Precision Phenomenology and Collider Physics

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Computer algebra and particle physics,

DESY Zeuthen, April 4, 2005

Collider-physics $\equiv$ perturbative QCD

1. Precise predictions for hard pp processes involving “standard particles" like W, Z, jets, top, Higgs

   NNLO partonic cross sections
   - few particles, but high order

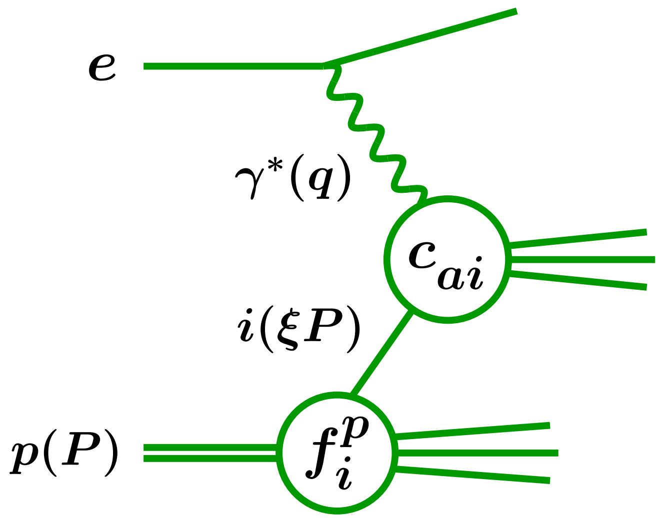
2. Predictions for multiparticle final states that occur at a high rate and form background to New Physics

   New methods for computing tree-amplitudes
   - many particles, but low order

1. Precise Predictions

Hard processes in perturbative QCD

Example: inclusive deep-inelastic scattering (DIS)



Kinematic variables

$$Q^2 = -q^2$$
$$x = Q^2/(2P\cdot q)$$

Lowest order : $x = \xi$

Structure functions F_a [up to $\mathcal{O}(1/Q^2)$]

$$F_a^p(x,Q^2) = \sum_i \left[c_{a,i}(\alpha_s(\mu^2), \mu^2/Q^2) \otimes f_i^p(\mu^2)\right](x)$$

Coefficient functions $c_{a,i}$, renormalization/factorization scale μ

Hard processes in perturbative QCD

Parton distributions f_i: evolution equations

$$\frac{d}{d\ln\mu^2} f_i(\xi,\mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2)\right](\xi)$$

Initial conditions incalculable in pert. QCD.

Splitting functions P, Coefficient functions c_a

$$\begin{aligned} P &= \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \ldots \\ c_a &= \alpha_s^{n_a}\left[c_a^{(0)} + \alpha_s c_a^{(1)} + \alpha_s^2 c_a^{(2)} + \ldots\right] \end{aligned}$$

NLO: standard approximation
NNLO: new emerging standard

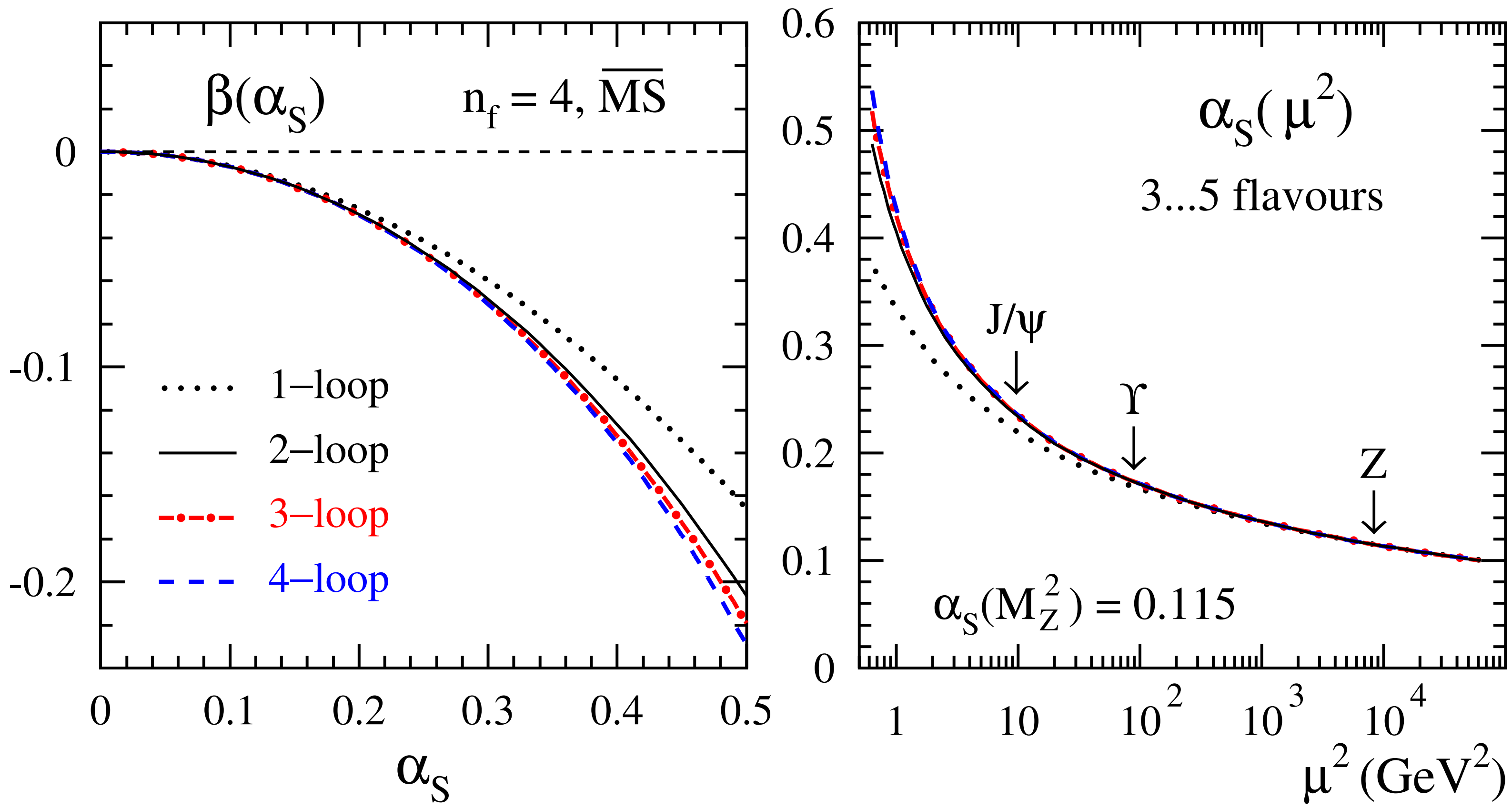
Moch, Vermaseren, Vogt

The running coupling in perturbative QCD

$$d\alpha_s/d\ln\mu^2 = -\beta_0\,\alpha_s^2 - \beta_1\,\alpha_s^3 - \beta_2\,\alpha_s^4 - \beta_3\,\alpha_s^5 - \ldots$$

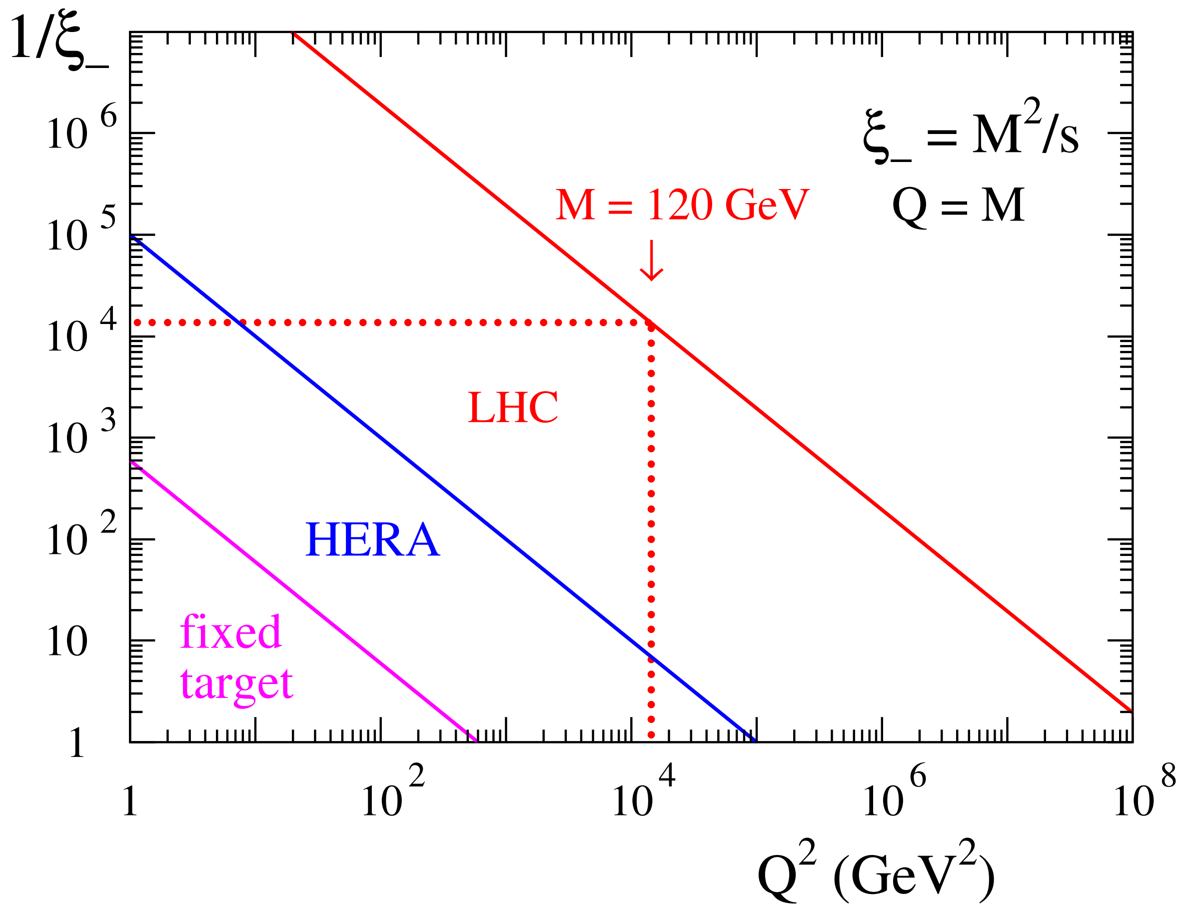
Four-loop coeff.:

van Ritbergen, Vermaseren, Larin; Czakon



Parton evolution from HERA to LHC

Kinematics: parton momenta $\xi_- < \xi < 1$ probed



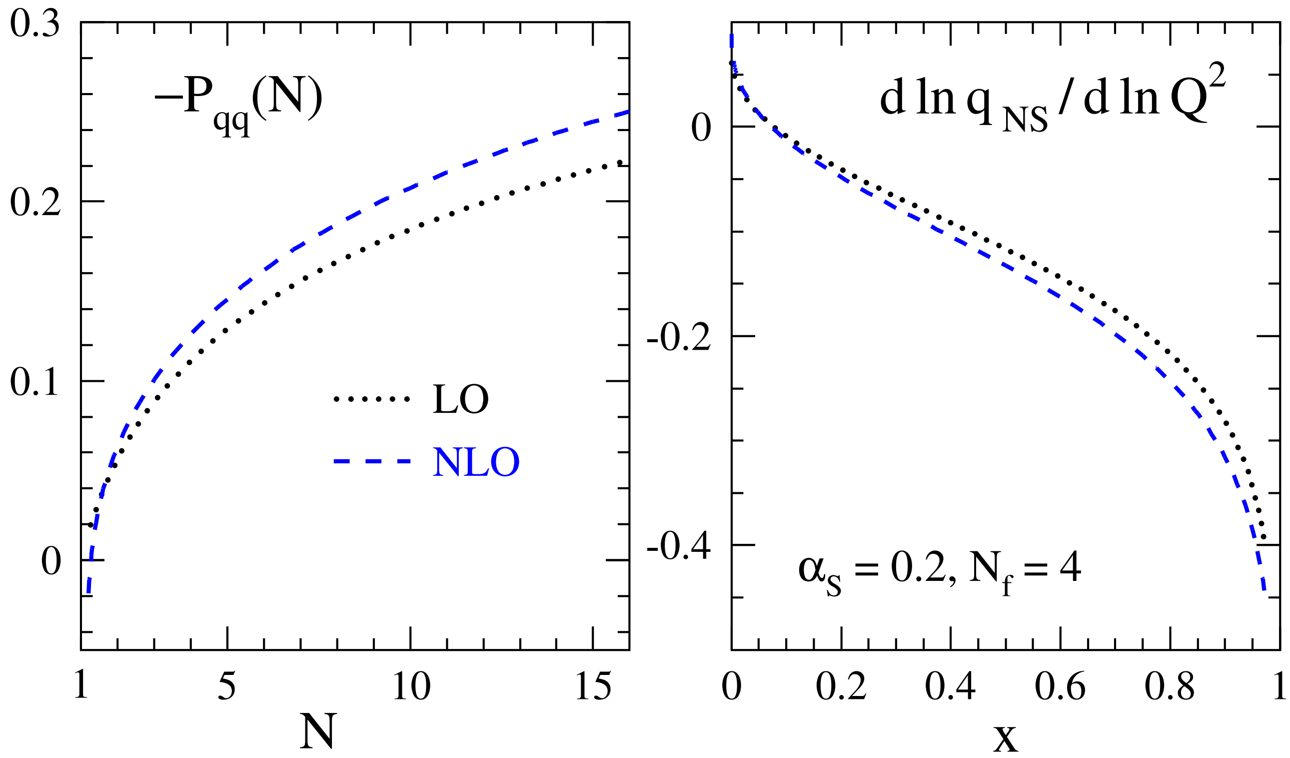
HERA $\longrightarrow$ LHC:

Q^2 evolution across up to three orders of magnitude

Parton evolution at large x

$A(N) = \int_0^1 dx\, x^{N-1} A(x)$.

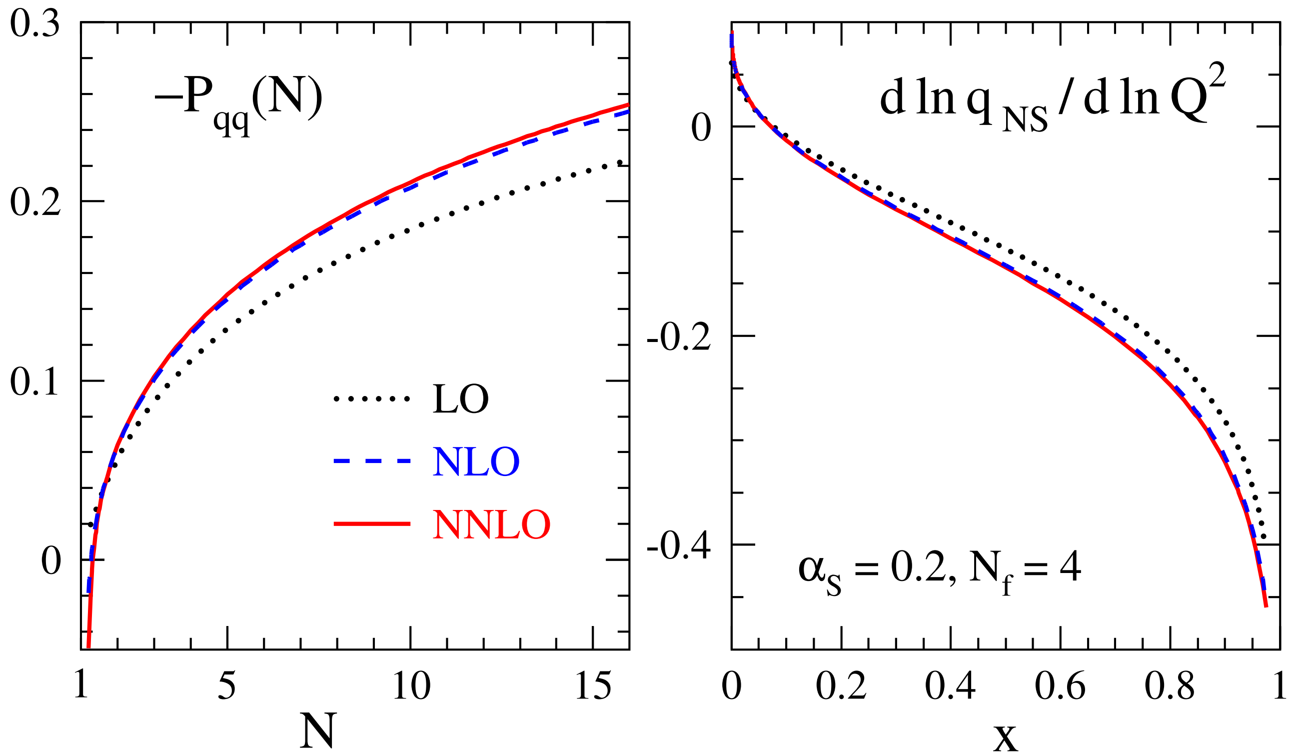
Non-singlet : $u+\bar{u} - (d+\bar{d})$ etc



Moch, Vermaseren, Vogt

Parton evolution at large x

$A(N) = \int_0^1 dx\, x^{N-1} A(x)\ .$ Non-singlet : $u+\bar{u} - (d+\bar{d})$ etc

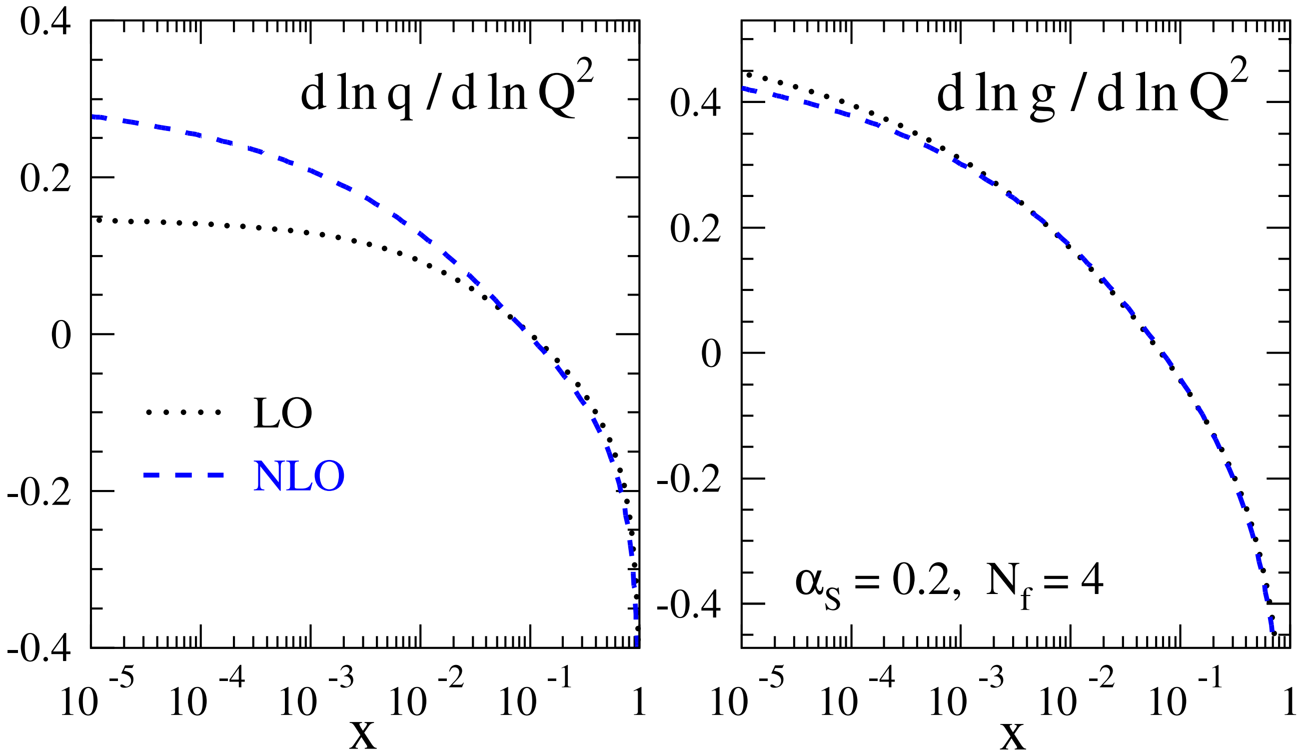


Moch, Vermaseren, Vogt

Perturbative expansion very benign: expect $< 1\%$ beyond NNLO

Parton evolution at small x

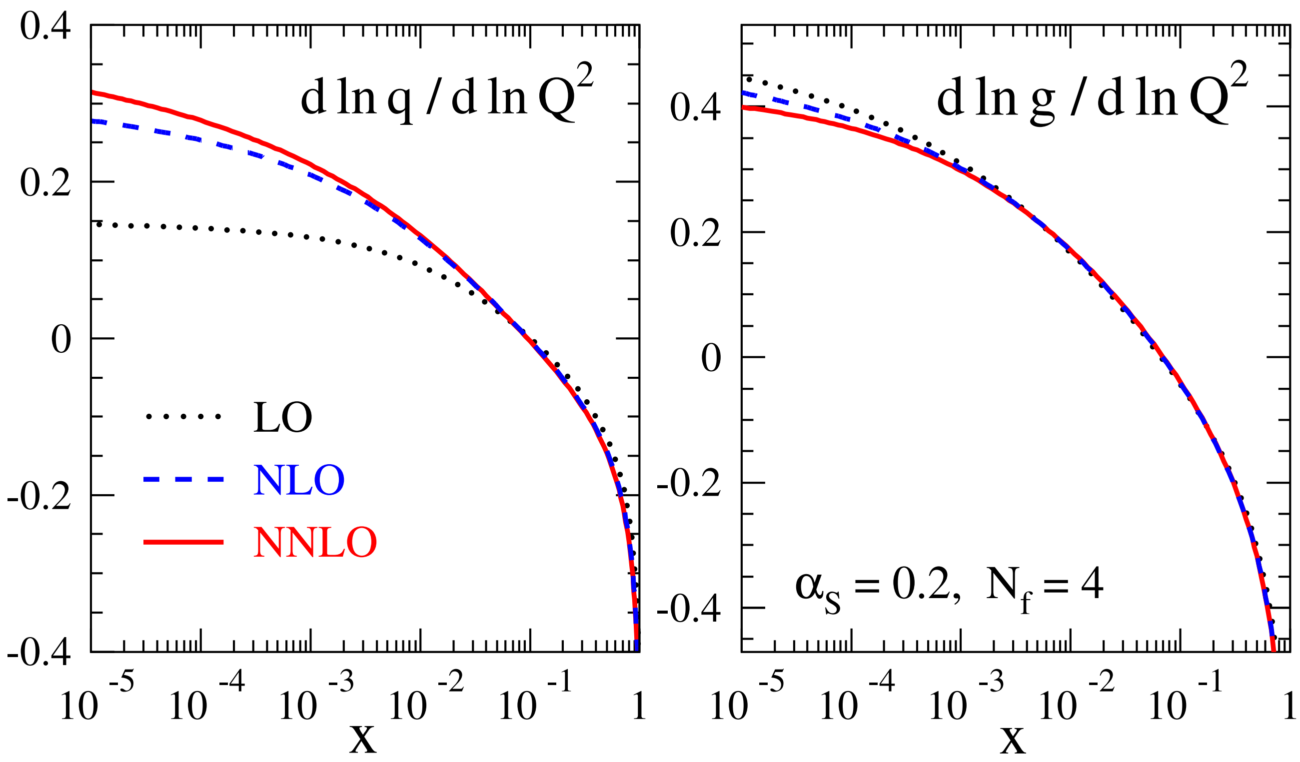
Scale derivatives of quark and gluon distributions at $Q^2 \approx 30$ GeV^2



Moch, Vermaseren, Vogt

Parton evolution at small x

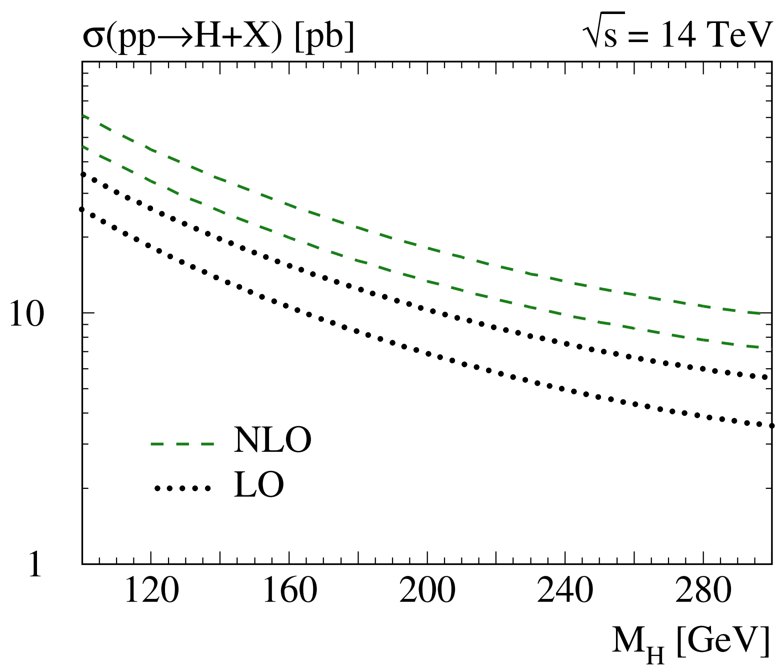
Scale derivatives of quark and gluon distributions at $Q^2 \approx 30$ GeV^2

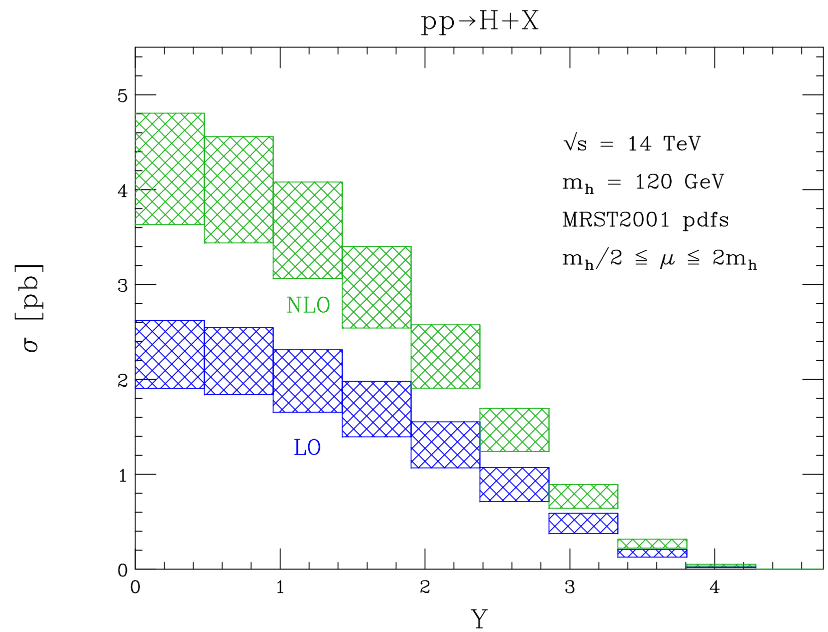


Moch, Vermaseren, Vogt

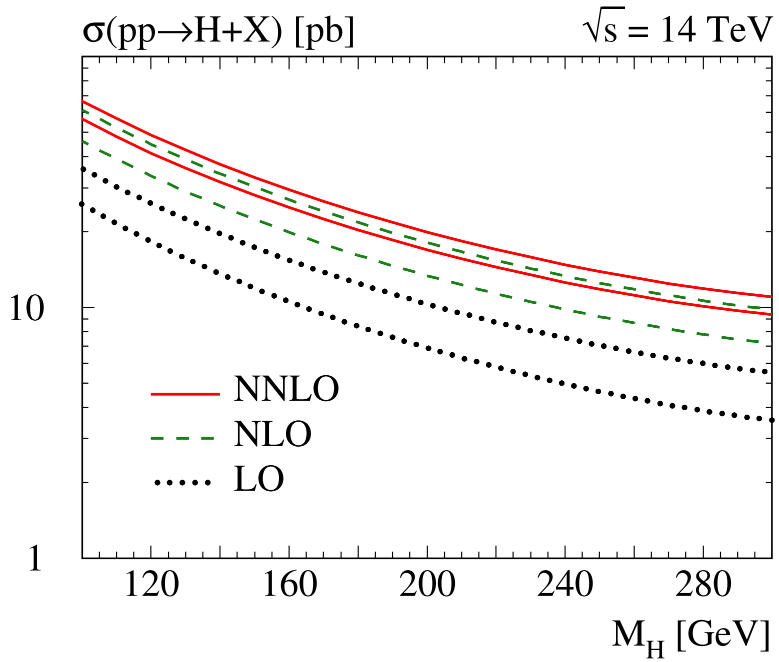
Expansion very stable except for very small momenta $x \lesssim 10^{-4}$

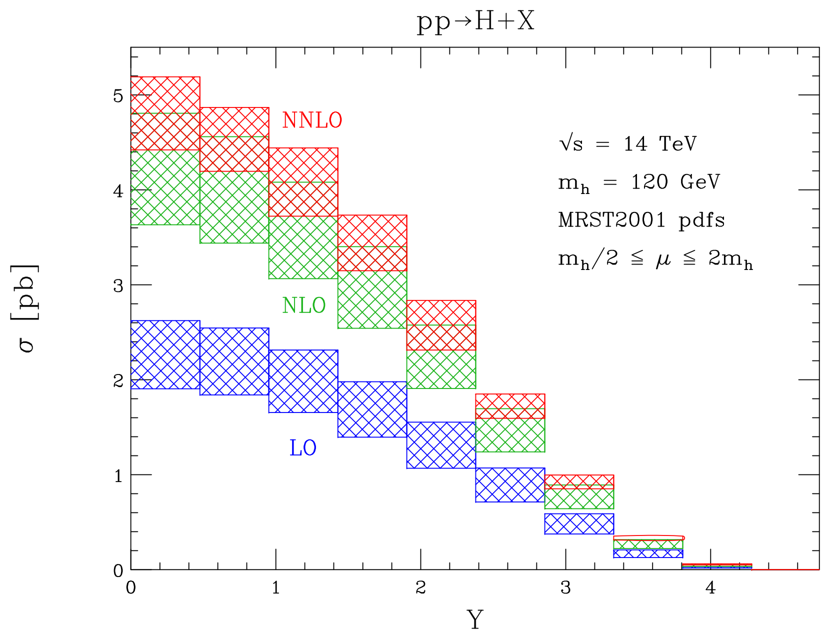
Higgs boson production at the LHC





Higgs boson production at the LHC





Total cross section

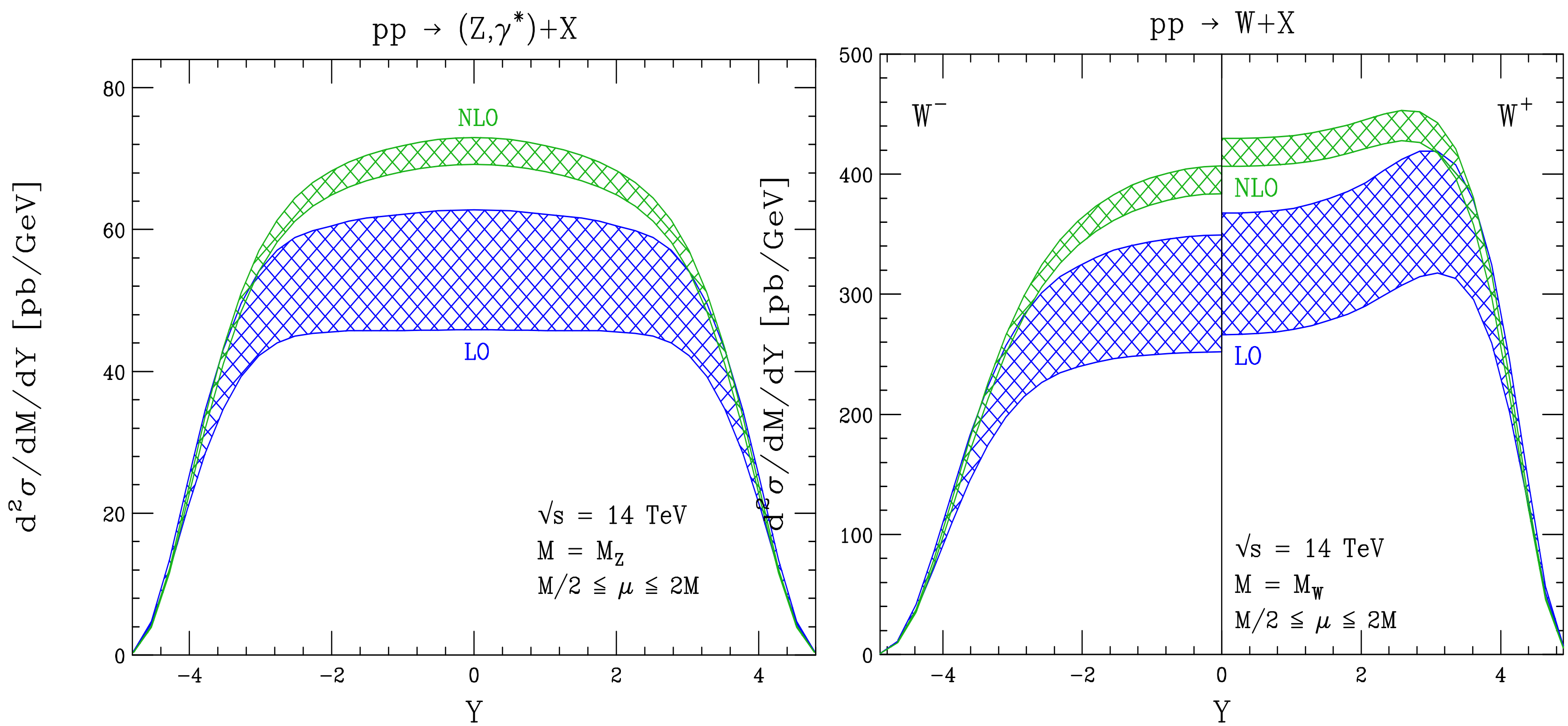
Harlander, Kilgore; Anastasiou, Melnikov, Petriello; . . .

Fully differential

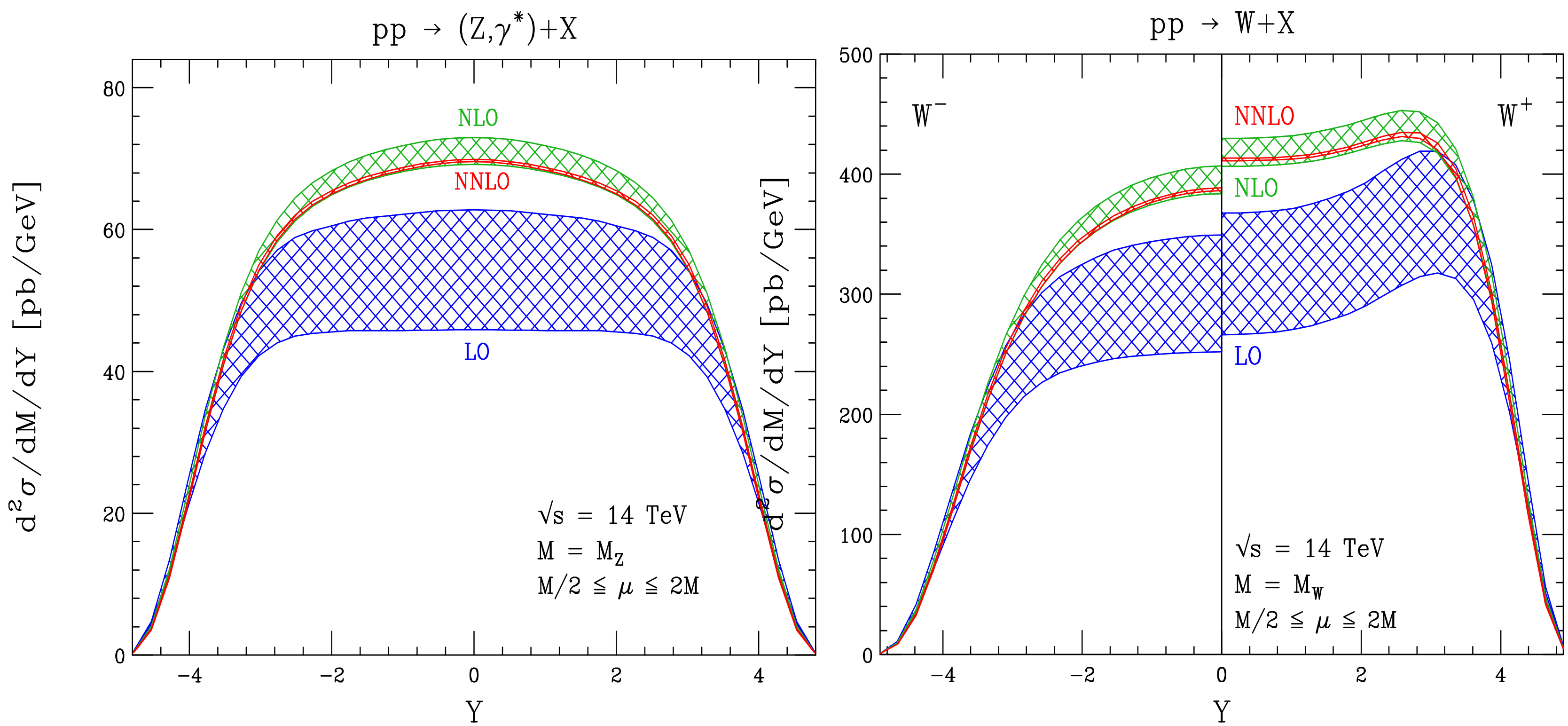
Anastasiou, Melnikov, Petriello

NNLO needed for reliable predictions

Gauge boson production at the LHC



Gauge boson production at the LHC



Gold-plated process

Anastasiou, Dixon, Melnikov, Petriello

NNLO perturbative accuracy better than 1%

⇒ use to determine parton-parton luminosities at the LHC

Jet production at NNLO

- $pp \to$ jet+X requires matrix elements for $2 \to 2$ at two-loops, $2 \to 3$ at one-loop and $2 \to 4$ at tree-level
  Two-loop amplitudes solved in past five years thanks to

  Smirnov, Tausk

- Techniques for handling infrared singularities
  Phase-space sector decomposition

  Binoth, Heinrich; Anastasiou, Melnikov, Petriello

  Subtraction terms

  Kosower; Weinzierl; Gehrmann-De Ridder, Gehrmann + NG; . . .

- First NNLO results for jets in e^+e^- annihilation
  Leading jet energy distribution in $e^+e^- \to 2$ jets

  Anastasiou, Melnikov, Petriello

  C_F^3 part of first moment of Thrust distribution

  Gehrmann-De Ridder, Gehrmann + NG

2. Multiparticle Production

Multiparticle production

In many cases the backgrounds to New Physics are standard model multiparticle final states
$\Rightarrow$ Whole raft of automated tree-level packages for generating cross section

e.g. MadEvent, ALPGEN, HELAC/PHEGAS, CompHEP, GRACE, ...

Example: Multi-jet production at the LHC using HELAC/PHEGAS

Draggiotis, Kleiss, Papadopoloulos

# of jets	2	3	4	5	6	7	8
# of dist.processes	10	14	28	36	64	78	130
total # of processes	126	206	621	861	1862	2326	4342
$\sigma(nb)$	-	91.41	6.54	0.458	0.030	0.0022	0.00021
$\%$ Gluonic	-	45.7	39.2	35.7	35.1	33.8	26.6

Sizeable cross sections for multi-jet events

Large uncertainty since $\sigma(n \text{ jets}) \sim \alpha_s^n$

Multiparticle production

The number of tree Feynman diagrams for an n gluon process increases very quickly with n

n	diagrams
4	4
5	25
6	220
7	2485
8	34300
9	559405
10	10525900

$\Rightarrow$ Feynman diagram evaluation is very inefficient for many legs
- too many diagrams, terms per diagram, kinematic variables

Insight from Twistor Space

- In a recent paper Witten made a striking proposal to relate perturbative gauge theory amplitudes to topological string theory in twistor space

Witten, hep-th/0312171

$\Rightarrow$ Advance in calculating tree amplitudes in massless gauge theories:

Cachazo, Svrcek and Witten, hep-th/0403047

Amplitudes constructed from scalar propagators and tree-level maximal helicity violating (MHV) amplitudes which are interpreted as new scalar vertices

$\Rightarrow$ New type of on-shell recursion relations

Britto, Cachazo and Feng, hep-th/0412308

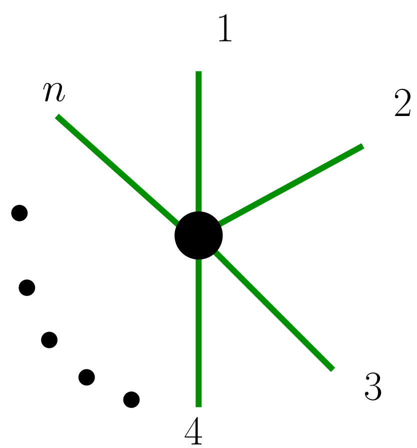
$\Rightarrow$ Recent developments in computing one-loop amplitudes in $\mathcal{N} = 4$ SuperYang Mills theory (as well as $\mathcal{N} = 1$ and maybe even QCD)

Colour Ordered Amplitudes

$$\mathcal{A}_n(1,\ldots,n) = \sum_{perms} Tr(T^{a_1}\ldots T^{a_n}) A_n(1,\ldots,n)$$

Colour-stripped amplitudes A_n: cyclically ordered permutations

Order of external gluons fixed

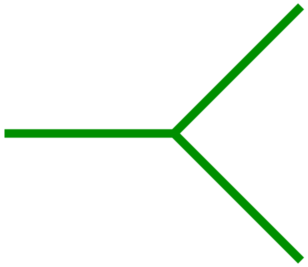


The subamplitudes A_n are
(a) gauge invariant
(b) have nice properties in the infrared limits.

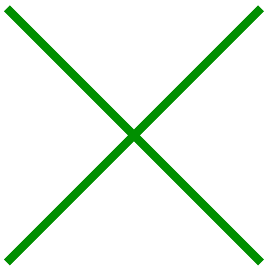
Can reconstruct the full amplitude $\mathcal{A}_n$ from A_n.
In the large N limit,

$$|\mathcal{A}_n(1,\ldots,n)|^2 \sim N^{n-2}(N^2-1) \sum_{perms} |A_n(1,\ldots,n)|^2$$

Colour Ordered Feynman Rules



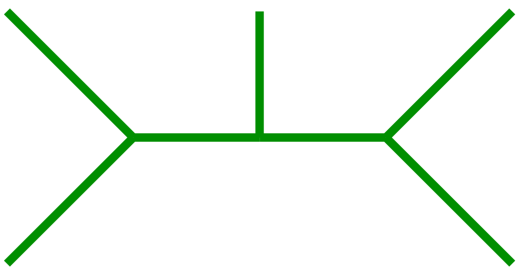
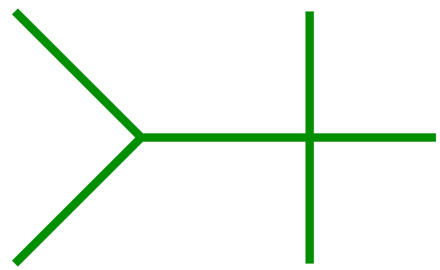
$$i\left(\left(p_1^{\mu_3}-p_2^{\mu_3}\right)g^{\mu_1\mu_2}+\left(p_2^{\mu_1}-p_3^{\mu_1}\right)g^{\mu_2\mu_3}+\left(p_3^{\mu_2}-p_1^{\mu_2}\right)g^{\mu_3\mu_1}\right)$$



$$-i\left(g^{\mu_1\mu_2}g^{\mu_3\mu_4}+g^{\mu_1\mu_4}g^{\mu_3\mu_2}-2g^{\mu_1\mu_3}g^{\mu_2\mu_4}\right)$$

Only calculate diagrams with cyclic colour ordering

Example:

$A_5 =$  $+$ 

i.e. 10 diagrams rather than 25

Power of colour ordering

n	diagrams	colour ordered diagrams
4	4	3
5	25	10
6	220	36
7	2485	133
8	34300	501
9	559405	1991
10	10525900	7335

$\Rightarrow$ Big reduction in number of diagrams

but still too many diagrams

Spinor Helicity Formalism

- Spinor for a massless fermion, momentum p:

$$\not{p}\, u(p) = 0, \qquad |p\pm\rangle = u_\pm(p) = \frac{1}{2}\left(1 \pm \gamma_5\right) u(p)$$

- Spinor products:

$$\begin{aligned} \langle ij \rangle &= \langle p_i - | p_j + \rangle = \overline{u_-(p_i)} u_+(p_j) \\ [ij] &= \langle p_i + | p_j - \rangle = \overline{u_+(p_i)} u_-(p_j) \end{aligned}$$

- Spinor products are complex numbers and have numerical representations
- Dot products

$$s_{ij} = (p_i + p_j)^2 = 2\, p_i \cdot p_j = \langle ij \rangle [ji]$$

Spinor Helicity Formalism

- Polarisation vector for a massless gauge boson, momentum p:

$$\epsilon_\mu^\pm(p,\eta) = \pm\frac{\langle p\pm|\gamma_\mu|\eta\pm\rangle}{\sqrt{2}\langle\eta\mp|p\pm\rangle}$$

- Easy to show that:

$$\epsilon^\pm\cdot\epsilon^{\pm *} = -1, \qquad p\cdot\epsilon(p,\eta) = 0, \qquad \epsilon^\pm\cdot\epsilon^{\mp *} = 0.$$

- η is a light-like axial gauge vector

$$\sum \epsilon_\mu^\pm(p,\eta)\epsilon_\nu^\pm(p,\eta) = -g_{\mu\nu} + \frac{p_\mu\eta_\nu + p_\nu\eta_\mu}{p\cdot\eta}$$

- amplitudes are η independent
sensible choice kills many diagrams

Spinor Helicity Formalism

- In Weyl (chiral) representation, each helicity state is represented by a bi-spinor ($a = 1, 2$)

$$u_+(p) = \lambda_{pa}, \qquad u_-(p) = \tilde{\lambda}_p^{\dot{a}},$$

$$\overline{u_+(p)} = \tilde{\lambda}_{p\dot{a}}, \qquad \overline{u_-(p)} = \lambda_p^a$$

so that

$$\begin{aligned} \langle ij\rangle &= \overline{u_-(p_i)}u_+(p_j) = \lambda_i^a \lambda_{ja} = \epsilon_{ab}\lambda_i^a\lambda_j^b \\ [ij] &= \overline{u_+(p_i)}u_-(p_j) = \tilde{\lambda}_{i\dot{a}}\tilde{\lambda}_j^{\dot{a}} = -\epsilon_{\dot{a}\dot{b}}\tilde{\lambda}_i^{\dot{a}}\tilde{\lambda}_j^{\dot{b}} \end{aligned}$$

- We can write massless vector

$$p_{a\dot{a}} \equiv p_\mu \sigma^\mu_{a\dot{a}} = \lambda_{pa}\tilde{\lambda}_{p\dot{a}}$$

Spinor Helicity Formalism

- Polarisation vectors for particle i:

$$\varepsilon^-_{ia\dot{a}} = \frac{\lambda_{ia}\tilde{\eta}_{\dot{a}}}{[\tilde{\lambda}_i\tilde{\eta}]}, \qquad \varepsilon^+_{ia\dot{a}} = \frac{\eta_a\tilde{\lambda}_{i\dot{a}}}{\langle\eta\lambda_i\rangle}$$

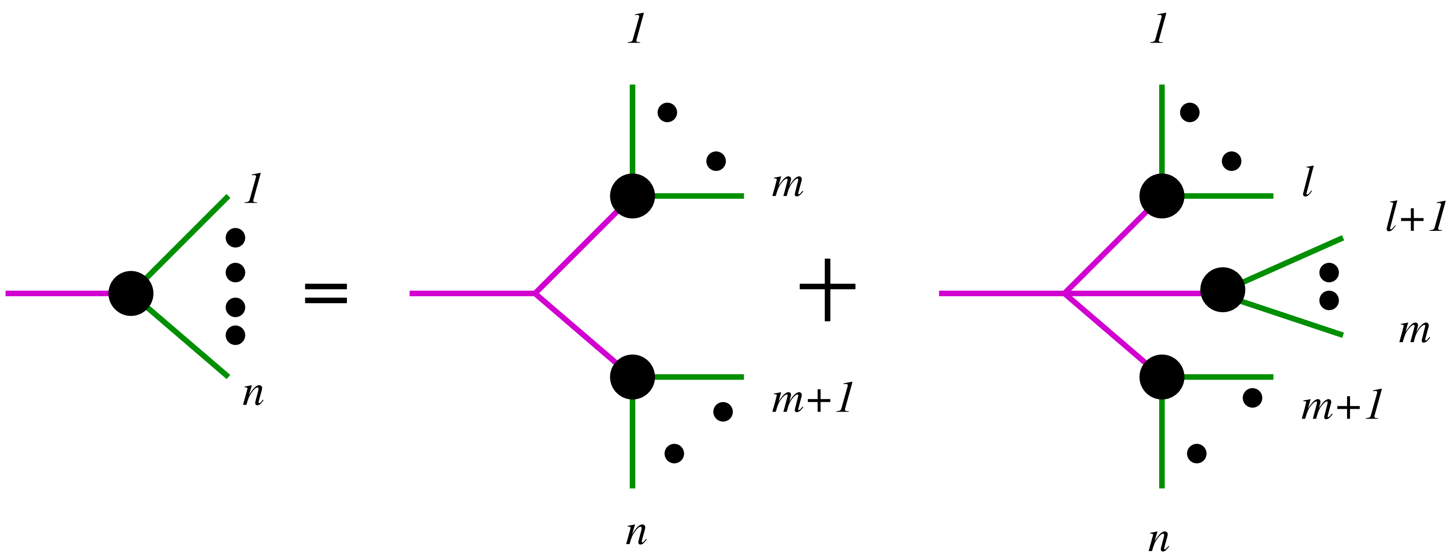
- For real momenta in Minkowski space,

$$\tilde{\lambda} = \lambda^*$$

- For space-time signature $(+,+,-,-)$, $\tilde{\lambda}, \lambda$ are real and independent
- Amplitudes are functions of the λ_i and $\tilde{\lambda}_i$

Recursion relations

Full amplitudes can be built up from simpler amplitudes with fewer particles



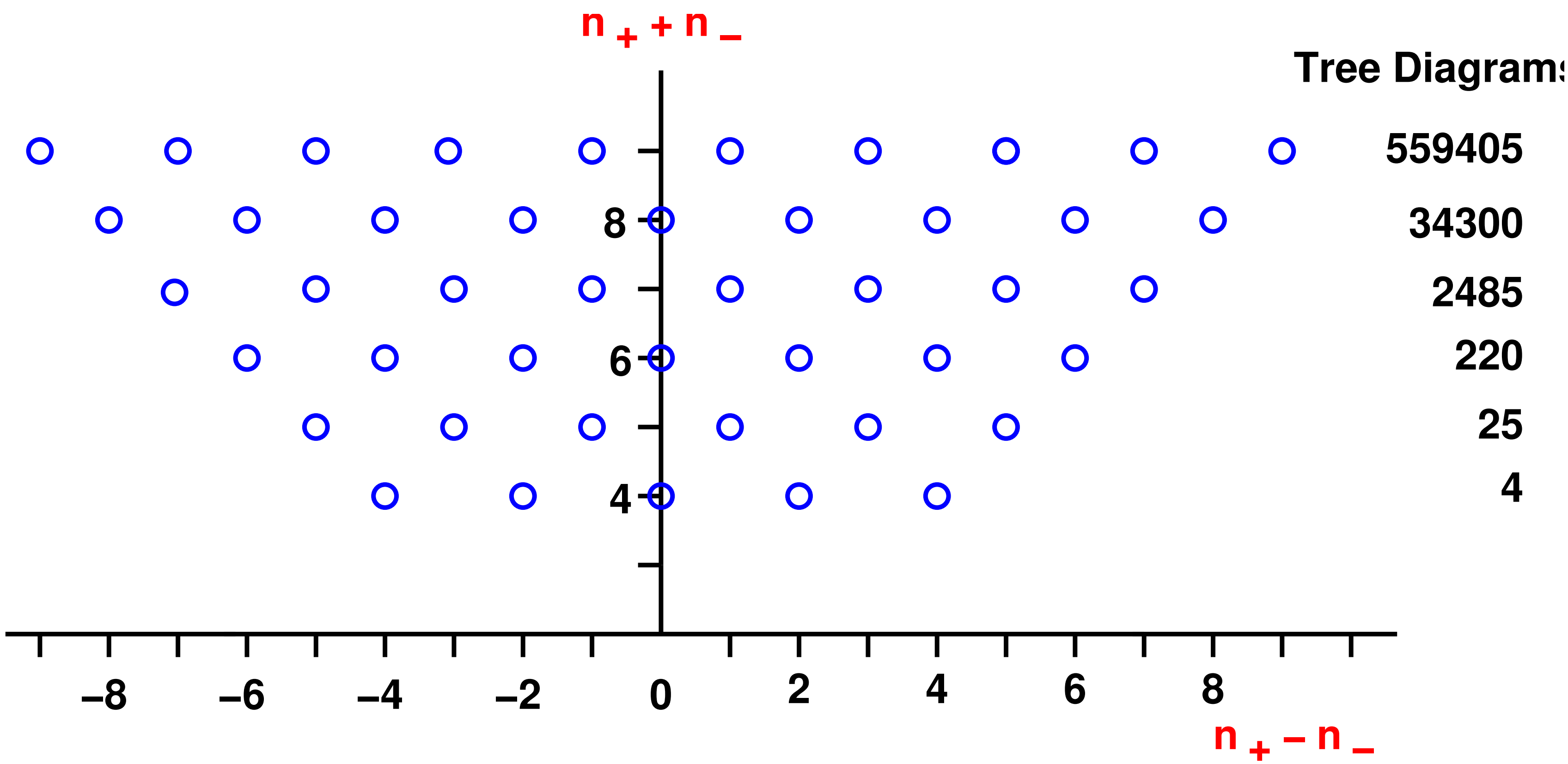
Purple gluons are off-shell, green gluons are on-shell.
This is a recursion relation built from off-shell currents.

Berends, Giele

Particularly suited to numerical solution

ALPGEN, HELAC/PHEGAS

Gluonic helicity amplitudes



Each row describes scattering with n_+ positive helicities and n_- negative helicities.
Each circle represents an allowed helicity configuration - from all positive on the right to all negative on the left

Gluonic helicity amplitudes

For example, the result of computing the 25 diagrams for the five-gluon process yields

$$A_5(1^\pm,2^+,3^+,4^+,5^+) \quad = \quad 0$$
$$A_5(1^-,2^-,3^+,4^+,5^+) \quad = \quad \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

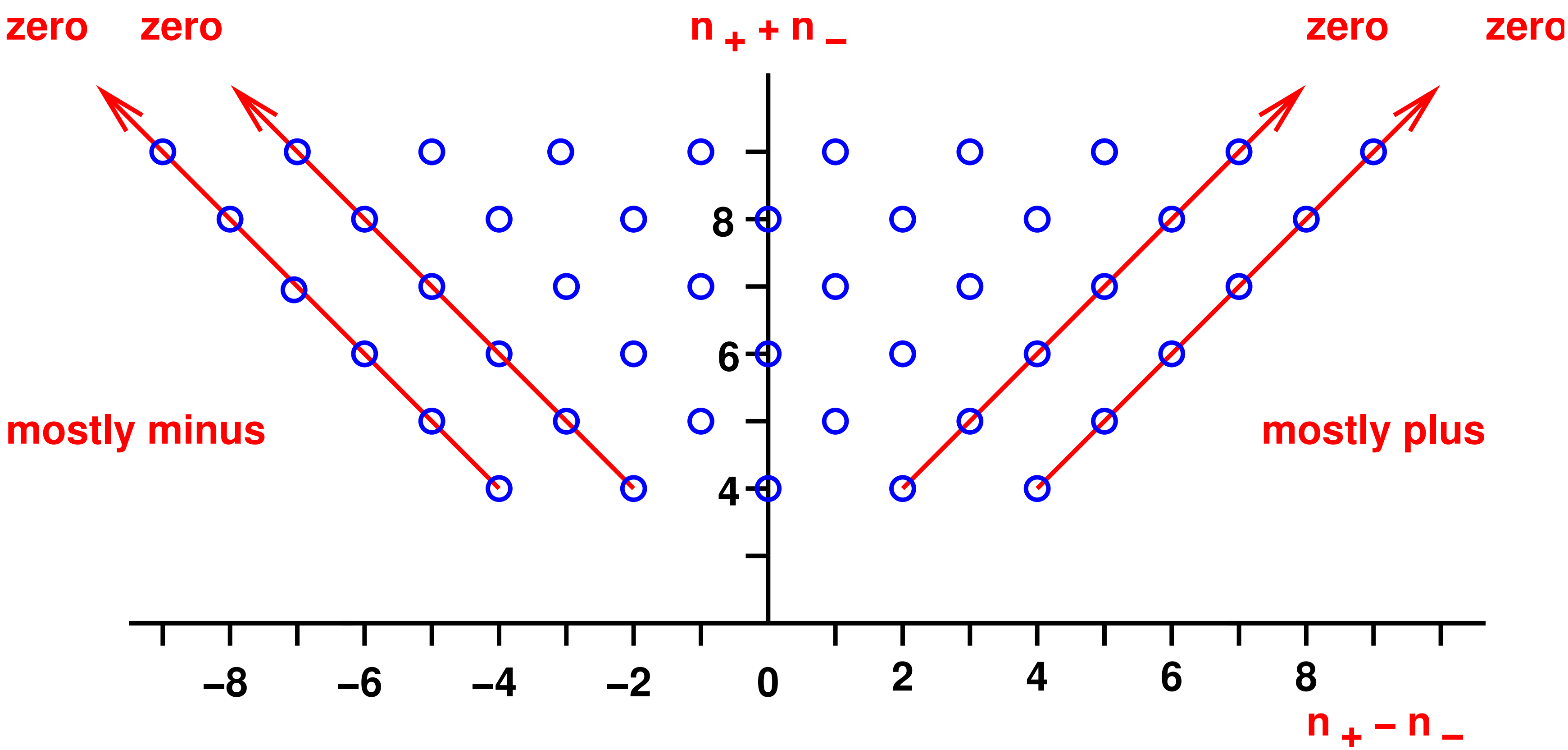
In fact, for n point amplitudes,

$$A_n(1^\pm,2^+,3^+,,\ldots,n^+) \quad = \quad 0$$
$$A_n(1^-,2^-,3^+,\ldots,n^+) \quad = \quad \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

Maximally helicity violating (MHV) amplitudes

Parke, Taylor; Berends, Giele

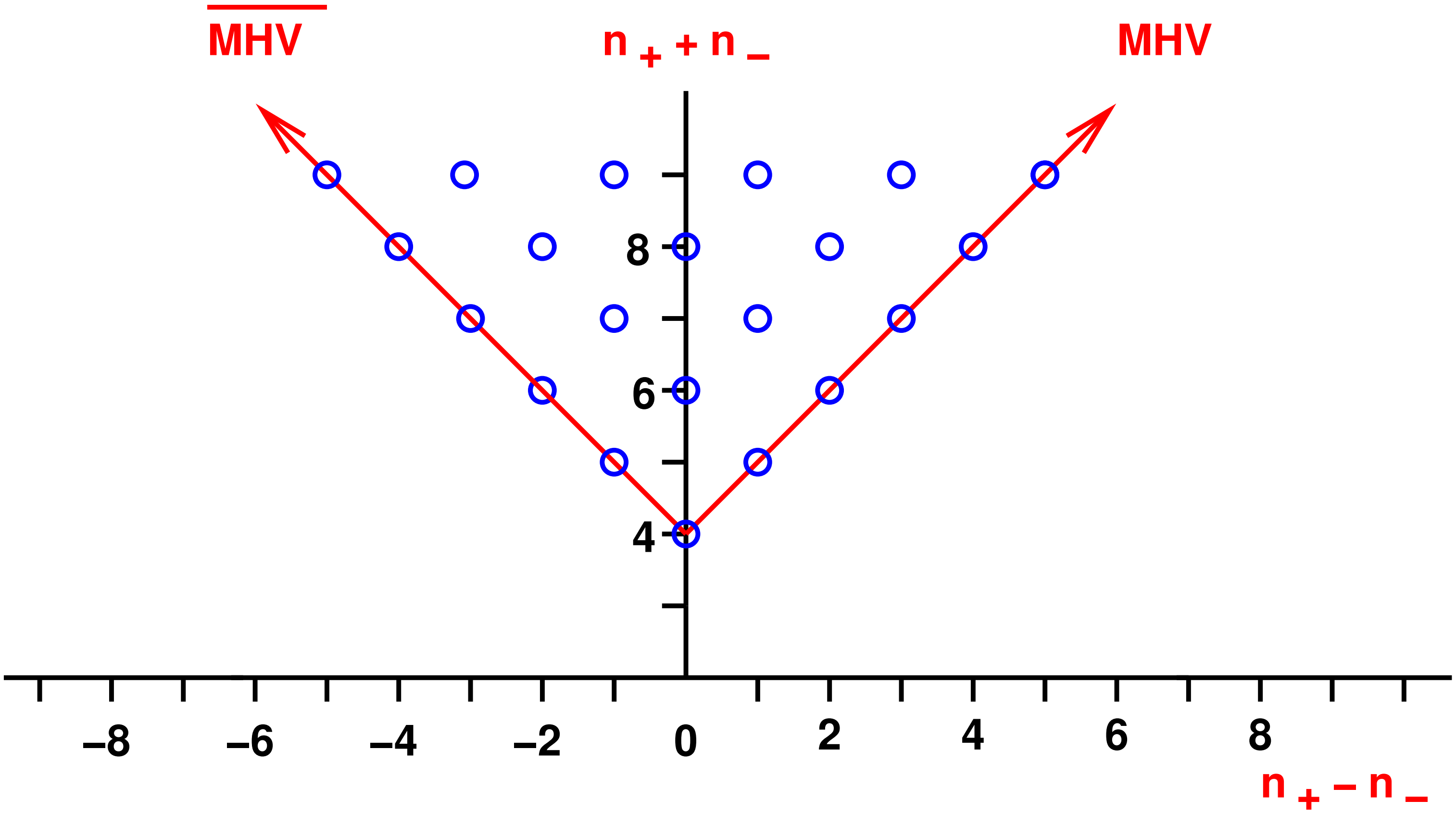
Gluonic helicity amplitudes



$$A_n(1^{\pm}, 2^+, 3^+, , \ldots, n^+) = 0$$

effective tree-level supersymmetry

Gluonic helicity amplitudes



$$A_n(1^-, 2^-, 3^+, \ldots, n^+) \quad = \quad \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}$$

Specific helicity amplitudes

For phenomenological purposes, all possible helicity amplitudes are needed - and which are usually much more complicated. For example, the 220 six gluon diagrams contributing to NMHV amplitudes (3- and 3+ helicities) can be written as

$$A_6 = 8g^4 \Bigg[\frac{\alpha^2}{s_{123}s_{12}s_{23}s_{34}s_{45}s_{56}} + \frac{\beta^2}{s_{234}s_{23}s_{34}s_{45}s_{56}s_{61}} + \frac{\gamma^2}{s_{345}s_{34}s_{45}s_{56}s_{61}s_{12}} + \frac{s_{123}\beta\gamma + s_{234}\gamma\alpha + s_{345}\alpha\beta}{s_{12}s_{23}s_{34}s_{45}s_{56}s_{61}} \Bigg]$$

where for $A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+)$,

$$\alpha = 0, \qquad \beta = \langle 23 \rangle [56] \langle 4 | \not{2} + \not{3} | 1 \rangle, \qquad \gamma = \langle 12 \rangle [45] \langle 6 | \not{1} + \not{2} | 3 \rangle,$$

Hidden structure is uncovered in twistor space

Twistor Space

Twistor space:

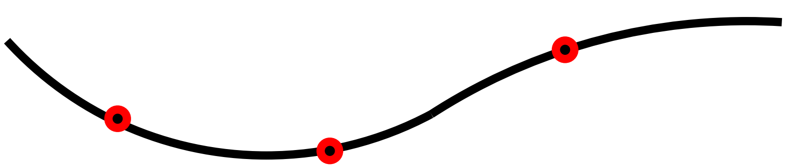
Penrose, 1967

Amplitudes in twistor space obtained by Fourier transform with respect to positive helicity spinors,

$$\tilde{A}(\lambda_i, \mu_i) = \int \prod_i \frac{d^2\tilde{\lambda}_i}{(2\pi)^2} \exp\left(i \sum_j \mu_j^{\dot{a}} \tilde{\lambda}_{j\dot{a}} \right) A(\lambda_i, \tilde{\lambda}_i)$$

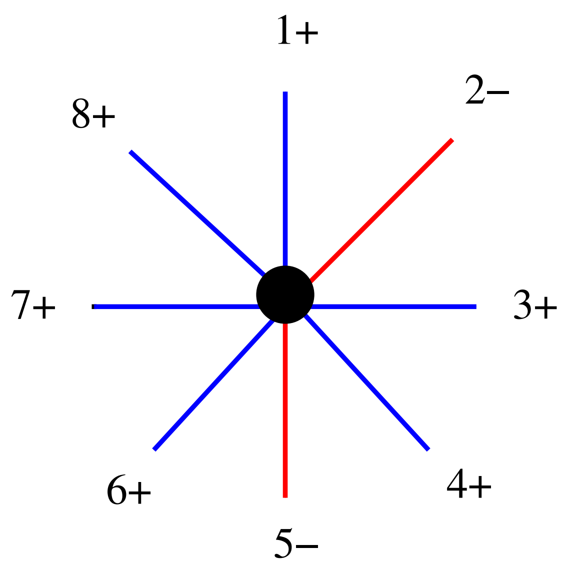
Witten observed that in twistor space external points lie on certain algebraic curves
$\Rightarrow$ degree of curve is related to the number of negative helicities and loops

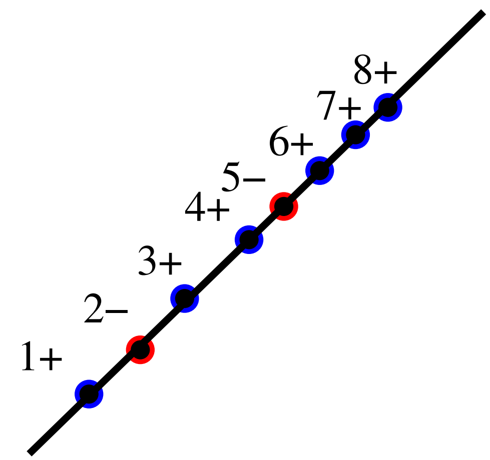
$$d = n_- - 1 + l$$



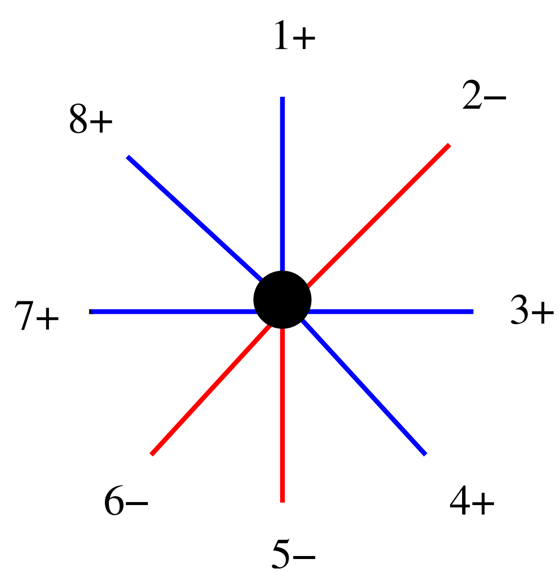
Twistor Space

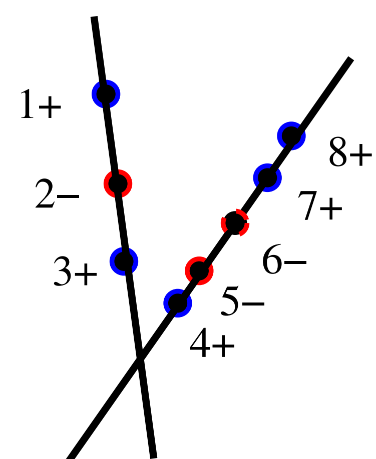
MHV





NMHV



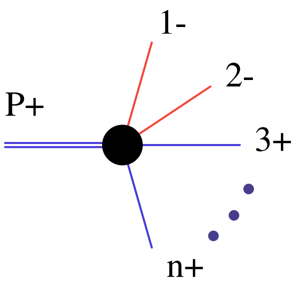


MHV rules

Start from MHV amplitude and define off-shell vertices

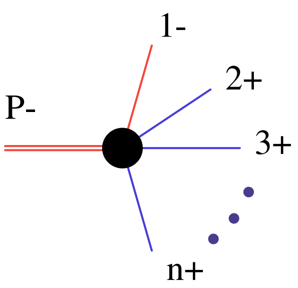
Cachazo, Svrcek and Witten

$$V(1^-,2^-,3^+,\ldots,n^+,P^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\cdots\langle n-1n\rangle\langle nP\rangle\langle P1\rangle}$$



and

$$V(1^-,2^+,3^+,\ldots,n^+,P^-) = \frac{\langle 1P\rangle^4}{\langle 12\rangle\cdots\langle n-1n\rangle\langle nP\rangle\langle P1\rangle}$$



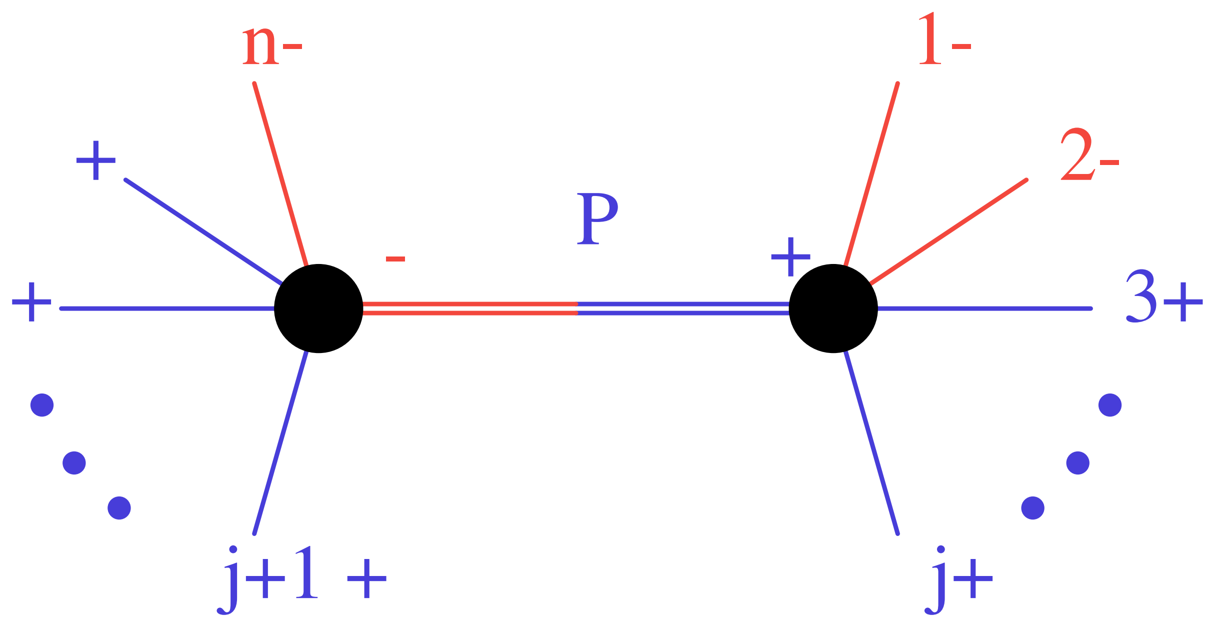
Crucial step is off-shell continuation $P^2 \neq 0$:

$$\langle iP\rangle = \frac{\langle i^-|\not{P}|\eta^-\rangle}{[P\eta]} = \sum_j \frac{\langle i^-|\not{j}|\eta^-\rangle}{[P\eta]}$$

where $P = \sum_j j$ and η is lightlike auxiliary vector

MHV rules

Must connect up a positive helicity off-shell line with a negative helicity off-shell line



Connecting two MHV's $\Rightarrow$ amplitude with 3 negative helicities
Connecting three MHV's $\Rightarrow$ amplitude with 4 negative helicities
etc.

Example: six gluon scattering

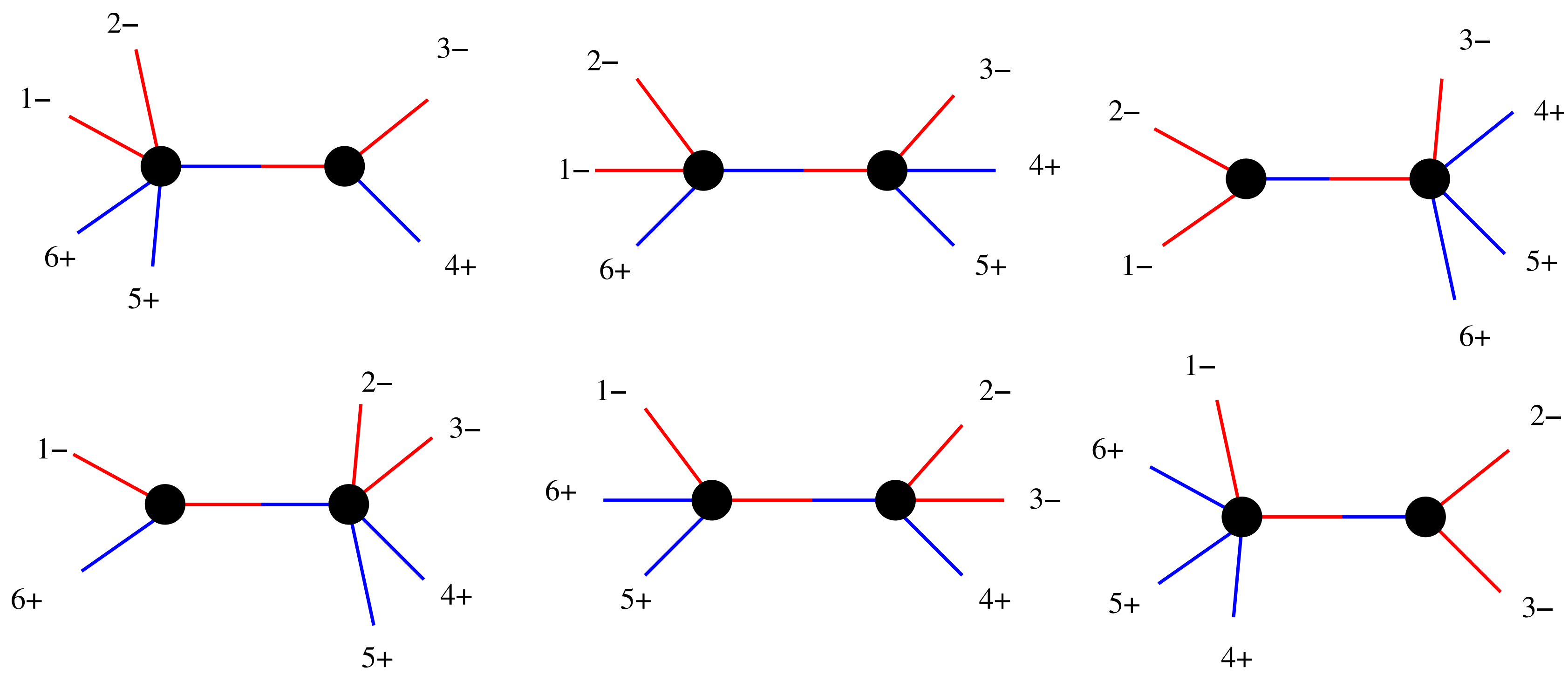
As an example, lets use the MHV rules to calculate one of the first non-MHV amplitudes

$$A_6(1^-,2^-,3^-,4^+,5^+,6^+)$$

Step 1 Draw all the allowed MHV diagrams

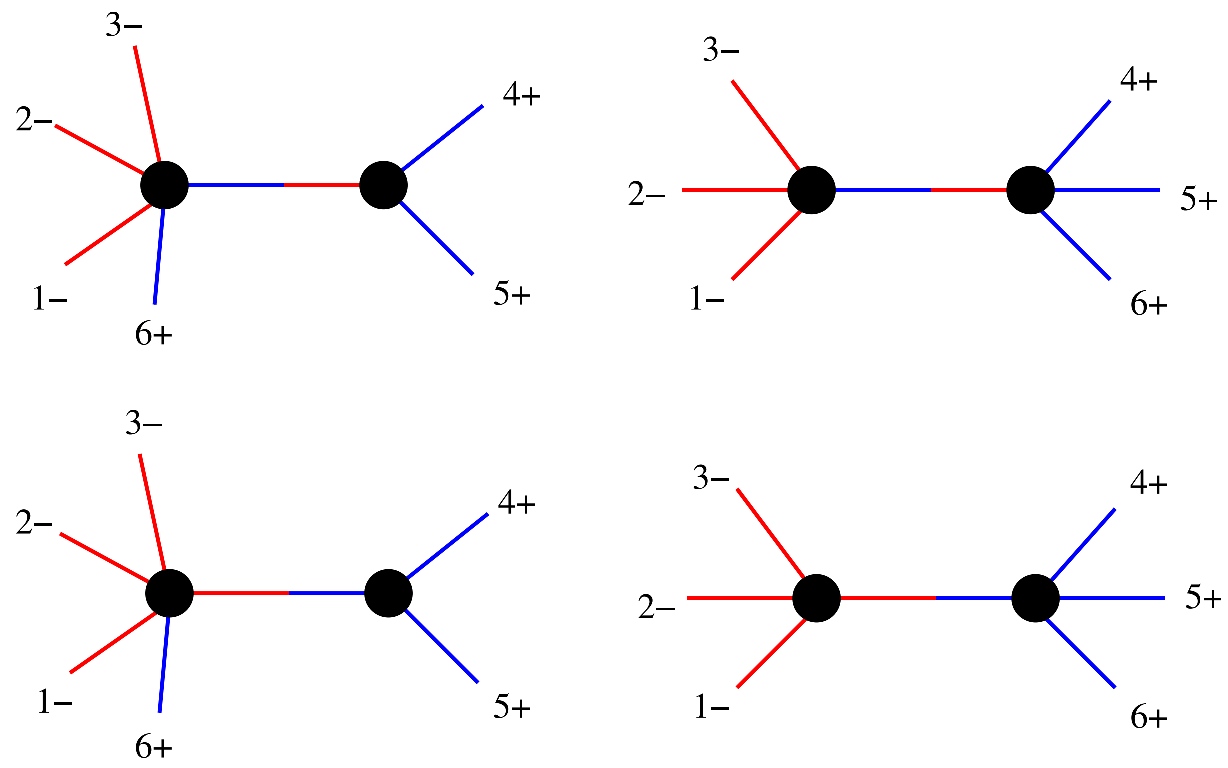
Example: six gluon scattering

There are six MHV graphs



Example: six gluon scattering

Some graphs are not allowed e.g.



Example: six gluon scattering

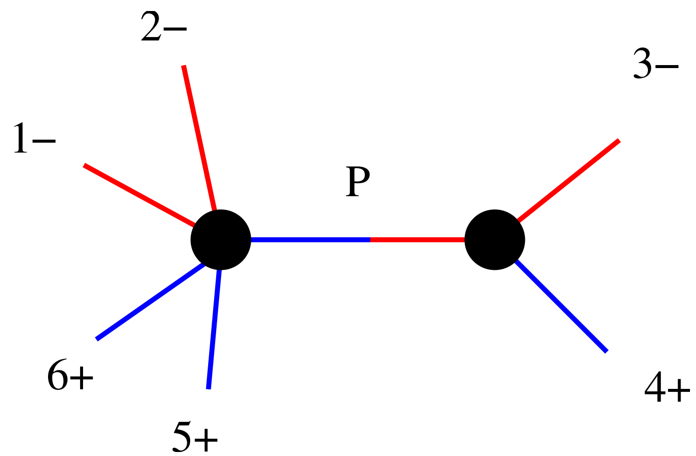
As an example, lets use the MHV rules to calculate one of the first non-MHV amplitudes

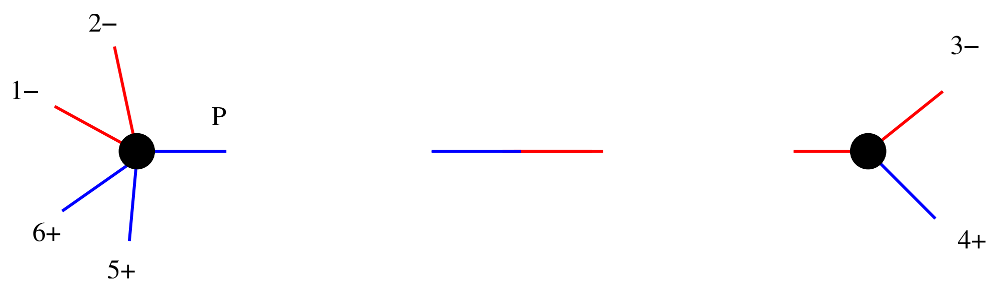
$$A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+)$$

Step 1 Draw all the allowed MHV diagrams

Step 2 Apply MHV rules to each diagram

Example: six gluon scattering: diagram 1

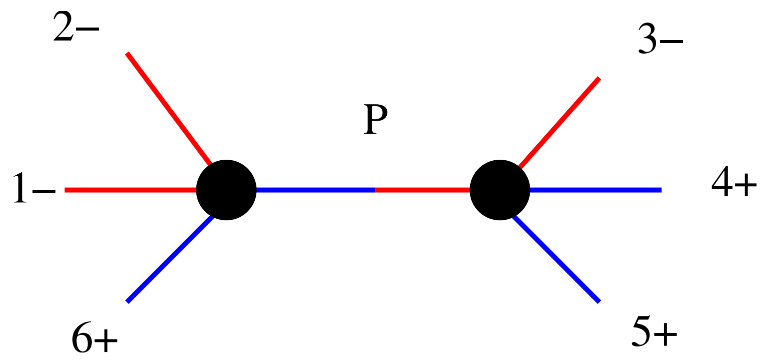


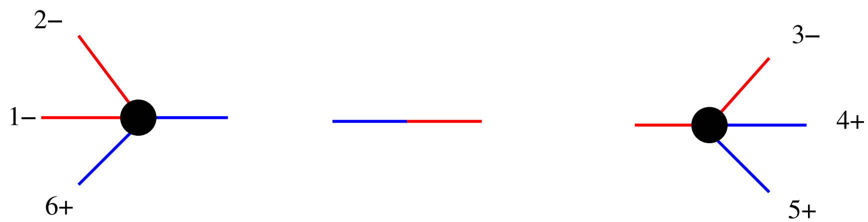


$$\frac{\langle 12\rangle^4}{\langle 56\rangle\langle 61\rangle\langle 12\rangle\langle 2|P|\eta\rangle\langle 5|P|\eta\rangle} \times \frac{1}{s_{34}} \times \frac{\langle 3|P|\eta\rangle^4}{\langle 34\rangle\langle 4|P|\eta\rangle\langle 3|P|\eta\rangle}$$

with $P = 3 + 4 = -(1 + 2 + 5 + 6)$

Example: six gluon scattering: diagram 2





$$\frac{\langle 12\rangle^4}{\langle 61\rangle\langle 12\rangle\langle 2|P|\eta\rangle\langle 6|P|\eta\rangle} \times \frac{1}{s_{345}} \times \frac{\langle 3|P|\eta\rangle^4}{\langle 34\rangle\langle 45\rangle\langle 5|P|\eta\rangle\langle 3|P|\eta\rangle}$$

with $P = 3 + 4 + 5 = -(1 + 2 + 6)$

Example: six gluon scattering

As an example, lets use the MHV rules to calculate one of the first non-MHV amplitudes

$$A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+)$$

Step 1 Draw all the allowed MHV diagrams

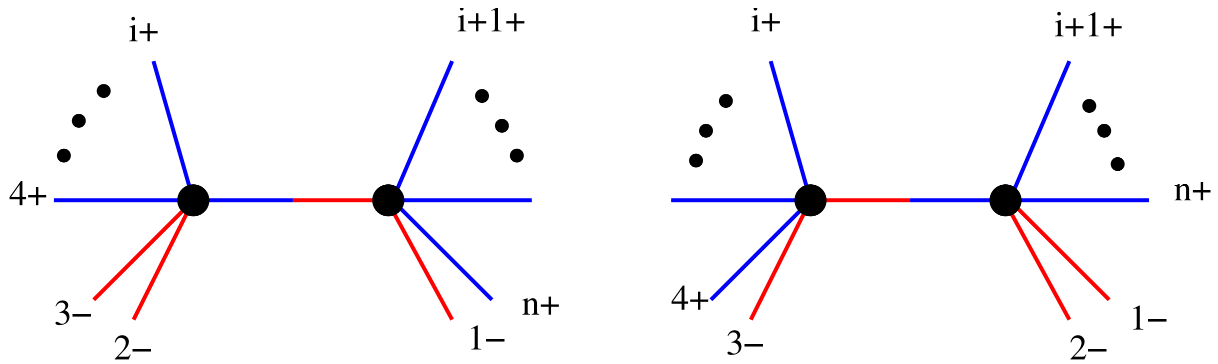
Step 2 Apply MHV rules to each diagram

Step 3 Add up diagrams and check η independence

Next-to MHV amplitude for n gluons

Simplest case: $A_n(1^-, 2^-, 3^-, 4^+, \ldots, n^+)$ $2(n-3)$ graphs

Cachazo, Svrcek and Witten



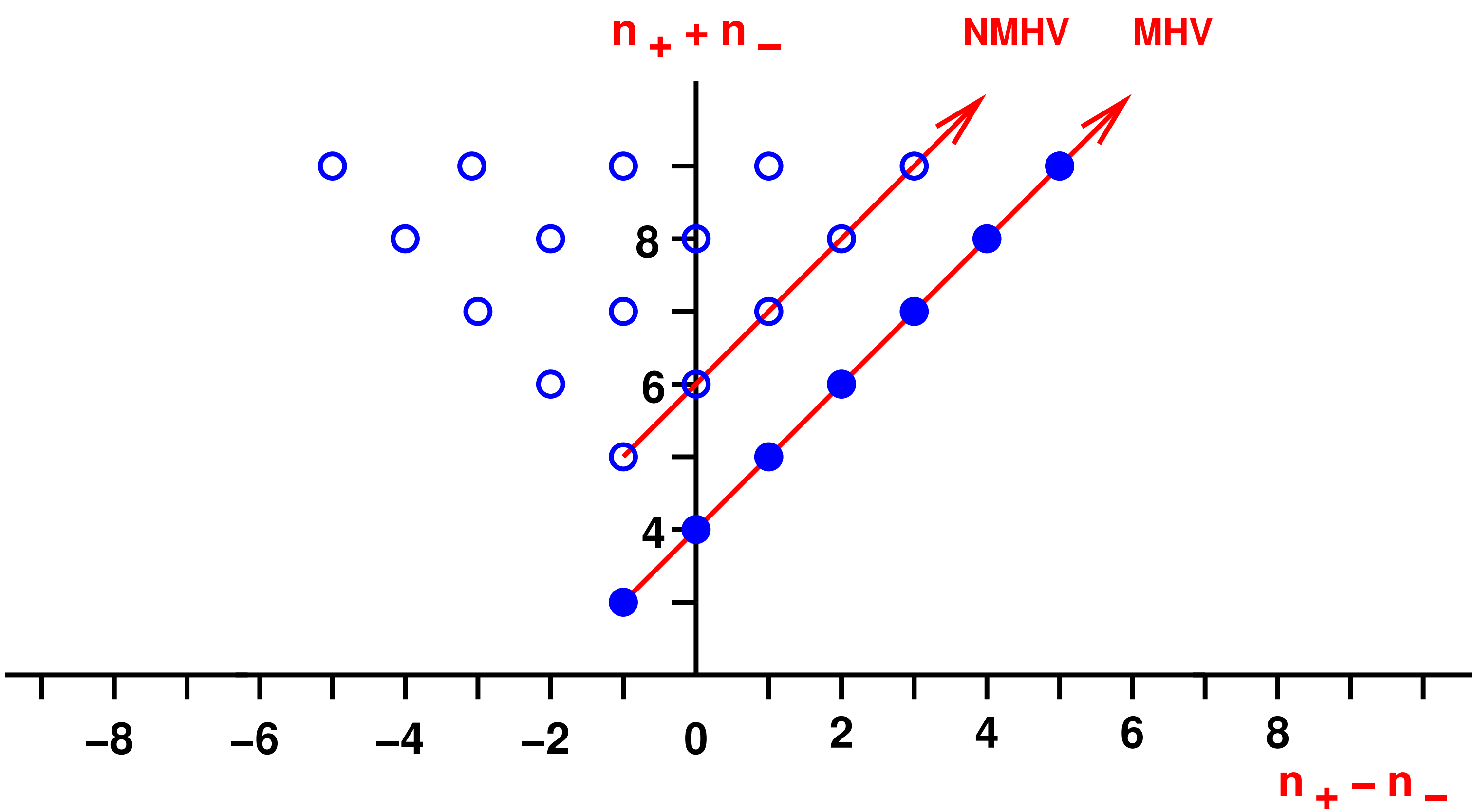
$$A = \sum_{i=3}^{n-1} \frac{\langle 1(2,i)\rangle^3}{\langle (2,i)i+1\rangle\langle i+1 i+2\rangle \ldots \langle n1\rangle} \frac{1}{s^2_{2,i}} \frac{\langle 23\rangle^3}{\langle (2,i)2\rangle\langle 34\rangle \cdots \langle i(2,i)\rangle}$$
$$+ \sum_{i=4}^{n} \frac{\langle 12\rangle^3}{\langle 2(3,i)\rangle\langle (3,i)i+1\rangle \ldots \langle n1\rangle} \frac{1}{s^2_{3,i}} \frac{\langle (3,i)3\rangle^3}{\langle 34\rangle \cdots \langle i-1 i\rangle\rangle\langle i(3,i)\rangle}.$$

where $(k,i) = k + \cdots + i$ and the off-shell continuation is suppressed

$\Rightarrow$ Lorentz invariant and gauge invariant expressions

Generating all the tree amplitudes

Amplitudes with $i-$ and $j+$ helicities



- MHV rules always adds one negative helicity and any number of positive helicities
  $\Rightarrow$ maps out all allowed tree amplitudes

Other processes

MHV rules have been generalised to many other processes

- √ with massless fermions - quarks, gluinos

  Georgiou and Khoze; Wu and Zhu; Georgiou, EWNG and Khoze

- √ with massless scalars - squarks

  Georgiou, EWNG and Khoze; Khoze

- √ with an external Higgs boson

  Dixon, EWNG, Khoze; Badger, EWNG, Khoze

- √ with an external weak boson

  Bern, Forde, Kosower and Mastrolia

Has provided new results for n-particle amplitudes
Also useful for studying infrared properties of amplitudes

Birthwright, EWNG, Khoze and Marquard

Processes with fermions

Similar colour decomposition

$$\mathcal{A}_n(1,\ldots,\Lambda_r,\Lambda_s,\ldots,n) = \sum_{perms} (T^{a_1}\ldots T^{a_n})_{r,s} A_n(\Lambda_r,1,\ldots,n,\Lambda_s)$$

MHV amplitude with 2 fermions and $n-2$ gluons

$$A_n(g_t^-,\Lambda_r^-,\Lambda_s^+) = \frac{\langle tr\rangle^3\langle ts\rangle}{\prod_{i=1}^n\langle i\; i+1\rangle}$$

MHV amplitude with 4 fermions and $n-4$ gluons

$$A_n(\Lambda_r^-,\Lambda_s^+,\Lambda_t^-,\Lambda_u^+) = \frac{\langle rt\rangle^3\langle su\rangle}{\prod_{i=1}^n\langle i\; i+1\rangle}$$

$\Rightarrow$ similar scalar graph construction for fermionic amplitudes

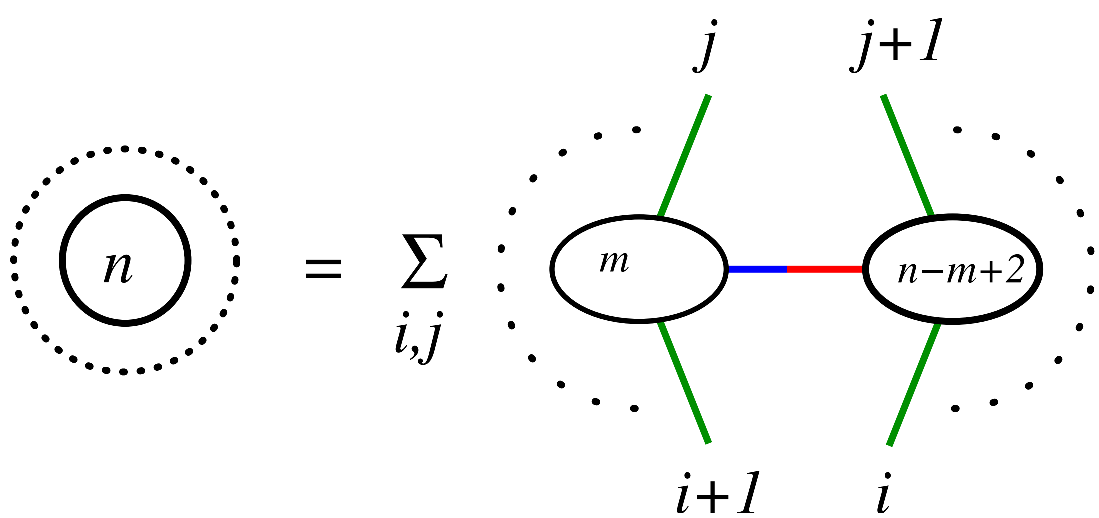
Georgiou and Khoze; Wu and Zhu; Georgiou, EWNG and Khoze

Recursive MHV amplitudes

As the number of negative helicity legs grows, the number of MHV diagrams grows

$\Rightarrow$ Use previously computed on-shell NMHV amplitudes as building blocks for recursion relation

Bena, Bern and Kosower

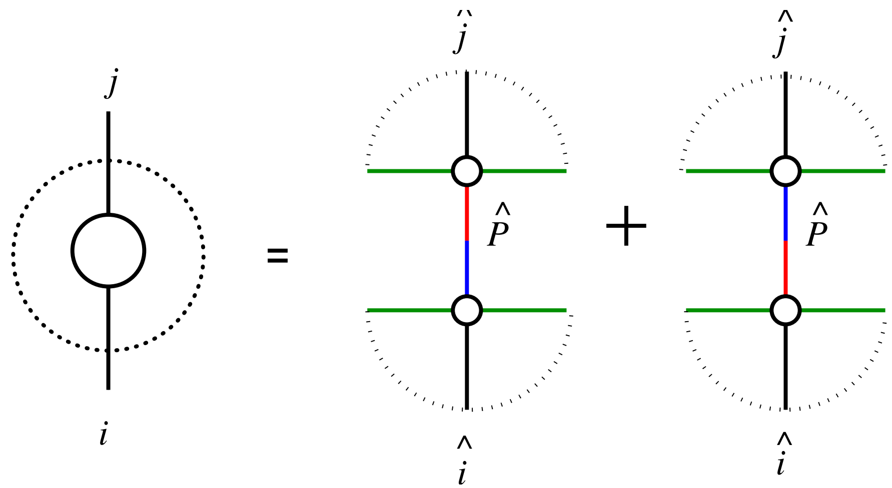


connected by same off-shell continuation as before.
Each blob is an amplitude with fewer particles and fewer negative helicities.

$\Rightarrow$ easily programmed

BCF recursion relations

Based on experience with one-loop amplitudes, Britto, Cachazo and Feng proposed a new set of on-shell recursion relations



Britto, Cachazo and Feng
Britto, Cachazo, Feng and Witten

hatted momenta are shifted to put on-shell

$$\hat{i} = i + z\eta, \qquad \hat{j} = j - z\eta, \qquad \hat{P} = P + z\eta$$

$\Rightarrow$ each vertex is an on-shell amplitude

BCF recursion relations

- It turns out that the shift η is not a momentum, but

$$\eta = \lambda_i \tilde{\lambda}_j \qquad OR \qquad \eta = \lambda_j \tilde{\lambda}_i$$

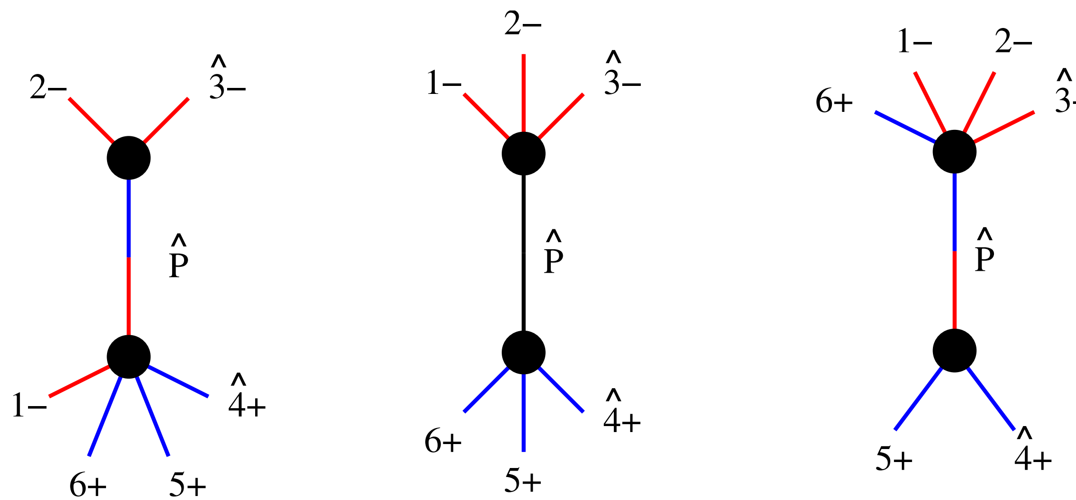
- The parameter z is given by

$$z = \frac{P^2}{\langle j P i]}$$

- Easy to prove that recursion relation is valid using complex analysis
- Requires on-shell three-point vertex contributions - both MHV and $\overline{\mathrm{MHV}}$.

BCF - six gluon example

If we select 3 and 4 to be the special gluons, there are only three diagrams (for any helicities)



For this helicity assignment, the middle one is zero!.

$A_6(1^-,2^-,3^-,4^+,5^+,6^+)$

$$= \frac{1}{\langle 5|\not{3}+\not{4}|2\rangle}\left(\frac{\langle 1|\not{2}+\not{3}|4\rangle^3}{[23][34]\langle 56\rangle\langle 61\rangle s_{234}} + \frac{\langle 3|\not{4}+\not{5}|6\rangle^3}{[61][12]\langle 34\rangle\langle 45\rangle s_{345}}\right)$$

Extremely compact (and correct) results for up to 8 gluons

Other processes

BCF recursion relations have been generalised to other processes

- √ with massless fermions - quarks, gluinos

  Luo and Wen

- √ gravitons

  Bedford, Brandhuber, Spence and Travaglini; Cachazo and Svrcek

There is nothing (in principle) to stop this approach being applied to particles with mass.

One loop amplitudes

- So far, supersymmetry was not a major factor - tree level amplitudes same for $\mathcal{N}=4$ $\mathcal{N}=1$ and QCD
- Not true at the loop level due to circulating states

$$
\begin{aligned}
A_n^{\mathcal{N}=4} &= A_n^{[1]} + 4A_n^{[1/2]} + 3A_n^{[0]} \\
A_n^{\mathcal{N}=1,chiral} &= A_n^{[1/2]} + A_n^{[0]} \\
A_n^{glue} &= A_n^{\mathcal{N}=4} - 4A_n^{\mathcal{N}=1,chiral} + A_n^{[0]}
\end{aligned}
$$

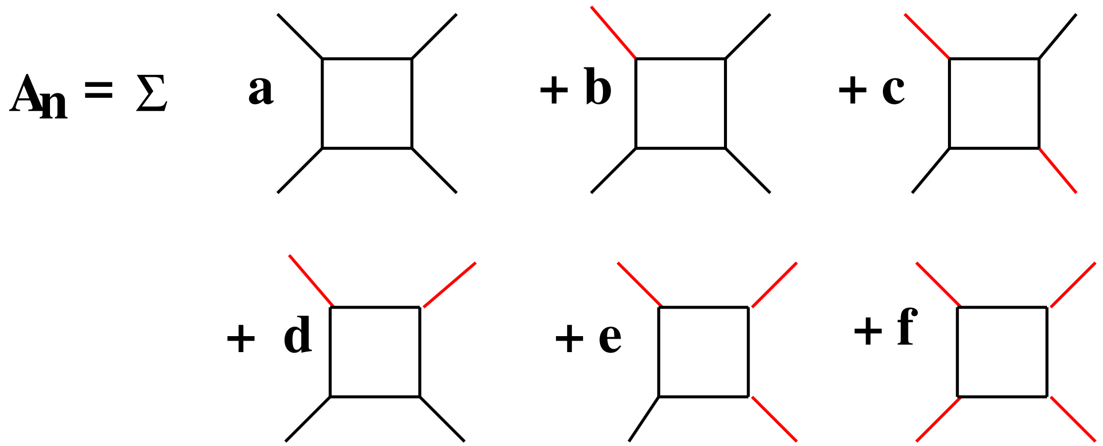
- All plus and nearly all-plus amplitudes do not vanish for non-supersymmetric QCD
- A lot of progress by a lot of people

SUSY QCD loops

- ✓ $\mathcal{N}=4$ and $\mathcal{N}=1$ one-loop amplitudes are constructible from their 4-dimensional cuts
  $\Rightarrow$ employ unitarity techniques

  Bern, Dixon, Dunbar, Kosower

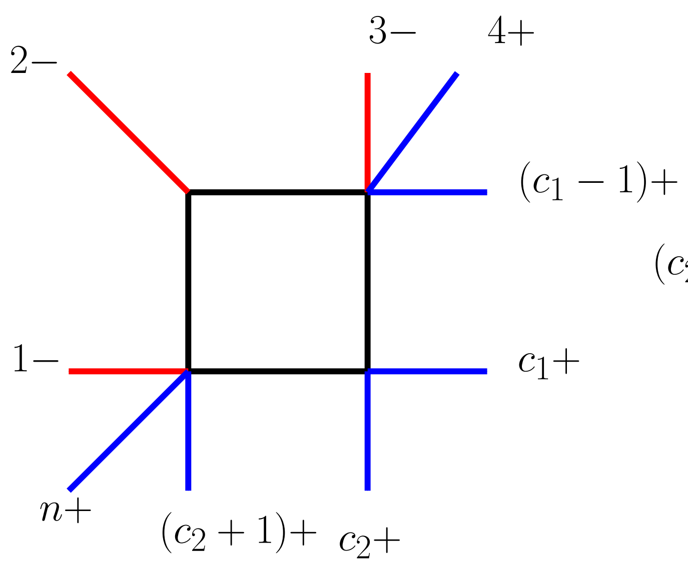
- ✓ For $\mathcal{N}=4$ all amplitudes are a linear combination of known box integrals

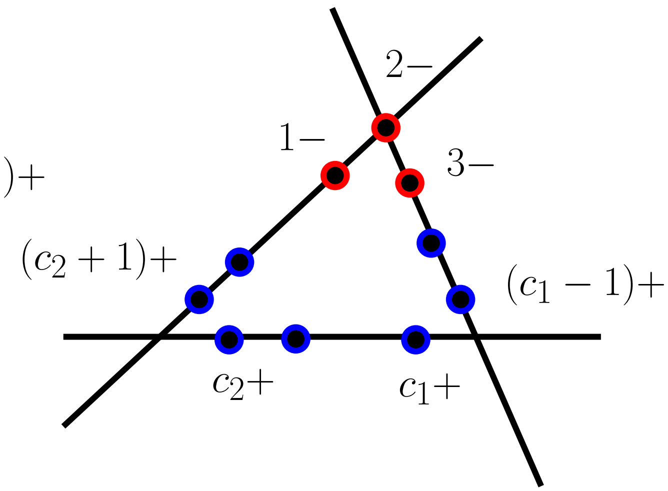


Twistor space interpretation

- Coefficients of boxes have very interesting structures.

Bern, Del Duca, Dixon, Kosower; Britto, Cachazo, Feng



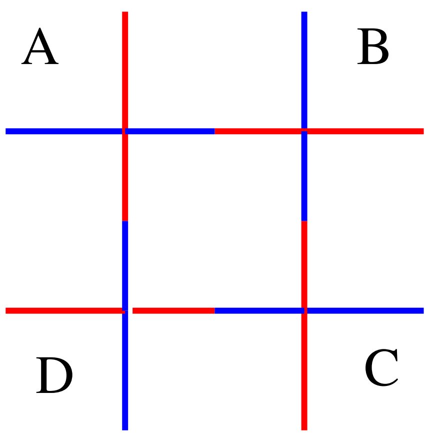


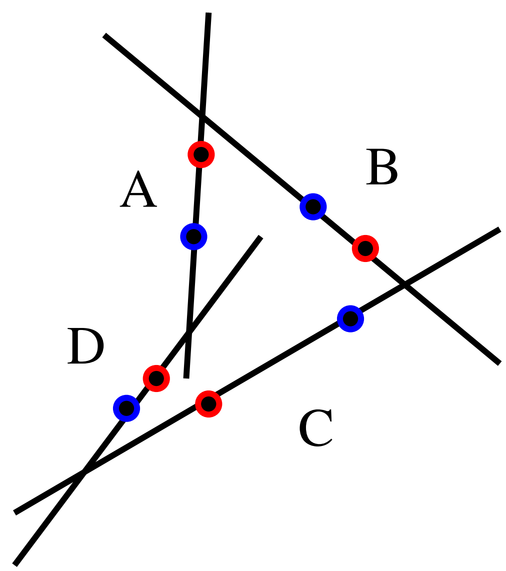
Twistor space interpretation

- Four mass box first appears in eight-point amplitude with four negative and four positive helicities

Bern, Dixon, Kosower

e.g.





QCD loops

QCD amplitudes more complicated

(a) Not 4-dimensional cut constructible. Rational function contribution not probed by 4-d cut

(b) All plus and almost all plus amplitudes not zero - but rational functions. Not protected by SWI.

Nevertheless, all four-point and five-point amplitudes known: Recent progress

√ On-shell recurrence relations for all plus and almost all plus amplitudes

Bern, Dixon and Kosower

Recursion relations complicated by double pole terms and boundary terms

√ Scalar six-point NMHV amplitudes

Bidder, Bjerrum-Bohr, Dunbar and Perkins

Computed parts of six-point QCD amplitudes that are obtainable using 4-dimensional cut constructibility

Summary - Precise predictions

Last few years has seen substantial progress in pQCD NNLO pQCD for collider phenomenology is becoming new standard

- Inclusive DIS coefficient functions completed
- Unpolarised three-loop splitting functions completed
- Differential distributions for Higgs and gauge bosons completed
- NNLO Jet cross sections on horizon for e^+e^- - and then pp/ep
- NNLO heavy quarks still a long way away

Summary - New rules for tree-level amplitudes

- MHV rules — Cachazo, Svrcek and Witten
  - √ New way of computing amplitudes with gluons and massless quarks
  - √ Higgs coupling to massless quarks and gluons

    Dixon, EWNG, Khoze; Badger, EWNG, Khoze
  - √ Vector bosons coupling to massless quarks

    Bern, Forde, Kosower and Mastrolia
- BCF recursion relations — Britto, Cachazo and Feng; Britto, Cachazo, Feng and Witten
  - √ Extended to quarks

    Luo and Wen
  - √ and gravitons

    Bedford, Brandhuber, Travaglini, Spence; Cachazo, Svrcek

Summary - New rules for one-loop amplitudes

- √ $\mathcal{N} = 4$ amplitudes
  almost at the point where coefficients of boxes can be read off - using quadruple cuts and holomorphic anomaly

  Britto, Cachazo and Feng

- ⇒ All NMHV amplitudes

  Bern, Dixon and Kosower

- √ $\mathcal{N} = 1$ MHV amplitudes and 6-point NMHV amplitudes
- √ Application to one-loop gravity

  Bern, Bjerrum-Bohr, Dunbar

- ? QCD amplitudes

  Bedford, Brandhuber, Spence and Travaglini; Bern, Dixon and Kosower; Bidder, Bjerrum-Bohr, Dunbar and Perkins

A very exciting and rapidly developing field
Expect more important results soon