

The results of our new fits of the CCFR data with twist-4 contributions, fixed through the IRR model of Ref.[2], are presented in Table 2.

Table 2
The results for $\Lambda^{(4)}_{\overline{MS}}$ and the IRR coefficient A_2' [GeV^2] from the CCFR'97 data, cutten at $Q^2 > 5\ GeV^2$ in different orders for Q_0^2=20 GeV^2.

	$\Lambda^{(4)}_{\overline{MS}}$ (MeV)	A_2'(HT)	χ^2/points
LO	433±52	-0.33±0.06	82.8/86
NLO	369±45	-0.12±0.06	81.8/86
NNLO	326±35	-0.01±0.05	76.9/86
N^{3}LO	340±37	-0.04±0.05	77.2/86

When the twist-4 parameters are not taken into account, the effects of the NNLO corrections are smaller than in the case of our previous analysis of Refs.[4,5] (see Table 1). However, they are still sizable in the case when twist-4 contributions are fixed through the IRR model. They have the tendency to make the value of $A_2^{'}$ comparable with zero. As the result, the NNLO value of $\Lambda^{(4)}_{\overline{MS}}$ is the same in the cases of both neglecting and retaining twist-4 terms.

At Fig.1 we present the extraction of the x-shape of the twist-4 terms from the LO, NLO, NNLO and expanded Padé fits with $Q_0^2 = 20\ GeV^2$ for the unfixed twist-4 contribution. For $\Lambda^{(4)}_{\overline{MS}}$ we got: 331±162 MeV (LO level), 440± 183 MeV (NLO level), 372± 133 MeV (NNLO level) and 371 ± 127 MeV (expanded [0/2] Padé)

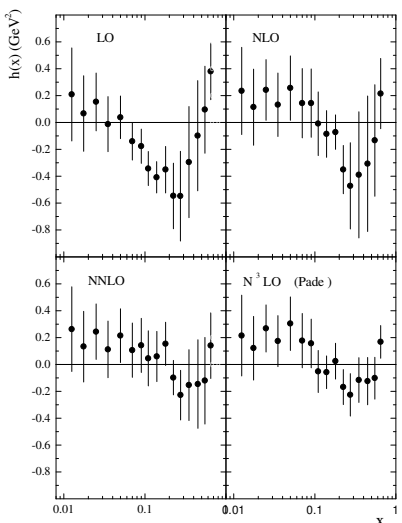

Figure 1. The x-shape of $h(x)$ extracted from the fits of CCFR'97 for $Q_0^2 = 20\ GeV^2$.

One can see that taking into account of the higher order perturbative corrections is decreasing the amplitude of the variation of $h(x)$. This observation is in agreement with the results of Refs.[4,5], obtained for the case of $Q_0^2 = 5\ GeV^2$. However, the change of factorization scale allows to detect the remaining twist-4 structure even at the NNLO. It is relatively stable to the application of the [0/2] Padé approximations method.

When the twist-4 terms are fixed through the IRR model we obtain

$$\begin{aligned} \alpha_s(M_Z)|_{NLO} &= 0.120 \pm 0.002(stat) \\ &\quad \pm 0.005(syst) \pm 0.004(th.) \\ \alpha_s(M_Z)|_{NNLO} &= 0.118 \pm 0.002(stat) \\ &\quad \pm 0.005(syst) \pm 0.003(th.) \end{aligned} \tag{4}$$

and

$$\begin{aligned} \alpha_s(M_Z)|_{NLO} &= 0.123^{+0.008}_{-0.010}(stat) \\ &\quad \pm 0.005(syst) \pm 0.004(th.) \\ \alpha_s(M_Z)|_{NNLO} &= 0.121^{+0.007}_{-0.009}(stat) \\ &\quad \pm 0.005(syst) \pm 0.003(th) \end{aligned} \tag{5}$$

when the twist-4 terms parameters $h(x_i)$ are free. The systematic errors are taken from the CCFR experimental analysis and the theoretical (th.) ambiguities are dominated by the uncertainty in the choice of the matching point in the NLO, NNLO and N^{3}LO variants of the $\overline{MS}$-matching condition [13], derived following the lines of Ref.[14]. It was estimated by varying b-quark threshold from $M_b = m_b$ to $M_b = 6.5 m_b$ [15] and is of over ± 0.002.

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