



Nobelpreis 2004



photo PRB

David Politzer



photo PRB

Frank Wilczek

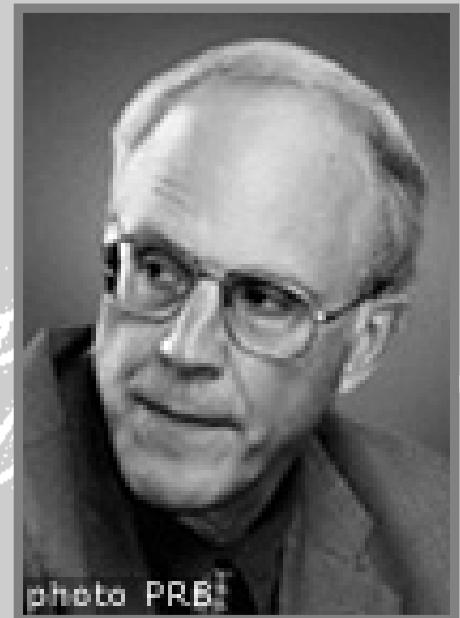


photo PRB

David Gross

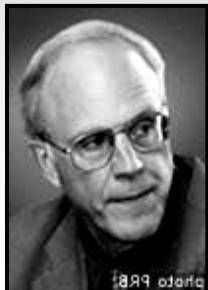


Th. Naumann DESY



Physik-Nobelpreis 2004

für die Entdeckung der asymptotischen Freiheit
in der Theorie der starken Wechselwirkung an:



David J. Gross

Kavli Inst. for Theoretical Physics,
Univ. of California, Santa Barbara,



H. David Politzer

California Inst. of Technology (Caltech),
Pasadena, und



Frank Wilczek

Massachusetts Inst. of Technology (MIT),
Cambridge, alle USA.



Physik-Nobelpreis 2004

- Das Problem: QED
 - Die Ladung wird unendlich
- Die Lösung: QCD
 - Die Kopplung läuft: asymptot. Freiheit
 - Die Kernkraft wird stark: Confinement
- Der Test + :
 - Laufende Kopplung
 - Protonen und Gluonen
 - QCD-Vakuum: Glueballs + Quagma
- Präzision
 - Störungstheorie + Gitter-QCD
- Der Weg zur Ur-Kraft
 - Super-Symmetrie + Große Vereinigung

Nobelpreise



Elektromagnetismus

- 1948 Tomonaga, Schwinger, Feynman: Quantum Electro Dynamics 1965

Schwache Kraft

- 1934 Fermi: Theory of Beta Decay 1938
- 1954 Yang, Lee: Gauge Theory of Weak Interactions 1957
- 1971 Glashow, Salam, Weinberg: Theory of Electroweak Interactions 1979
- 1971 t'Hooft, Veltman: Renormalization of Electroweak Interaction 1999

Starke Kraft

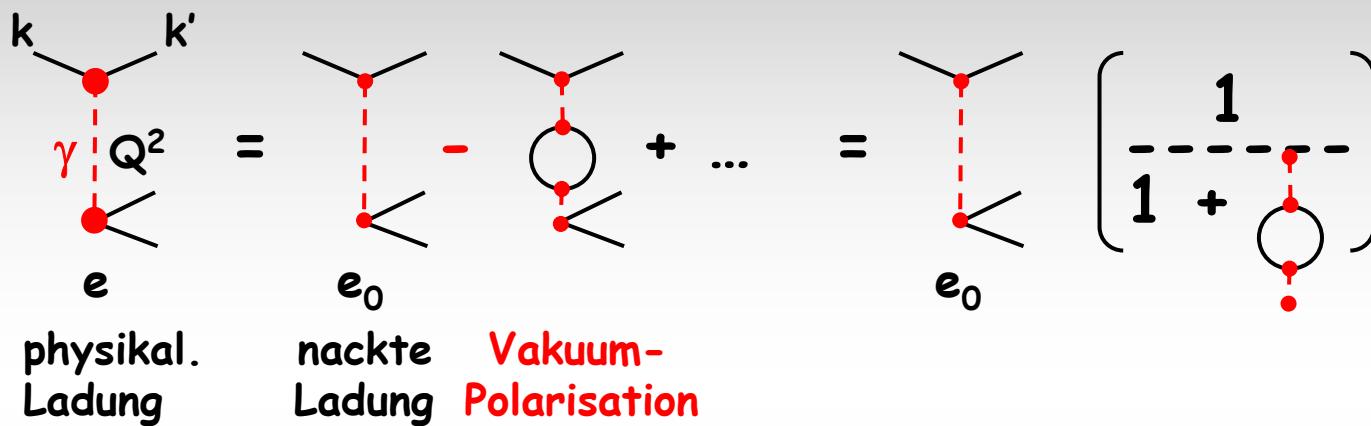
- 1935 Yukawa: Theory of Nuclear Forces 1949
- 1964 Gell-Mann: Symmetries of elementary particles: $SU(3)$ 1969
- 1969 Friedman, Kendall, Taylor: Quark discovery in ep scattering 1990
- 1971 Wilson: Theory of phase transformations: Renorm. group 1982

QED:

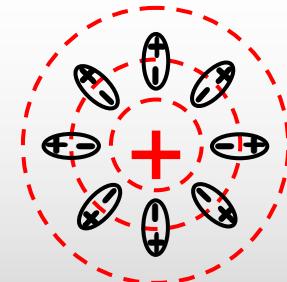
**laufende
Konstanten**

QED

Vakuum-Polarisation in e-e Streuung:



dielektr.
Screening:



klass. Elektron-Radius
 $r = \alpha/m_e \sim 3 \text{ fm}$
 Compton-Wellenlänge
 $\lambda_c = 1/m_e$

- Kopplung \sim Ladung 2 : $F = \alpha/r^2$ $\alpha = e^2/4\pi$
- infrarot stabil: $\alpha = 1/137$
 ultraviolet divergent - nackte Ladung unendlich !?
- Cutoff bei willkürl. Skala: Renormierung !
 Energie-Skala: $Q^2 = -(k-k')^2$
- betrachte nur Evolution von Energieskala Q zu Skala μ
 UV Divergenzen kürzen sich:

Die Evolution ist alles - das Ziel ist divergent.



Nobel Prize
for first
Quark Evidence.

Physics Today,
Jan. 1991.

ALRIGHT RUTH, I ABOUT GOT THIS ONE RENORMALIZED

Das Problem

- Landau 1955:
„weak coupling electrodynamics is ...
fundamentally logically incomplete.“
„within the limits of formal electrodynamics
a point interaction is equivalent ... to no interaction at all.“
- Dyson 1960:
“The correct theory will not be found within the next 100 years.”
- Feynman 1961:
“I still ... do not subscribe to the philosophy of renormalization.”
- Weinberg 1972, Gravitation + Cosmology:
„we encounter theoretical difficulties beyond the range
of modern statistical mechanics.“

Renormierung

- Petermann, Stückelberg 1951. Gell-Mann, Low 1954.
- Bogoljubov, Shirkov 1956. Callan, Symanzik 1970.
- Wilson 1971. Teilchen- u. Festkörperphysik. OPE. Gitter.
- Renormiere Ladung + Cutoff so, daß Physik nicht von willkürl. Energie-Skala μ abhängt:
- **β -Funktion:** $2\beta = \delta \alpha(\mu) / \delta \ln(\mu)$
- Renormierungsgruppen-Glg:

$$\beta=0 \text{ für } \mu \rightarrow 0 \quad \text{IR stabil} \quad \alpha(0) = 1/137$$

• Problem:

Punkt-Ww. heißt **UV stabil** ! $\alpha(\infty) = ?$

$\beta=0$ für $\mu \rightarrow \infty$

Gibt es asymptot. freie Feldtheorien ?

Gibt es Punkt-Ww. (an Elementarteilchen) ?

Die laufende Kopplung

$\alpha = e^2/4\pi$ Ladung ~ Kopplung = Feinstruktur-Konstante
Konstante nicht konstant:

$$2\beta = \partial \alpha(\mu) / \partial \ln \mu = \frac{2}{3\pi} \alpha^2 + \dots$$

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log \frac{Q^2}{\mu^2}}$$

$\alpha(E)$ running (or crawling):

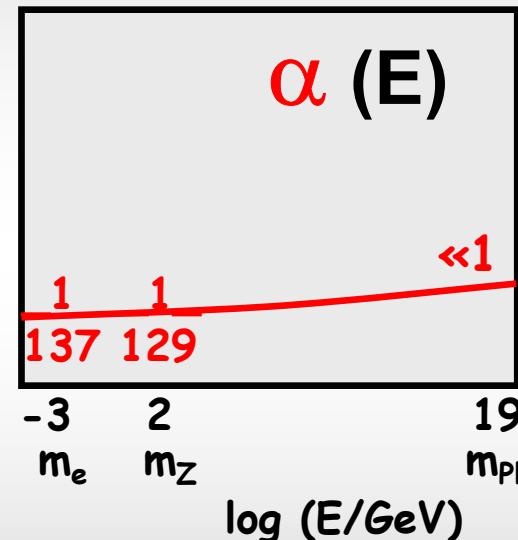
$$\partial \alpha(Q^2) / \partial Q^2 > 0$$

$$\alpha(m_e = 0.5 \text{ MeV}) = 1/137$$

$$\alpha(m_Z = 91 \text{ GeV}) = 1/128.9 \quad \text{CERN LEP}$$

$$\alpha(m_{Pl} = 10^{19} \text{ GeV}) \ll 1 \quad \text{mehr Fermion-Loops}$$

$$\alpha(\infty) \quad \text{undefiniert} \quad \text{keine elektr. Punkt-Ww.}$$



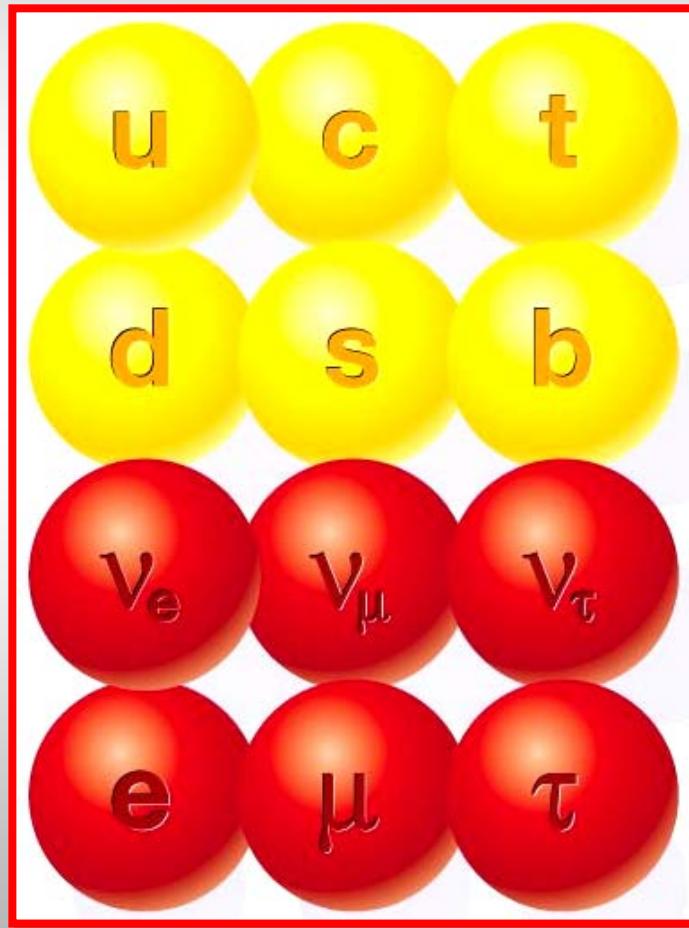
Das Standard Modell

Die Bausteine

3 Familien

QUARKS

LEPTONEN



Up

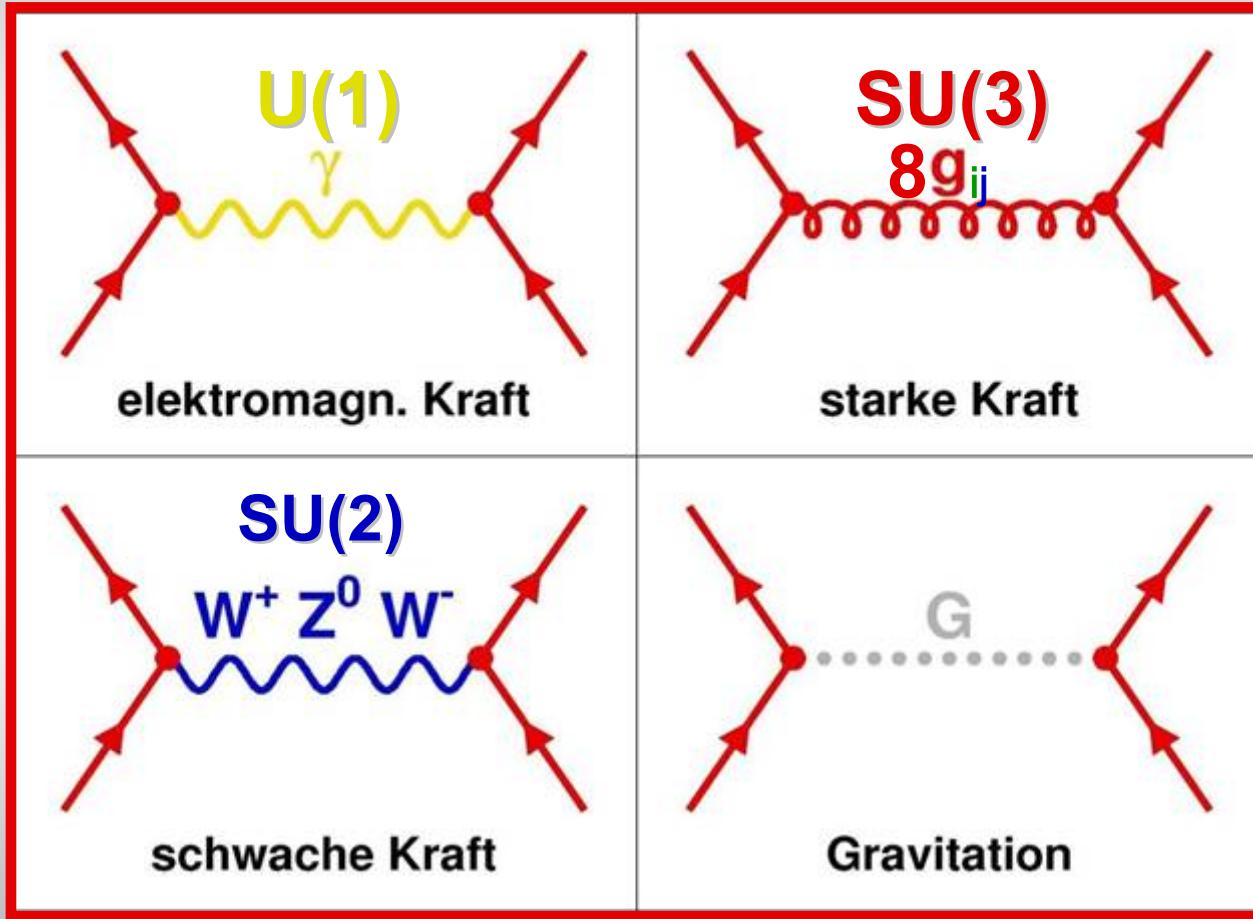
x2 = 6 Flavors

Down

Neutrinos

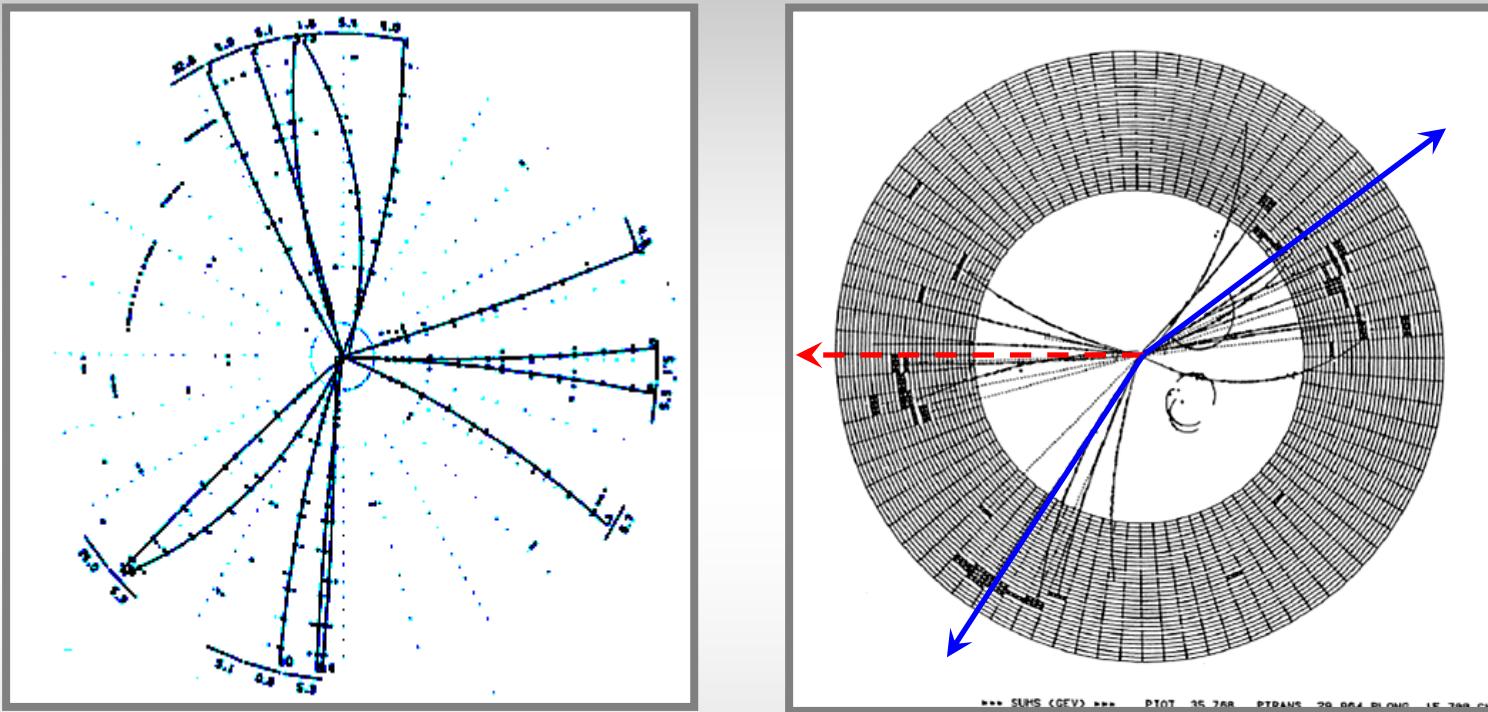
Elektronen

Die Kräfte

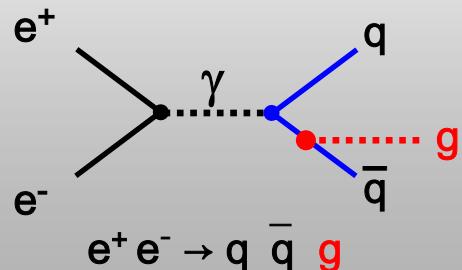




Das Gluon



Entdeckt 1979 bei PETRA am DESY in



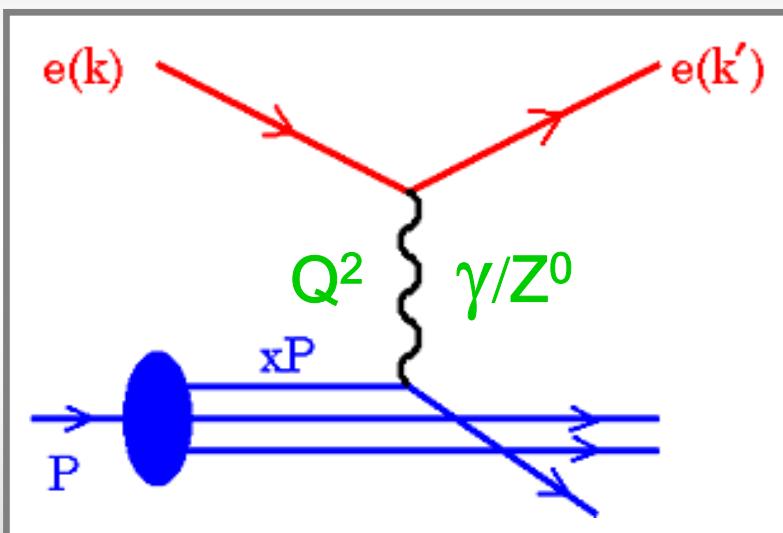
Kinematik

Energie² im Massenschwerpunkt-System:

$$s = (k + P)^2 = 4 E_e E_p$$

$$s = 4 \cdot 27.6 \text{ GeV} \cdot 920 \text{ GeV} = (319 \text{ GeV})^2$$

HERA:



$$x = Q^2 / (2P \cdot q)$$

Bjorken
Impuls-Anteil des Partons im Proton
(Quark, Gluon)

$Q^2 = -q^2 = - (k - k')^2$
Quadrat des
Viererimpuls-Übertrags

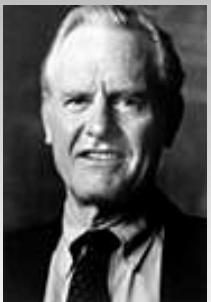
Rutherford 1911:
Photon-Propagator
 $d\sigma / dQ^2 = 4\pi\alpha^2 / Q^4$
 $d\sigma / d\cos(\theta) = \pi\alpha^2 / 2E^2 \sin^4(\theta/2)$

Mott 1929, Elektron-Spin:
 $\sigma_{\text{Mott}} = \cos^2(\theta/2) \sigma_{\text{Ruth.}}$

Quarks + Scaling



Friedman



Kendall



Taylor

Nobelpreis



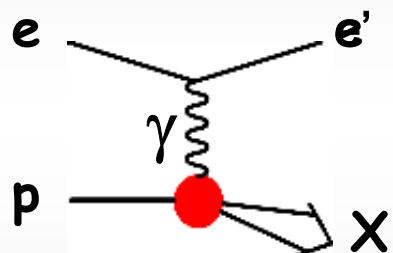
1990

Stanford Linear Accelerator

USA, 1968-71:

Elektron-Proton

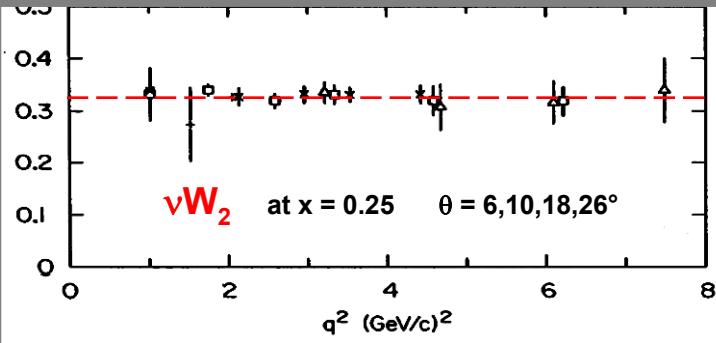
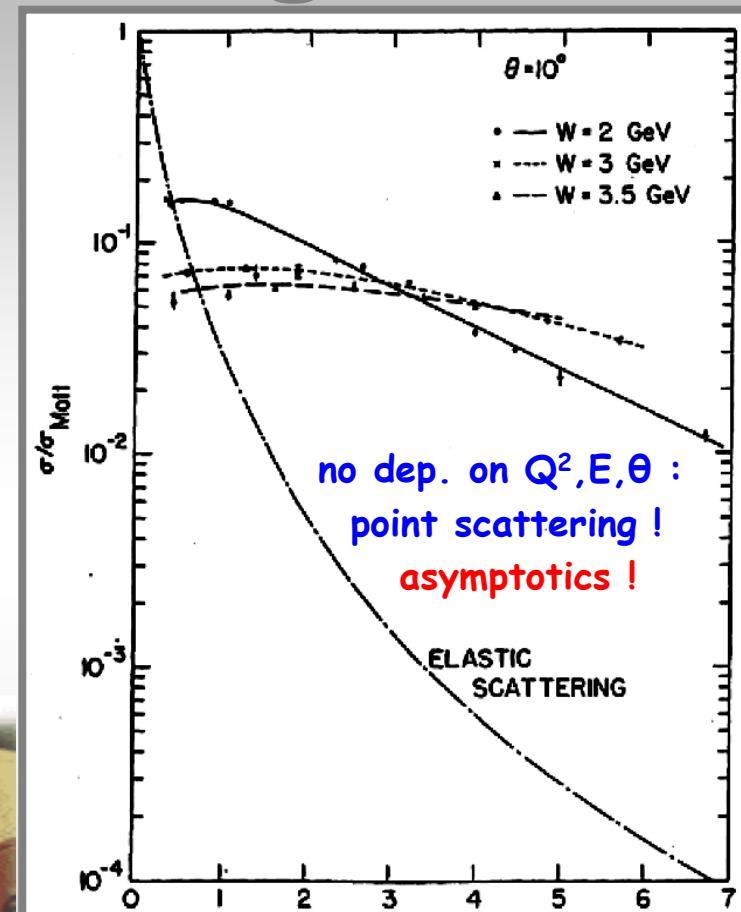
tief inelastische Streuung



$E = 1.5 - 20 \text{ GeV}$

$\theta = 6^\circ - 26^\circ$

$Q^2 = 1 - 7 \text{ GeV}^2$



Callan-Gross Relation

Parton Spin	Photon Polarisation
0,1	$\sigma_T = 0$ longitudinal
1/2	$\sigma_L = 0$ transversal

$$\sigma_L / \sigma_T = 0$$

elektr. = magnet. Formfaktor (Spin flip)

$$F_2(x) = 2x F_1(x) \quad \text{Callan-Gross}$$

Bausteine des Protons:
punktformig + Spin=1/2

Feynman: Parton-Modell

erst QCD Gluon-Strahlung
gibt transversale Freiheitsgrade:

$$F_2(x) \neq 2x F_1(x) \quad \sigma_L > 0$$

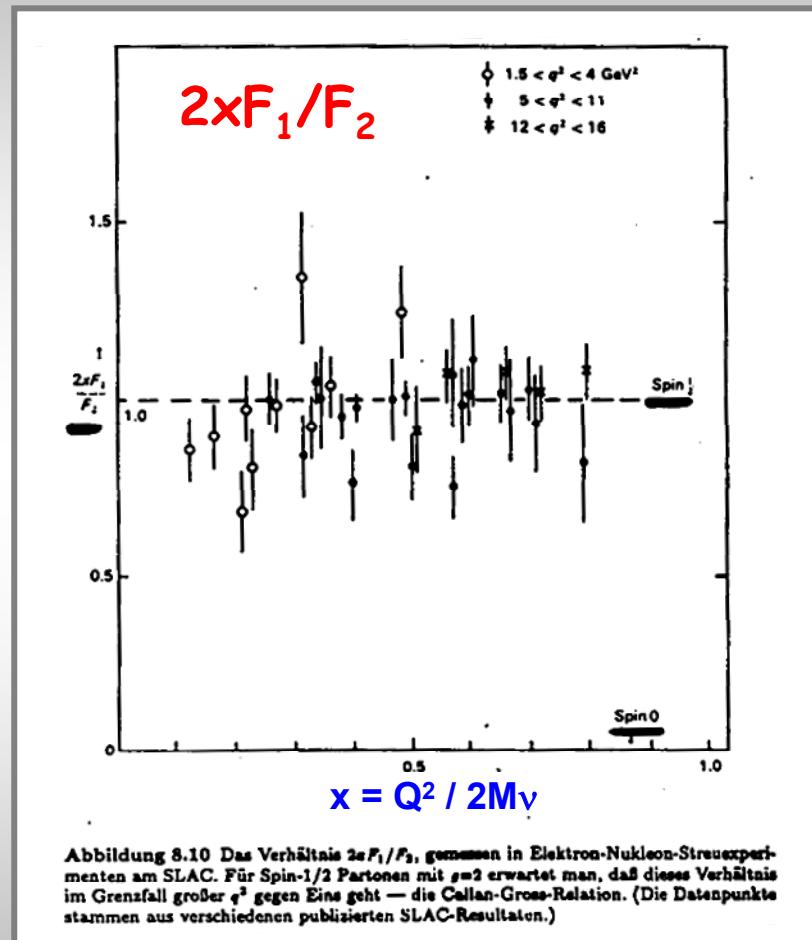
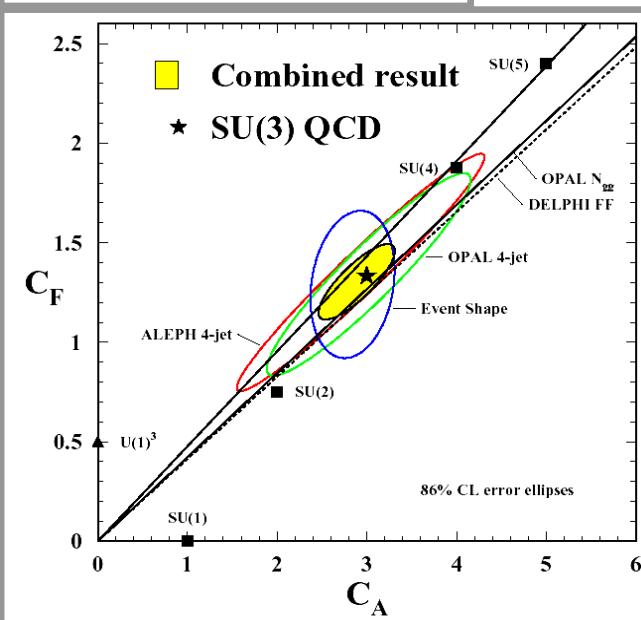
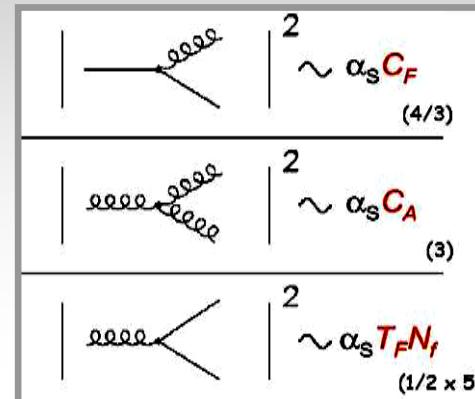
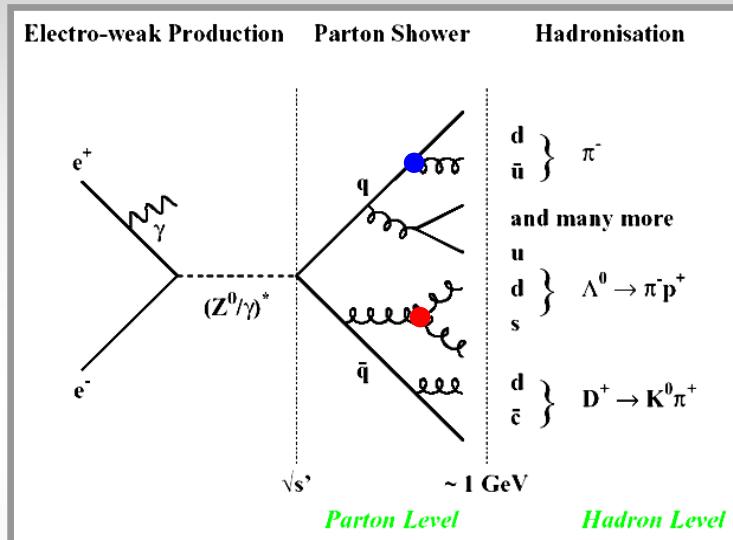


Abbildung 8.10 Das Verhältnis $2x F_1 / F_2$, gemessen in Elektron-Nukleon-Streuexperimenten am SLAC. Für Spin-1/2 Partonen mit $g=2$ erwartet man, daß dieses Verhältnis im Grenzfall großer q^2 gegen Eins geht — die Callan-Gross-Relation. (Die Datenpunkte stammen aus verschiedenen publizierten SLAC-Resultaten.)

QCD = SU(3) ?

CERN LEP: $e^+e^- \rightarrow 4 \text{ jets}$ event shape in NLO QCD



**SU(N) Casimir-Operatoren
QCD Farb-Faktoren**

$$C_A$$

$$C_F$$

$$\text{SU}(3): \quad 3 \quad \frac{4}{3}$$

$$\text{Expt.:} \quad 3.0 \pm 0.5 \quad 1.3 \pm 0.3$$

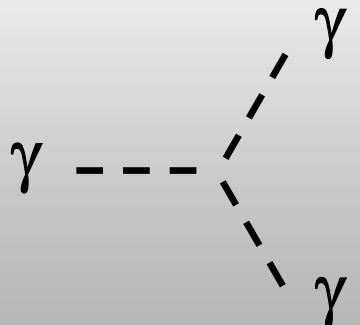
Quanten-Chromo-Dynamik

• QED

- $U(1)$, abelsch
- 1 Ladungs-Typ
- 1 Photon:
- elektr. **neutral**
- **keine** Photon-Photon

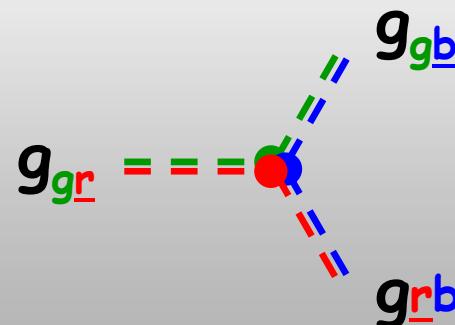
Selbst-Kopplung:

- Licht klumpt nicht ...



• QCD

- $SU(3)_{\text{COLOR}}$ nicht-abelsch
 - 3 Ladungs-Typen: **r,g,b**
 - $\{3\} \otimes \{\bar{3}\} = \{1\} \oplus \{8\}$: 8 Gluonen:
 - tragen Farb-Ladungen
 - Gluon-Gluon
- Selbst-Kopplung**
- Gluonium, Glueballs



Der QCD Lagrangian

$$\mathcal{L} = \frac{1}{4g^2} G_{\mu\nu}^a G^{a\mu\nu} + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

where $G_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + i f_{bc}^a A_\mu^b A_\nu^c$

and $D_\mu \equiv \partial_\mu + i t^a A_\mu^a$

That's it!

j ... quark flavors a,b,c ... 3 colors μ,ν ... space-time

F.Wilczek, Physics Today, August 2000.

1973:
D.Gross, F.Wilczek, D.Politzer
Gibt es
Asymptotisch freie Eichtheorien ?

**W.+P.: Doktoranden, 22+24 Jahre alt,
erste Publikation !**

D.J. Gross, F. Wilczek, "Ultraviolet Behavior of Non-Abelian Gauge Theories",
Phys.Rev.Letters 30 **1343** (1973).

H.D. Politzer, "Reliable Perturbative Results for Strong Interactions",
Phys.Rev.Letters 30 **1346** (1973).

D.J. Gross, F. Wilczek, "Asymptotically Free Gauge Theories. I",
Phys.Rev. D8 3633 (1973).

H.D. Politzer, "Asymptotic Freedom: An Approach to Strong Interactions",
Phys.Rep. 14 129 (1974).

β -Funktion nicht-abelscher Eichtheorien

entwickle nach Potenzen d. Kopplung + finde Nullstellen

$$\mu \frac{\partial \alpha_s}{\partial \mu} = 2\beta(\alpha_s) = -\frac{\beta_0}{2\pi} \alpha_s^2 - \frac{\beta_1}{4\pi^2} \alpha_s^3 - \frac{\beta_2}{64\pi^3} \alpha_s^4 - \dots$$

$$\beta_0 = (11 N_c - 2 N_f) / 3$$

N_c ... Zahl d. Farben N_f ... Zahl d. Flavors

Casimir-Operatoren d. Eich-Gruppe $SU(N)$

für $N_f \leq 16$ Fermion-Flavors gewinnen N_c Boson-Colors:

$$\partial \alpha(\mu) / \partial \ln \mu < 0$$

Nicht-abelsche Eichtheorien asymptotisch frei !

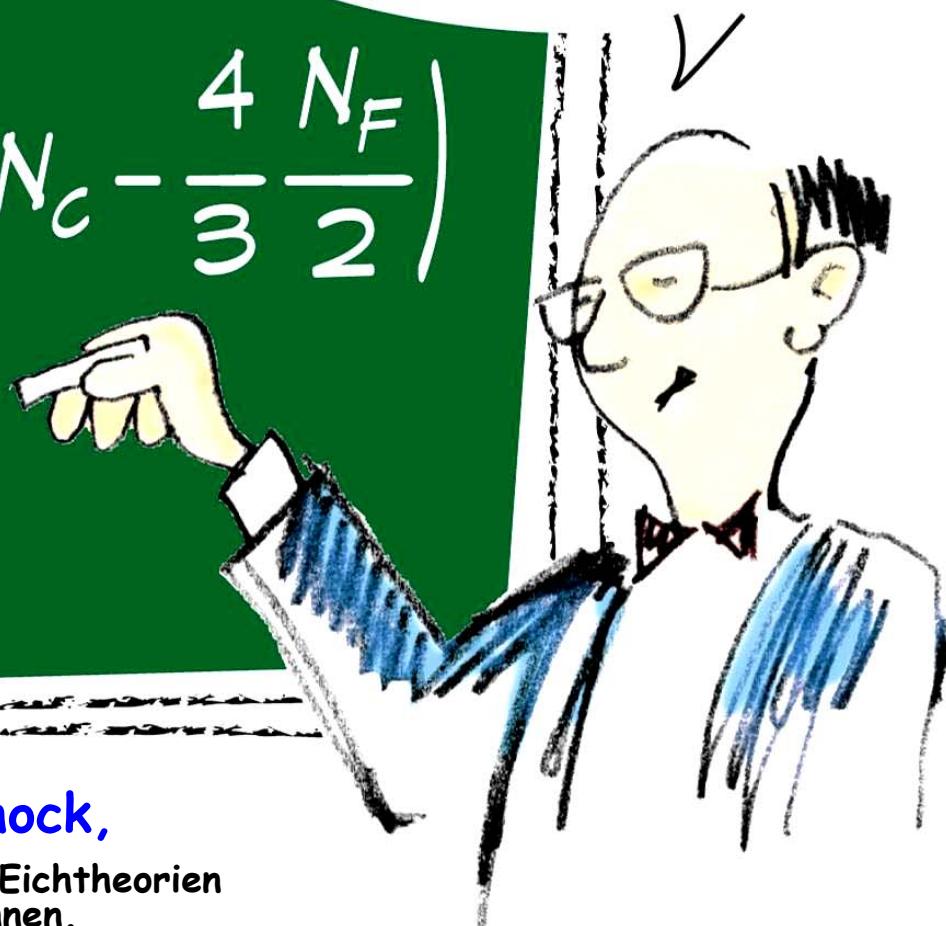
D.Gross, Zinnowitz 2004:

The discovery of asymptotic freedom was totally unexpected ...

Field theory was not wrong.

In QCD and the Standard Model
the beta function is indeed
negative!

$$\beta(g) = \frac{-g^3}{16\pi^2} \left(\frac{11}{3}N_c - \frac{4}{3}\frac{N_F}{2} \right)$$



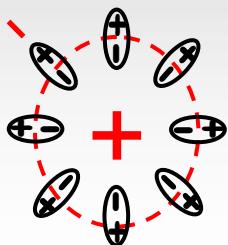
G.t'Hooft: Es war wie ein Schock,

als man entdeckte, daß die nicht-abelschen Eichtheorien
einen negativen β -Koeffizienten besitzen können.

(Lexikon d. Physik, Spektrum Verlag 2000)

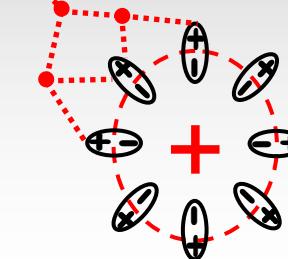
QED und QCD

Screening:



$$\gamma = \dots - q + \dots$$

Anti-Screening:



$$g = \dots - q + g + \dots$$

$$-2N_F + 11N_c$$

$$b_0 = -\frac{4}{3}$$

$$\partial \alpha(Q^2) / \partial Q^2 > 0$$

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 + b_0 \frac{\alpha(\mu^2)}{4\pi} \log \frac{Q^2}{\mu^2}}$$

$$b_0 = \frac{(-2N_F + 11N_c)}{3}$$

$$\partial \alpha(Q^2) / \partial Q^2 < 0$$

Gluon masselos !

$SU(2)_W: m_W > 10^5 m_e$

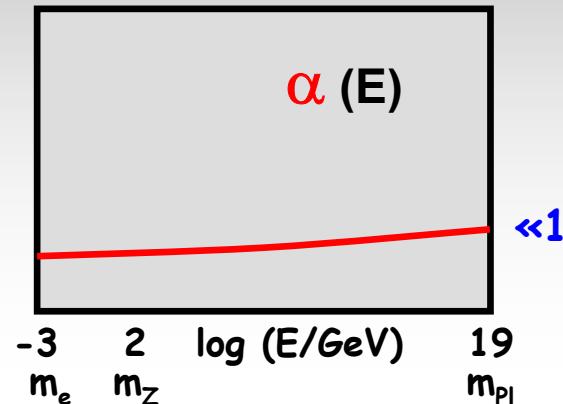
QED und QCD

QED:

$$\partial \alpha(\mu^2) / \partial \mu^2 > 0$$

Screening

IR:
 $\alpha = 1/137$



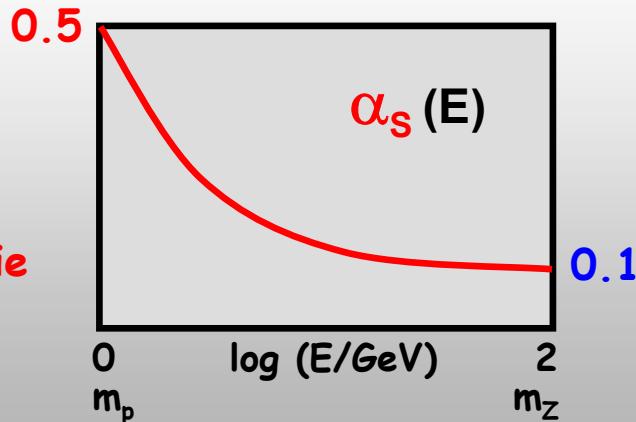
UV:
 Landau-Singularität

QCD:

$$\partial \alpha(\mu^2) / \partial \mu^2 < 0$$

Anti-Screening

IR:
 $\alpha_s \rightarrow \infty$
 Kollaps d.
 Störungstheorie
 infrarote
 Sklaverei
 Confinement



UV:
 $\alpha_s \rightarrow 0$
 asymptot.
 Freiheit

Confinement

statt $\alpha_s(\mu^2)$ definiere

$$\Lambda = \mu \exp [-2\pi/(b_0 \alpha_s(\mu^2))]$$

$$\alpha_s(Q^2) = \frac{4\pi}{9 \ln (Q^2/\Lambda^2)} + \dots \quad (N_F=3)$$

$$\alpha_s(Q^2 \rightarrow \Lambda^2) \rightarrow \infty$$

Kollaps der Störungstheorie
Kernkraft schließt sich ein
infrarote Sklaverei:
keine freien Quarks !

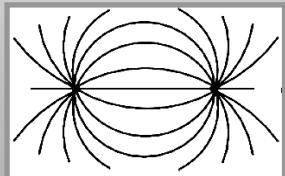
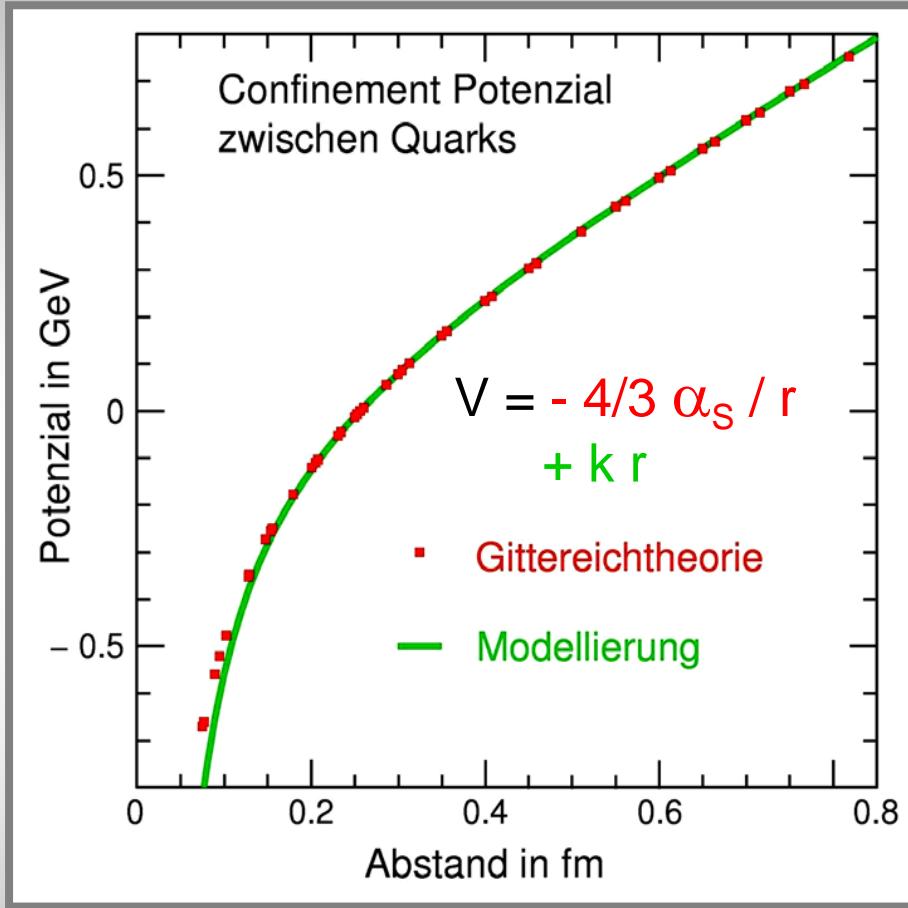
$\hbar c \approx 200 \text{ MeV} \cdot \text{fm}$
QCD-Skala • Protonradius

Proton = QCD Black Hole

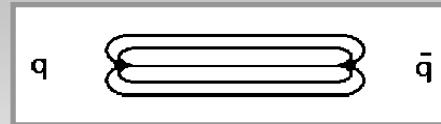
Quarks
are
born free,
but everywhere
they are
in chains.

F. Wilczek
Nobel talk
2004

Quarkonium



schwere Quarks = kurze Distanz:
 $V = -4/3 \alpha_s/r$ Coulomb-Kraft
 asymptot. Freiheit



Hadron-Radius:
Confinement

Color-String:
 konstante Kraft= Energie/Länge:
 $k = 1 \text{ GeV / fm}$

Spektroskopie
 gebundener Zustände
 schwerer Quarks:

$$\Psi, \Psi', \Psi'', \dots = (c \bar{c})$$

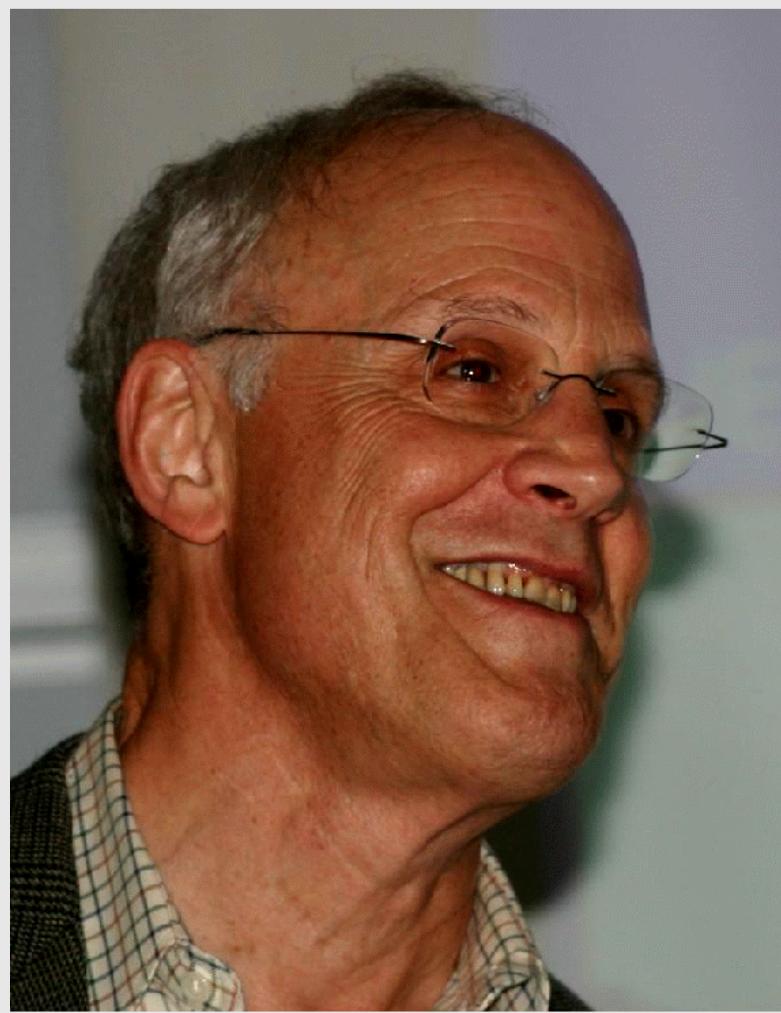
$$Y, Y', Y'', \dots = (b \bar{b})$$

wie Positronium

Appelquist, Politzer 1974.



D.Gross 2004:



Experiments
are performing
tests of QCD with
amazing precision ...

D.Gross, Loops & Legs,
DESY Theory Workshop,
Zinnowitz, Germany,
April 2004.

Experimente

von
infraroter Sklaverei
zur
asymptotischen Freiheit

Die laufende Kopplung

$$\alpha_s(Q^2) = \frac{4\pi}{9 \ln(Q^2/\Lambda^2)} + \dots$$

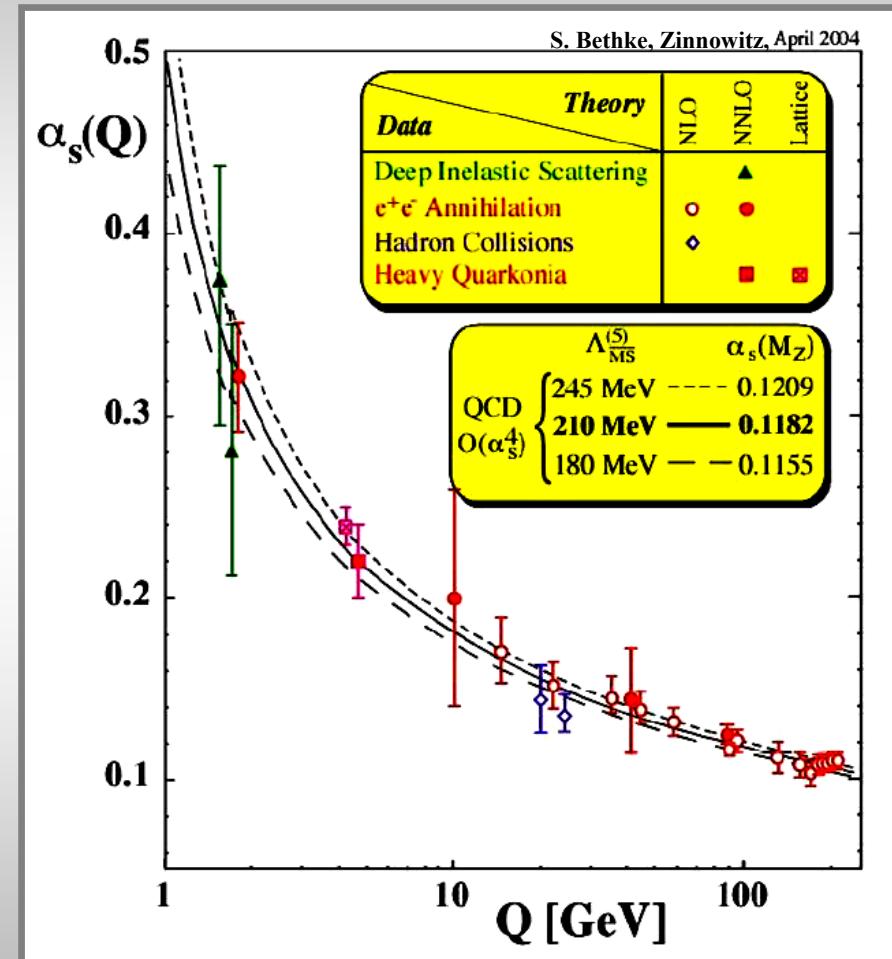
$$\alpha_s(M_Z) = 0.1182 \pm 0.0027$$

QCD Skala:

$$\Lambda = 210 \pm 30 \text{ MeV} \quad (\text{MS}, N_F=5)$$

$$\hbar c \approx 200 \text{ MeV} \cdot \text{fm}$$

QCD-Skala • Protonradius



Frank Wilczek:

The most dramatic of these [tests], that protons viewed at ever higher resolution would appear more and more as field energy (soft glue), was only



clearly verified at HERA
twenty years later.

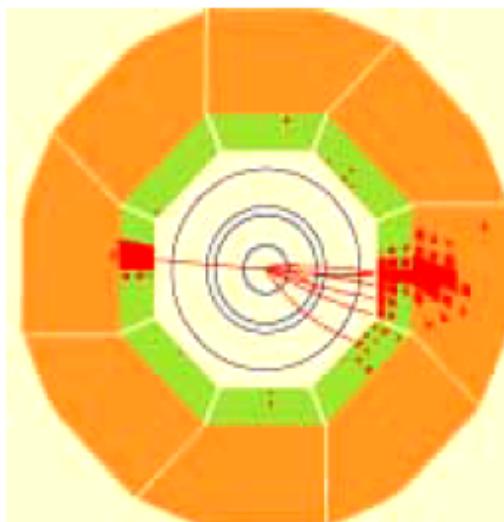


F.Wilczek
publication list
[www. 2001.](http://www.2001.de)



The Nobel Prize in Physics 2004



Picture: the DESY-laboratory, Hamburg

The theory shows its true colours

The aftermath of a high-energy collision between a proton and an electron, as seen by the H1 experiment at the DESY laboratory in Hamburg. The experiment is shown in cross-section, perpendicular to colliding beams of protons and electrons. The electron has struck one of the quarks in a proton. An impressive shower of particles - providing information about the struck quark - is spontaneously produced from the energy stored in the gluon force-field. The charged particles in the shower bend in the experiment's strong magnetic field.

And you're glue

Nature, Vol. 400, 1 July 1999.

Frank Wilczek

It's a widely believed half-truth that protons and neutrons are made out of quarks. Actually, physicists are increasingly discovering that it's considerably less than half the truth. The modern theory of the strong force, which binds quarks inside protons and neutrons, and these particles in turn to make atomic nuclei, is quantum chromodynamics (QCD). The other ingredients of QCD, the colour gluons, were once conceived as mere paste that somehow links together more substantial stuff (their name reflects this). No longer. On closer inspection, the quarks appear as the showier, but gluons as the weightier and more dynamic, constituents of matter. Definitive images¹ from a microscope capable of looking inside protons, the HERA accelerator in Hamburg, Germany, reveal as well that there is more to gluons than meets the eye.

To understand these evolving views, you must consider how one goes about looking inside a proton, to 'see' what it is made of. An ordinary microscope, using ordinary light, is woefully inadequate, because the wavelength of light is about one billion times larger than the size of the proton. Even fancy electron or scanning tunnelling microscopes can barely resolve single atoms, and fall far short of seeing the nucleus inside. The right tool for the job is a high-energy accelerator. They produce virtual photons of very short wavelength (and lifetime), that can be used to take snapshots of the proton's interior (Box 1, overleaf).

There's a catch, however, to this seemingly straightforward procedure. You get to see only what the virtual photon allows you to see. And because the photons couple only to electrically charged particles, constituents of





6
km

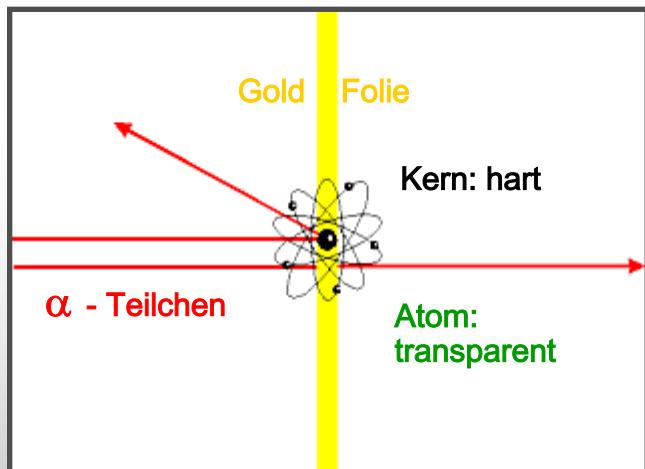
WR 88 QR tesla



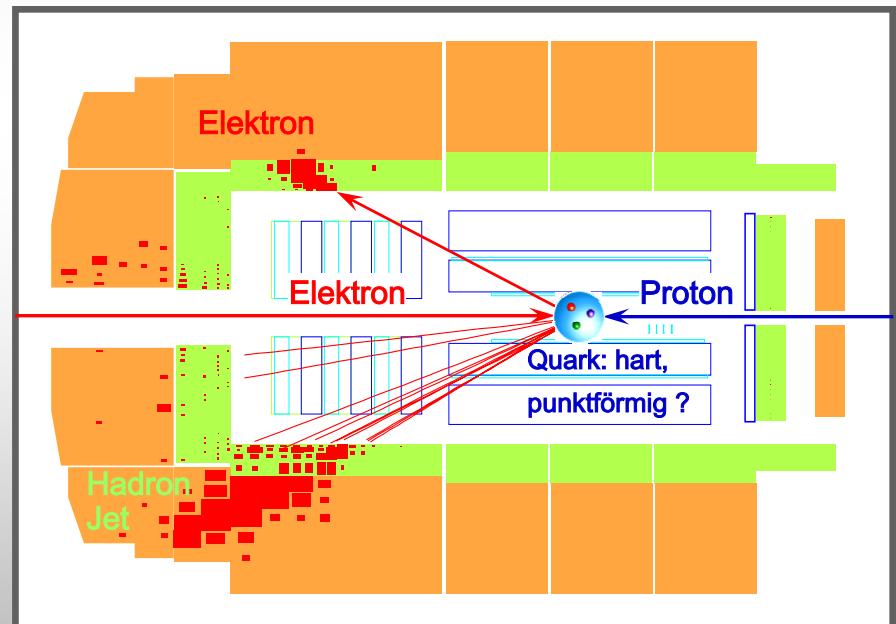
ZEUS

Struktur der Materie

Rutherford 1910 :
Entdeckung des Atomkerns

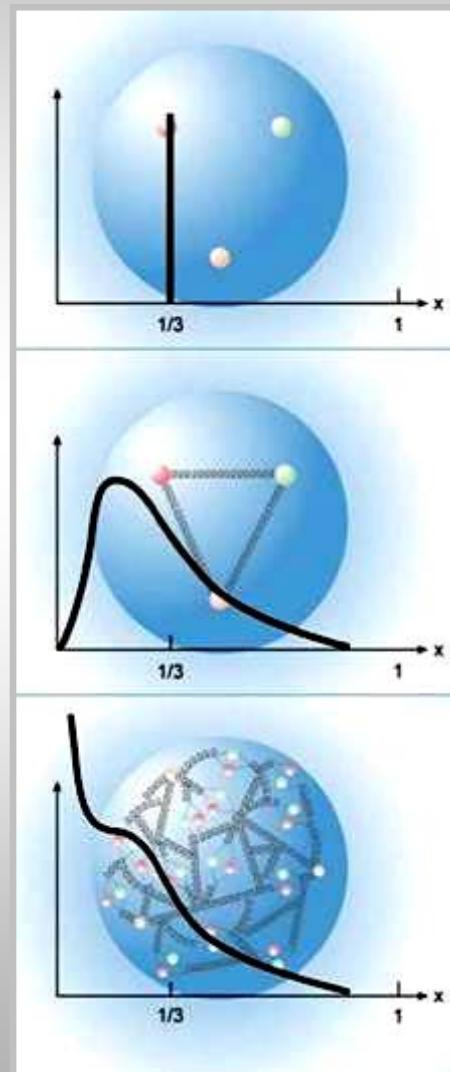


HERA 2000 :
Struktur des Protons



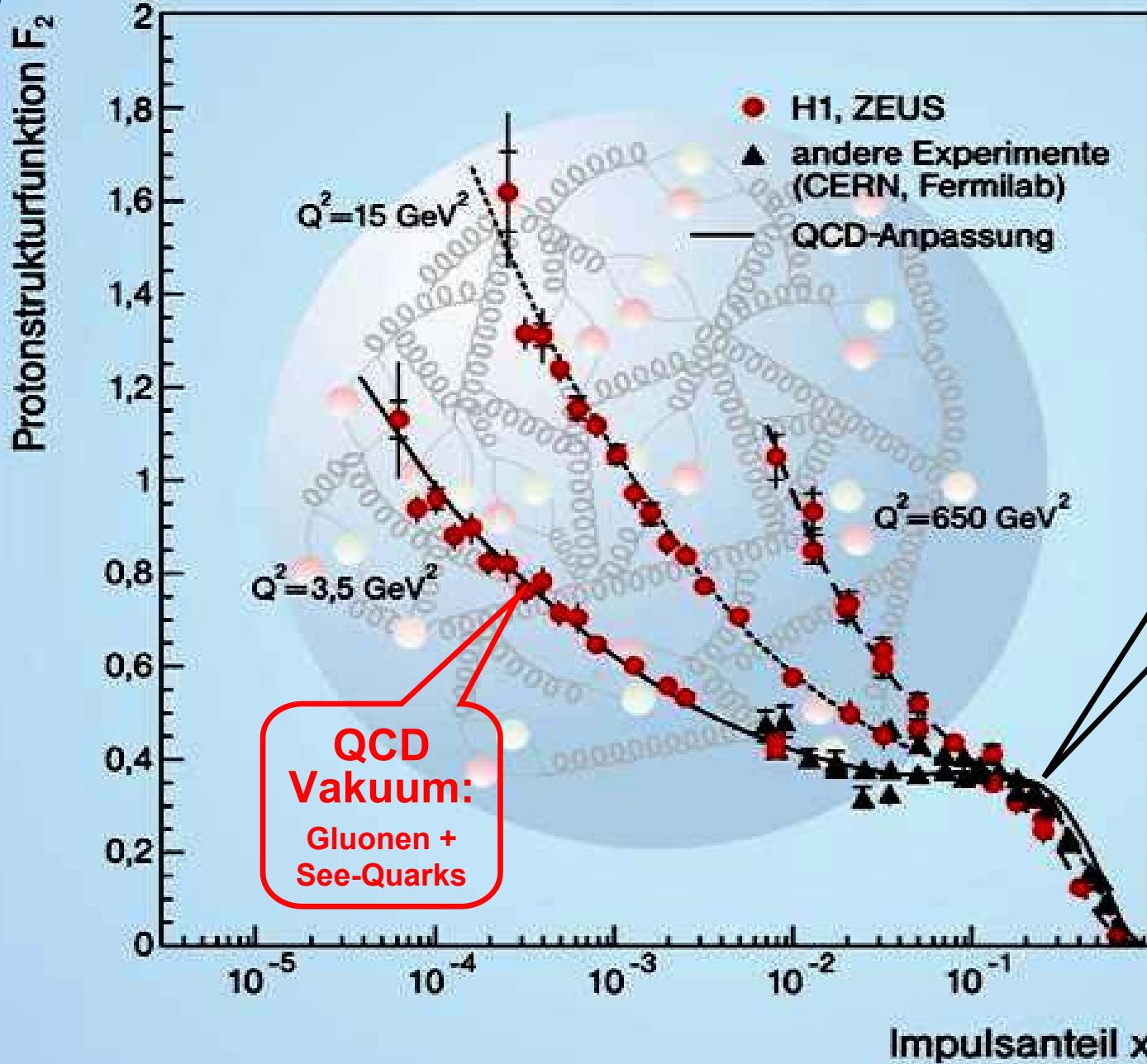
Proton-Struktur

QCD
Order



- **SLAC 1968:**
drei Valenzquarks:
 $x = 1/3$
- drei gebundene Quarks:
 $\sim p/2$ in 3 Quarks +
 $\sim p/2$ in N Gluonen
Impuls-Anteil des Quarks im Hadron:
 $x \sim 1/6$
- **HERA 1994:**
 $x > 0.1$: Valenzquarks
 $x \ll 0.1$: Seequarks + Gluonen :
reine QCD !

Proton-Struktur





Das Vakuum kocht ...



1974 - an der Wiege der QCD:

PHYSICAL REVIEW D

VOLUME 10, NUMBER 5

1 SEPTEMBER 1974

Possible non-Regge behavior of electroproduction structure functions*

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(Received 15 April 1974)

26 September 1996

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PHYSICS LETTERS B

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Physics Letters B 385 (1996) 411–414

On the asymptotic behaviour of $F_2(x, Q^2)$

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Abstract

We discuss how the proton structure function $F_2(x, Q^2)$ is described in the HERA kinematic range by double asymptotic expressions for low x and large Q^2 .

Proton-Struktur:

Aufblasen des Protons
aus dem QCD Vakuum

QCD-Asymptotik

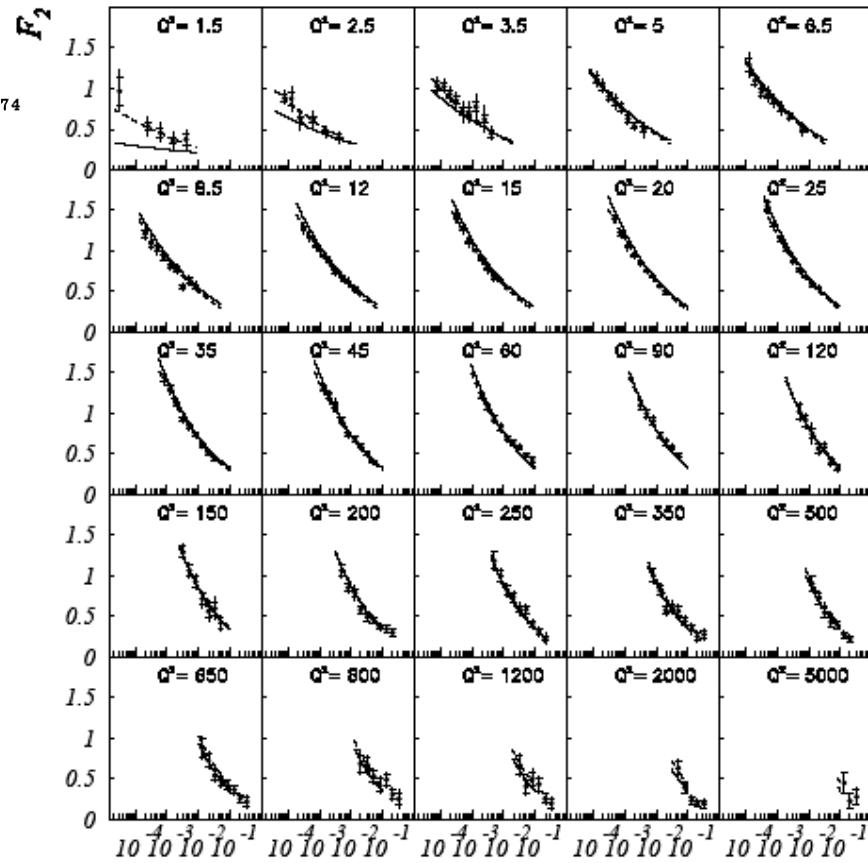
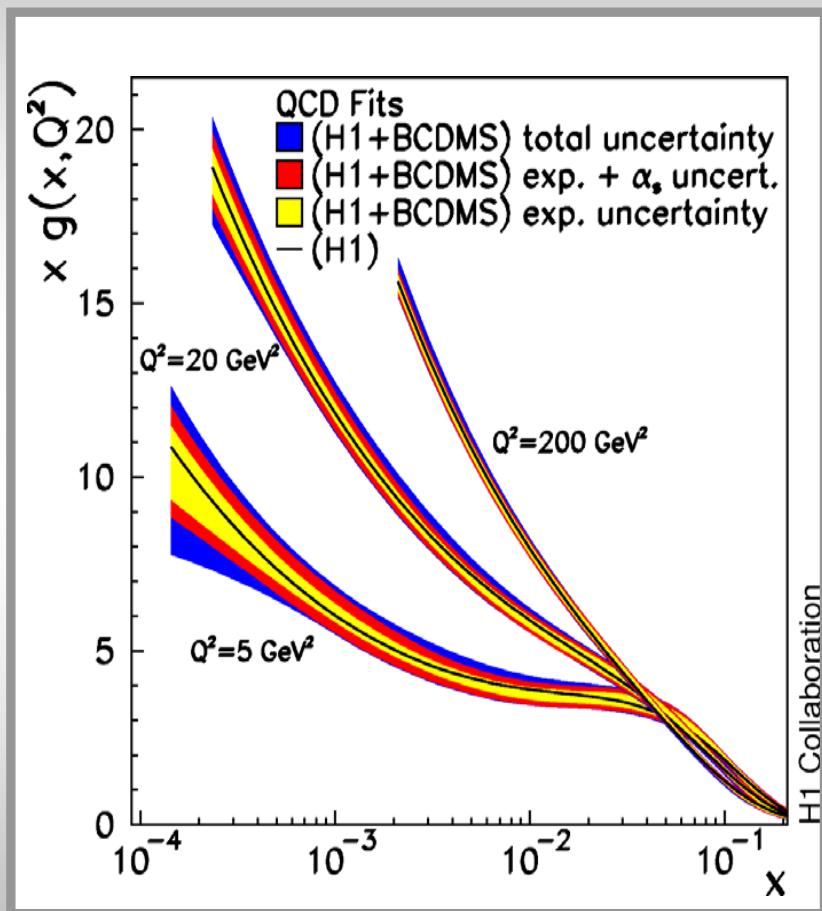


Figure 1: The proton structure function $F_2(x, Q^2)$ as measured by the H1 experiment at HERA together with a fit to the NLO double asymptotic expression (1) (full line) for $Q^2 > 5 \text{ GeV}^2$ and with a fit to the modified DLL expression $F_2 = N_f e^{-\gamma\sqrt{T}/\bar{\epsilon}}$ (dashed line) in the full Q^2 range.



Gluon-Dichte

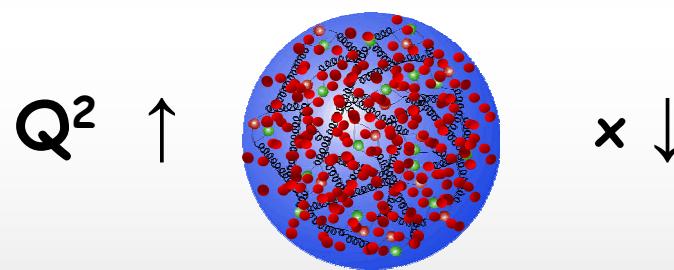


Gluon-Selbstkopplung

treibt Proton-Struktur und $\alpha_s(Q^2)$

$$\frac{\partial\sigma(x, Q^2)}{\partial \ln Q^2} \sim \alpha_s(Q^2) x g(x, Q^2)$$

Test d. nicht-abelschen QCD :
Aufkochen des QCD Vakuums:

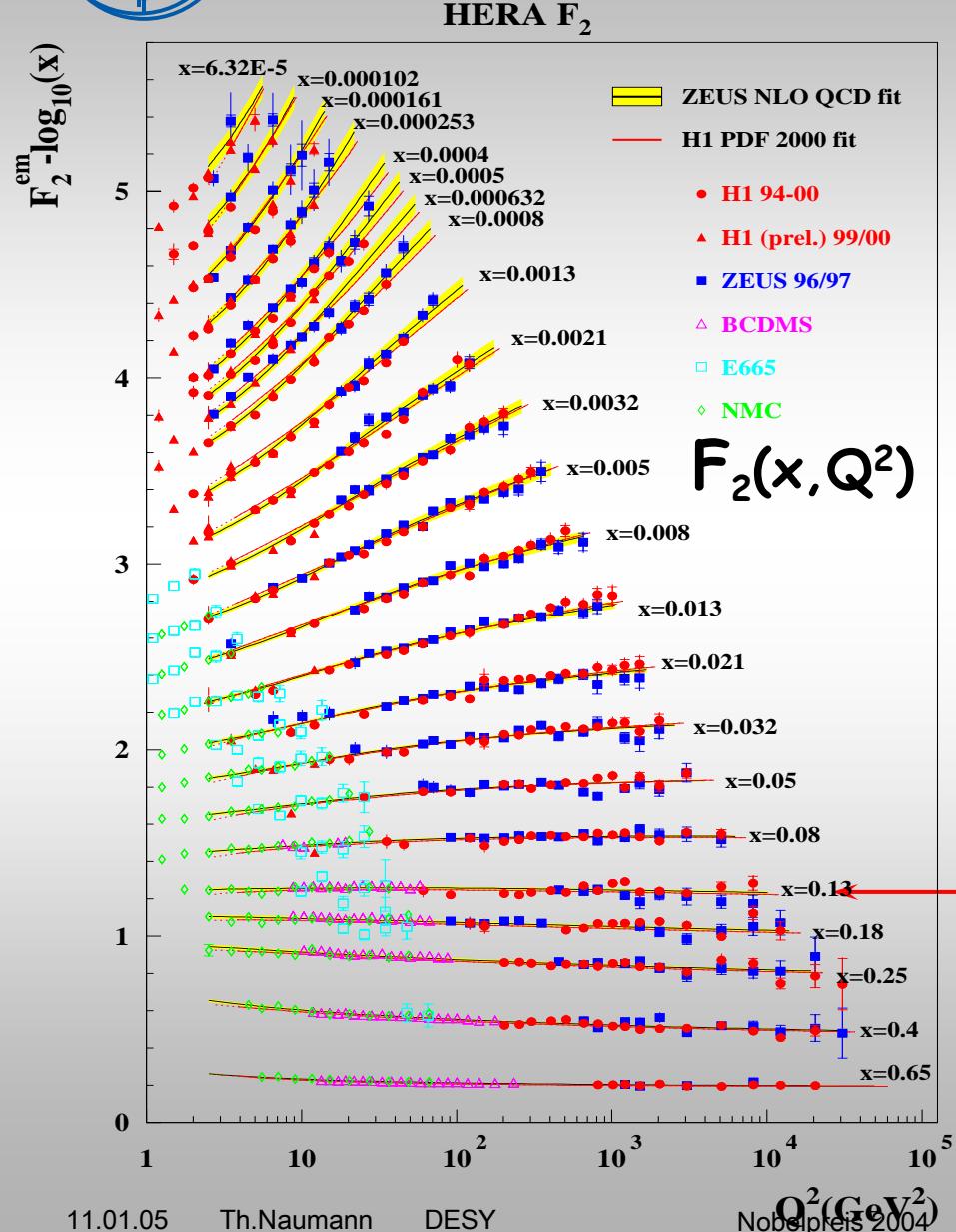


$\alpha_s \rightarrow 1$: Kollaps d. Störungstheorie ?

- Gluonen = Bosonen
Gluon-Saturation ? Shadowing ?
- Gluonium, Glue-Balls ?
- Quark-Gluon-Plasma ?



Proton Strukturfunktion



D. Gross:
tests of QCD with
amazing precision ...

x klein:
Anregung d.
QCD Vakuums

QCD-Fits d. log.
Skalenverletzungen:

- $\alpha_s(Q^2)$
- $xg(x, Q^2)$
- $xq(x, Q^2)$

Scaling

x groß:
Gluon-Abstrahlung



Die starke Kopplung



DESY Experimente H1+ZEUS:

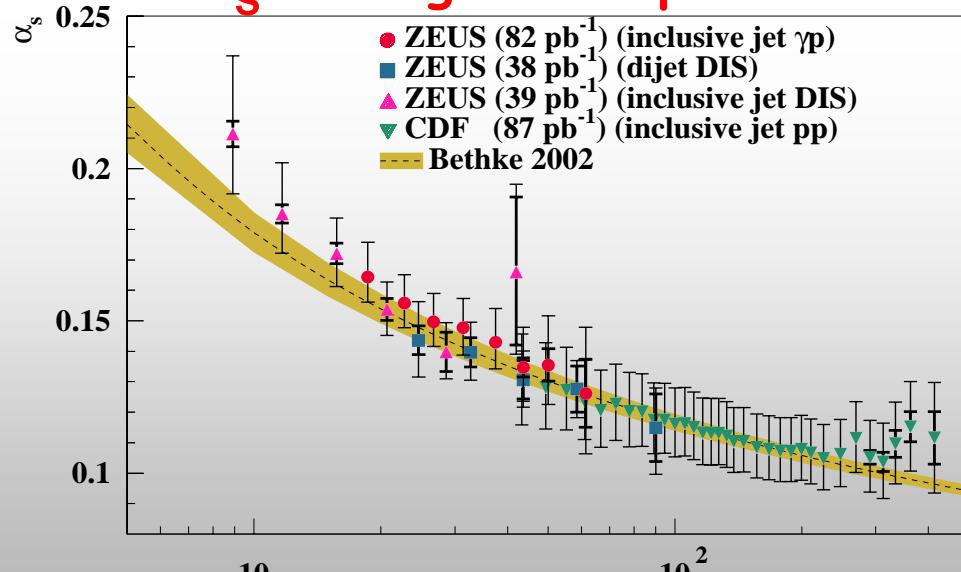
- QCD-Fit der Skalenverletzungen:

$$\alpha_s = 0.115 \pm 0.002 \text{ (exp.+fit)} \pm 0.005 \text{ (scale)}$$

- Jet-Wirkungsquerschnitte:

$$\alpha_s = 0.120 \pm 0.002 \text{ (exp.)} \pm 0.004 \text{ (syst.)}$$

α_s running in 1 Experiment:



$\alpha_s (\ln Q^2)$ crawling not running !

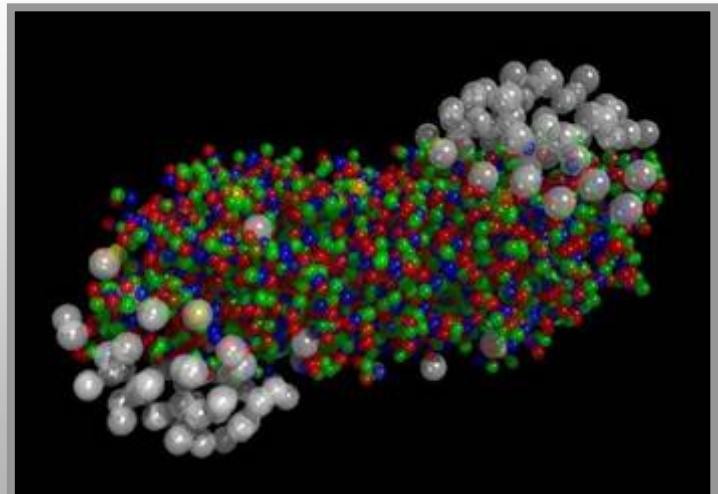
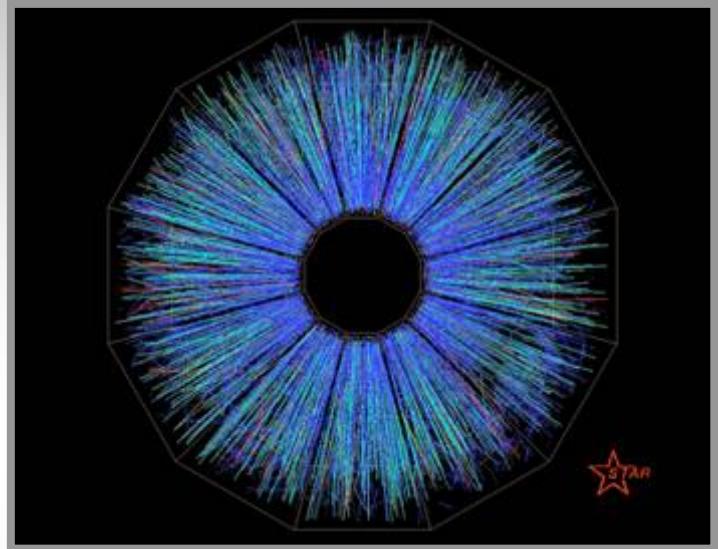
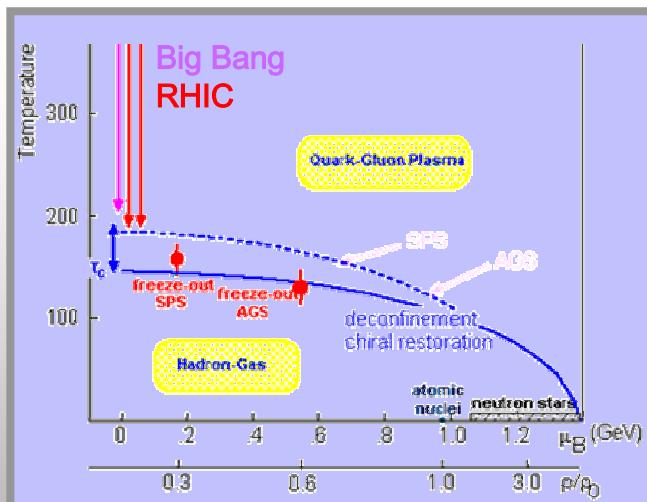
Quark-Gluon-Plasma

Gott schuf die Quarks frei

- $< \mu s$ nach dem Urknall
- danach QCD Confinement:
Phasenübergang Quagma - Hadron-Gas
Nukleonen frieren aus

RHIC am BNL, Brookhaven, USA:

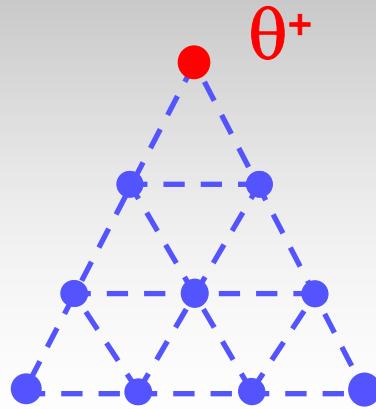
- Au-Au Kollisionen mit 100 GeV/Nukleon
>1000 Quarks, Thermalisierung:



GSI Darmstadt >2009



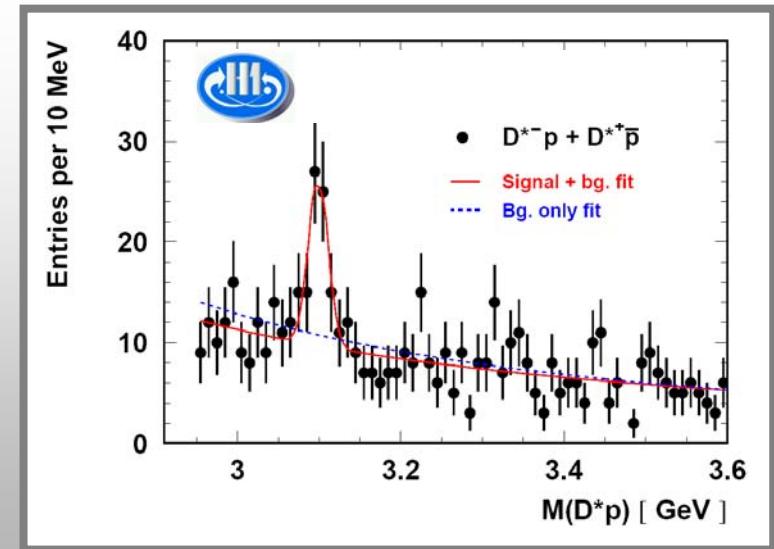
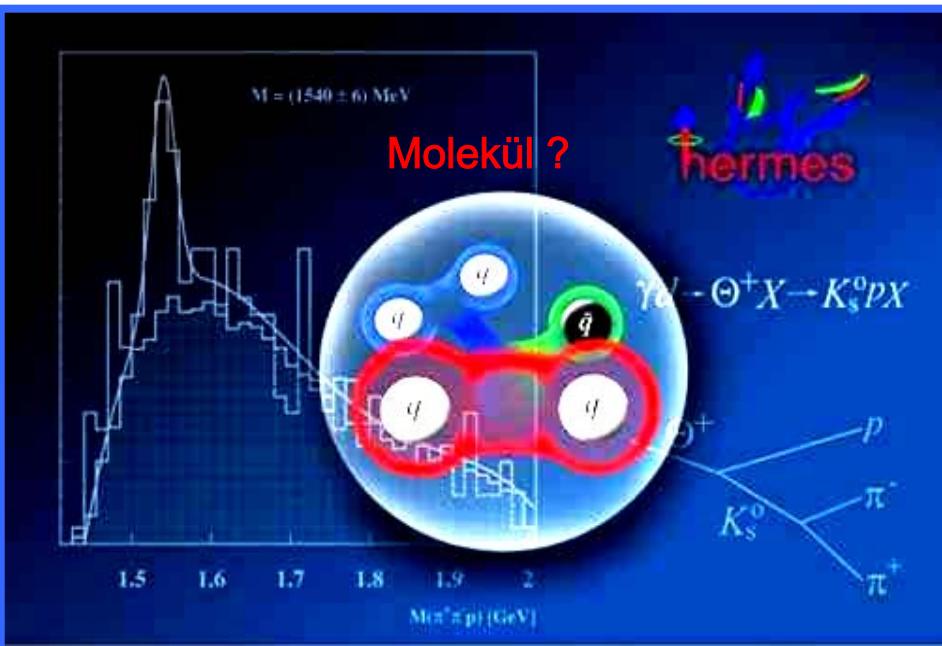
Penta-Quarks



Quarkmodell:

Baryon: (qqq) Meson: $(q\bar{q})$

Expt.: HERMES H1
Zustand $\Theta^+ = pK^0$ $\Theta_c = pD^*$:
exotisch: $(uudd\bar{s})$ $(uudd\bar{c})$

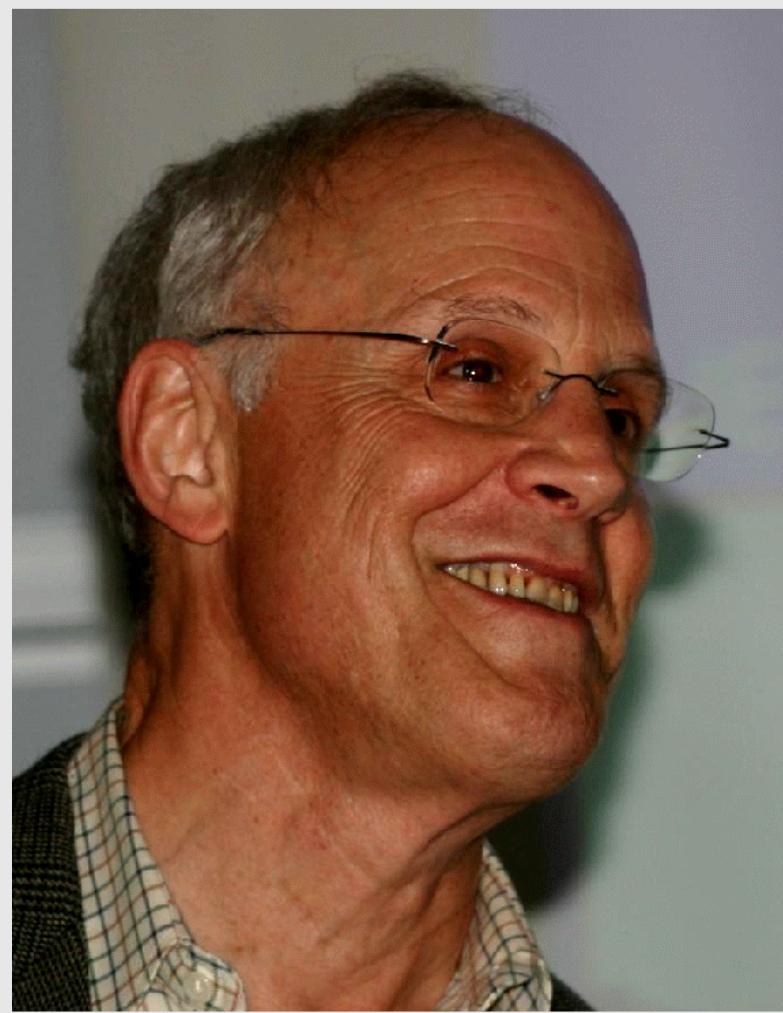


Präzision

für den Weg zur
Vereinigung



D.Gross 2004:



Theoretical calculations
of perturbative QCD
are truly heroic.

D.Gross, Loops & Legs,
DESY Theory Workshop,
Zinnowitz, Germany,
April 2004.

QCD in 2 Loops

$$\begin{aligned}\mu \frac{\partial \alpha_s}{\partial \mu} = 2\beta(\alpha_s) &= -\frac{\beta_0}{2\pi} \alpha_s^2 - \frac{\beta_1}{4\pi^2} \alpha_s^3 - \frac{\beta_2}{64\pi^3} \alpha_s^4 - \dots, \\ \beta_0 &= 11 - \frac{2}{3} n_f, \\ \beta_1 &= 51 - \frac{19}{3} n_f, \\ \beta_2 &= 2857 - \frac{5033}{9} n_f + \frac{325}{27} n_f^2;\end{aligned}$$

$$\begin{aligned}\alpha_s(\mu) &= \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)} \left[1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln[\ln(\mu^2/\Lambda^2)]}{\ln(\mu^2/\Lambda^2)} + \frac{4\beta_1^2}{\beta_0^4 \ln^2(\mu^2/\Lambda^2)} \right. \\ &\quad \times \left. \left(\left(\ln[\ln(\mu^2/\Lambda^2)] - \frac{1}{2} \right)^2 + \frac{\beta_2 \beta_0}{8\beta_1^2} - \frac{5}{4} \right) \right].\end{aligned}$$

β -Funktion in 3 Loops

$$\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$$

$$\beta_1 = \frac{34}{3}C_A^2 - 4C_F T_F n_f - \frac{20}{3}C_A T_F n_f$$

$$\begin{aligned}\beta_2 = & \frac{2857}{54}C_A^3 + 2C_F^2 T_F n_f - \frac{205}{9}C_F C_A T_F n_f \\ & - \frac{1415}{27}C_A^2 T_F n_f + \frac{44}{9}C_F T_F^2 n_f^2 + \frac{158}{27}C_A T_F^2 n_f^2\end{aligned}$$

$$\begin{aligned}\beta_3 = & C_A^4 \left(\frac{150653}{486} - \frac{44}{9}\zeta_3 \right) + C_A^3 T_F n_f \left(-\frac{39143}{81} + \frac{136}{3}\zeta_3 \right) \\ & + C_A^2 C_F T_F n_f \left(\frac{7073}{243} - \frac{656}{9}\zeta_3 \right) + C_A C_F^2 T_F n_f \left(-\frac{4204}{27} + \frac{352}{9}\zeta_3 \right) \\ & + 46C_F^3 T_F n_f + C_A^2 T_F^2 n_f^2 \left(\frac{7930}{81} + \frac{224}{9}\zeta_3 \right) + C_F^2 T_F^2 n_f^2 \left(\frac{1352}{27} - \frac{704}{9}\zeta_3 \right) \\ & + C_A C_F T_F^2 n_f^2 \left(\frac{17152}{243} + \frac{448}{9}\zeta_3 \right) + \frac{424}{243}C_A T_F^3 n_f^3 + \frac{1232}{243}C_F T_F^3 n_f^3 \\ & + \frac{d_A^{abcd} d_A^{abcd}}{N_A} \left(-\frac{80}{9} + \frac{704}{3}\zeta_3 \right) + n_f \frac{d_F^{abcd} d_A^{abcd}}{N_A} \left(\frac{512}{9} - \frac{1664}{3}\zeta_3 \right) \\ & + n_f^2 \frac{d_F^{abcd} d_F^{abcd}}{N_A} \left(-\frac{704}{9} + \frac{512}{3}\zeta_3 \right)\end{aligned}$$

$$q^2 \frac{\partial a_s}{\partial q^2} = -\beta_0 a_s^2 - \beta_1 a_s^3 - \beta_2 a_s^4 - \beta_3 a_s^5 + O(a_s^6)$$

Nicht-triviale Struktur des QCD-Vakuums

reine SU(3)



Loops+Legs 2004



β_1 1974

T.Jones
Liverpool



β_0 1973

D.Gross
Kavli Inst.



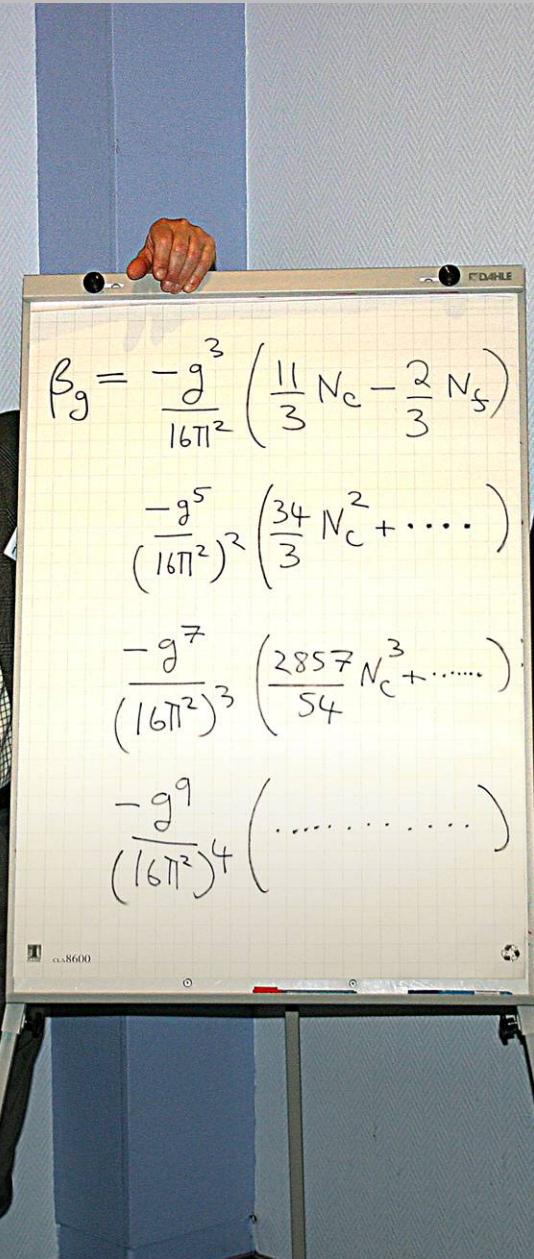
β_2 1981

O.Tarasov
DESY



β_3 1997

J.Vermaseren
NIKHEF





Störungstheorie

with

$$f_0(x) = \frac{1}{x}, \quad f_{\pm 1}(x) = \frac{1}{1 \mp x}. \quad (4.3)$$

A useful short-hand notation is

$$H_{w,\dots,w} = \sum_{n=0}^{\infty} \frac{x^n}{n!} H_{w,\dots,w}(x) = H_{(w+\dots,w+\dots)}(x). \quad (4.4)$$

For $w \leq 3$ the harmonic polylogarithms can be expressed in terms of standard polylogarithms; a complete list can be found in appendix A of Ref. [45]. All harmonic polylogarithms of weight $w = 4$ in this article can be expressed in terms of standard polylogarithms, Nielsen functions [74] or, by means of the defining relation (4.2), as one-dimensional integrals over these functions. A FORTRAN program for the functions up to weight $w = 4$ has been provided in Ref. [75].

For completeness we recall the one- and two-loop non-singular splitting functions [3, 8]

$$P_{\text{split}}^{(1)}(x) = C_F C_F \left(p_{\text{W}}(x) \left[\frac{57}{3} - \zeta_3 + \frac{11}{6} H_0 + H_{0,0} \right] + p_{\text{W}}(-x) \left[\zeta_3 + 2 H_{1,0} - H_{0,0} \right] \right. \quad (4.5)$$

1

2

$$\begin{aligned} P_{\text{split}}^{(1)}(x) &= 4 C_F C_F \left(p_{\text{W}}(x) \left[\frac{57}{3} - \zeta_3 + \frac{11}{6} H_0 + H_{0,0} \right] + p_{\text{W}}(-x) \left[\zeta_3 + 2 H_{1,0} - H_{0,0} \right] \right. \\ &\quad + \frac{14}{3} (1-x) \delta(1-x) \left[\frac{17}{24} - \frac{11}{3} \zeta_3 - 3 \zeta_5 \right] \left(-4 C_F \gamma_F \left(p_{\text{W}}(x) \left[\frac{5}{6} - \frac{1}{2} \zeta_3 \right] + \frac{2}{3} (1-x) \right. \right. \\ &\quad + \delta(1-x) \left[\frac{1}{12} + \frac{2}{3} \zeta_3 \right] \left. \right) + 4 C_F \gamma_F \left(p_{\text{W}}(x) \left[H_{1,0} - \frac{3}{4} H_0 + H_{2,0} \right] - 2 p_{\text{W}}(-x) \left[\zeta_3 + 2 H_{1,0} \right. \right. \\ &\quad \left. \left. - H_{0,0} \right] - H_0 (1 - \frac{1}{2} \zeta_3) H_0 - (1+x) H_0 + \delta(1-x) \left[\frac{3}{8} - 3 \zeta_3 + 6 \zeta_5 \right] \right), \\ P_{\text{split}}^{(1)}(-x) &= P_{\text{split}}^{(1)}(x) + 16 C_F \left(C_F - \frac{C_F}{2} \right) \left(p_{\text{W}}(-x) \left[\zeta_3 + 2 H_{1,0} - H_{0,0} \right] - 2(1-x) \right. \\ &\quad \left. - (1+x) H_0 \right). \end{aligned} \quad (4.6)$$

Here and in Eqs. (4.9)–(4.11) we suppress the argument x of the polylogarithms and use

$$p_{\text{W}}(x) = 2(1-x)^{-1} - 1 - x. \quad (4.8)$$

All divergences for $x = 1$ are understood in the sense of δ -distributions.

The three-loop splitting function for the evolution of the ‘plus’ combinations of quark densities in Eq. (2.2), corresponding to the anomalous dimension (3.8) reads

$$\begin{aligned} P_{\text{split}}^{(2)}(x) &= 16 C_F C_F \gamma_F \left(\frac{1}{3} p_{\text{W}}(x) \left[\frac{10}{3} - \zeta_3 - \frac{209}{36} H_0 - \frac{167}{18} H_1 + 2 H_0 H_{0,0} - 7 H_{0,0,0} - 2 H_{0,0,0,0} \right. \right. \\ &\quad + 3 H_{1,0,0,0} - H_1 + \frac{1}{3} p_{\text{W}}(-x) \left[\frac{2}{3} \zeta_3 - \frac{5}{3} H_0 - 2 H_{1,0} - 2 H_1 H_{0,0} - \frac{19}{3} H_{1,0,0} - H_{1,0,0,0} - H_{1,0,0,0,0} \right. \\ &\quad \left. \left. + 2 H_{1,0,-1} + \frac{1}{2} H_{0,0,2} + \frac{5}{3} H_{0,0} + H_{0,0,0,0} + H_1 \right] + (1-x) \left[\frac{1}{6} \zeta_2 - \frac{257}{54} H_2 - \frac{43}{18} H_3 - \frac{1}{6} H_4 - H_5 \right] \right. \\ &\quad \left. - H_6 \right) \\ &\quad - \frac{43}{4} \zeta_3 - \frac{5}{6} H_{1,0} - \frac{11}{12} H_0 \zeta_3 - \frac{1}{2} H_0 H_2 \zeta_3 - \frac{5}{6} H_0 H_3 \zeta_3 + 7 H_2 - \frac{1}{2} H_{2,0,0} + 3 H_3 + \frac{3}{4} H_4 \right] + \frac{1}{2} H_0 H_5 \zeta_3 \\ &\quad + \frac{1}{4} H_2^2 - \frac{8}{3} \zeta_3^2 + \frac{17}{3} H_0 + H_{1,0} - \frac{19}{2} H_0 + \frac{5}{3} H_0 H_2 - H_0 H_3 + \frac{13}{3} H_0 H_4 + \frac{2}{3} H_0 H_5 + H_{0,0,0,0} \end{aligned}$$

3

$$\begin{aligned} &\quad - \delta(1-x) \left[\frac{1657}{576} - \frac{281}{27} H_0 + \frac{1}{2} H_1 - \frac{97}{3} \zeta_3 - \frac{5}{2} \zeta_5 \right] + 16 C_F \gamma_F^2 \left(\frac{1}{3} p_{\text{W}}(x) \left[H_{1,0,0} - \frac{1}{3} + \frac{1}{2} H_0 \right] \right. \\ &\quad + (1-x) \left[\frac{13}{24} + \frac{1}{6} H_0 - \delta(1-x) \left[\frac{17}{24} - \frac{5}{2} \zeta_3 + \frac{5}{2} \zeta_5 \right] \right] + 16 C_F \gamma_F^2 \left(\frac{1}{3} p_{\text{W}}(-x) \left[\zeta_3 - 4 H_{1,0,0} \right. \right. \\ &\quad - \frac{55}{16} + \frac{5}{8} H_0 + H_0 H_2 + \frac{3}{2} H_0 - H_{0,0,0} - \frac{10}{3} H_0 - \frac{10}{3} H_2 - 2 H_{0,0} - 2 H_1 + \frac{1}{2} H_2 \\ &\quad - \frac{3}{2} H_3 + H_{2,0} + 2 H_4 - H_{1,0} + \frac{1}{3} H_{1,0,0} - H_{1,0,0,0} - 2 H_{1,0,-1} - \frac{1}{2} H_{0,0,2} - \frac{5}{3} H_0 H_0 - H_{0,0,0,0} + H_1 \right] \\ &\quad - (1-x) \left[\frac{10}{18} + \frac{15}{12} H_0 - \frac{1}{2} H_1 + \frac{1}{2} H_2 + H_{1,0} - \frac{1}{2} H_{1,0,0} - \frac{1}{24} H_{1,0,0,0} + \frac{1}{2} H_0 \right] + \frac{9}{4} H_0 \\ &\quad + \frac{7}{9} H_0 H_2 + H_0 H_3 - \frac{7}{16} \zeta_2 - \frac{25}{30} H_2 - \frac{17}{6} H_3 + \frac{9}{8} H_4 + \frac{7}{9} H_5 + \frac{1}{6} H_6 - H_{1,0} - H_{1,0,0} - H_{1,0,0,0} - H_{1,0,0,0,0} - H_{1,0,0,0,0,0} \\ &\quad + 6 H_1 - 7 H_2 + 12 H_{1,0} - 1,0 - H_{1,0,0} - \frac{3}{16} H_0 - \frac{3}{2} H_0 H_2 + H_0 H_3 + \frac{13}{2} H_0 H_4 - 2 H_0 H_5 + 8 H_{1,0} \\ &\quad + 12 H_1 H_5 + \delta(1-x) \left[\frac{93}{10} - \frac{93}{4} \zeta_3 - \frac{81}{2} H_0 - 15 H_{1,0} - 20 H_{1,0,0} + 2 H_{1,0,0,0} + 4 H_{1,0,0,0,0} + 4 H_{1,0,0,0,0,0} \right. \\ &\quad + 4 H_{1,0} + H_{1,0,0} + 2 H_1 + P_{\text{W}}(-x) \left[\frac{2}{3} \zeta_3 - \frac{9}{2} \zeta_5 - H_{1,0} - 3 H_0 + 32 H_1 H_5 + H_{1,0,0,0} + H_{1,0,0,0,0} + H_{1,0,0,0,0,0} \right. \\ &\quad - 26 H_{1,0,0,0} - 28 H_{1,0,0,0,0} + 2 H_{1,0,0,0,0,0} + 36 H_{1,0,0,0,0,0,0} + 36 H_{1,0,0,0,0,0,0,0} + 36 H_{1,0,0,0,0,0,0,0,0} \\ &\quad + 48 H_{1,0,0,0,0,0,0,0} + 40 H_{1,0,0,0,0,0,0,0,0} + 3 H_{1,0,0,0,0,0,0,0,0,0} - 22 H_{1,0,0,0,0,0,0,0,0} - 6 H_{1,0,0,0,0,0,0,0,0} - 6 H_{1,0,0,0,0,0,0,0,0} \\ &\quad - \frac{3}{2} H_0 H_5 - 13 H_0 H_6 - 14 H_0 H_7 - \frac{9}{2} H_0 H_8 + 6 H_0 H_{0,0,0} + 6 H_0 H_{0,0,0,0} + 3 H_1 H_3 + 2 H_{1,0} + 12 H_1 \\ &\quad + (1-x) \left[2 H_{1,0,0} - \frac{31}{3} + 4 H_{1,0,0,0} - 2 H_{1,0,0,0,0} + 3 H_0 H_2 - 3 H_0 H_3 + H_{1,0,0,0,0,0} - \frac{5}{2} H_2 \right] \\ &\quad + (1+x) \left[\frac{37}{10} - \frac{9}{2} \zeta_3 - \frac{93}{4} \zeta_5 - \frac{81}{2} H_0 - 15 H_{1,0} - 20 H_{1,0,0} + 2 H_{1,0,0,0} - 2 H_{1,0,0,0,0} - 2 H_{1,0,0,0,0,0} \right. \\ &\quad - 24 H_{1,0,0,0} - \frac{539}{16} H_0 - 28 H_0 H_2 + \frac{191}{8} H_0 H_3 - \frac{205}{4} H_0 H_4 - 3 H_1 H_5 - 3 H_2 H_0 - 3 H_3 H_0 + 13 H_3 \\ &\quad - H_4 + 4 C_F + 335 H_{1,0} + 4 H_{1,0,0} + 10 H_{1,0,0,0} + \frac{67}{2} H_0 + 6 H_0 H_2 + 19 H_0 H_3 - 25 H_0 H_4 - 17 H_0 H_5 - 17 H_0 H_6 \\ &\quad - 2 H_1 H_2 - 4 H_2 H_3 - 4 H_3 H_4 + (1-x) \left[\frac{29}{32} - 2 H_0 H_5 + \frac{9}{8} H_0 H_6 + \frac{18}{5} H_0 H_7 + \frac{17}{4} H_0 H_8 + 15 H_0 H_9 \right] \end{aligned} \quad (4.9)$$

$$\begin{aligned} P_{\text{split}}^{(2)}(-x) &= P_{\text{split}}^{(2)}(x) + 16 C_F C_F \left(C_F - \frac{C_F}{2} \right) \left(p_{\text{W}}(-x) \left[\frac{134}{3} \zeta_3 - 4 \zeta_5 - 11 \zeta_7 - 4 H_{1,0,0} \right. \right. \\ &\quad + 32 H_{1,0,0,0} + \frac{22}{3} H_{1,0,0,0,0} - 16 H_{1,0,0,0,0,0} - 32 H_{1,0,0,0,0,0,0} + \frac{44}{3} H_{1,0,0,0,0,0,0,0} - 64 H_{1,0,0,0,0,0,0,0} \\ &\quad + 32 H_{1,0,0,0,0,0} - 64 H_{1,0,0,0,0,0,0} - 12 H_{1,0,0,0,0,0,0,0} + \frac{268}{3} H_{1,0,0,0,0,0,0,0,0} - 22 H_{1,0,0,0,0,0,0,0,0} - \frac{44}{3} H_{1,0,0,0,0,0,0,0,0} \\ &\quad \left. \left. - 2 H_{1,0,0,0,0,0,0,0,0,0} \right] \right) \end{aligned} \quad (4.10)$$

The x -space counterpart of Eq. (3.8) for the evolution of the ‘minus’ combinations (2.2) is given by

$$P_{\text{split}}^{(2)}(-x) = P_{\text{split}}^{(2)}(x) + 16 C_F C_F \left(C_F - \frac{C_F}{2} \right) \left(p_{\text{W}}(-x) \left[\frac{134}{3} \zeta_3 - 4 \zeta_5 - 11 \zeta_7 - 4 H_{1,0,0} \right. \right. \\ \left. \left. + 32 H_{1,0,0,0} + \frac{22}{3} H_{1,0,0,0,0} - 16 H_{1,0,0,0,0,0} - 32 H_{1,0,0,0,0,0,0} + \frac{44}{3} H_{1,0,0,0,0,0,0,0} - 64 H_{1,0,0,0,0,0,0,0} \right. \right. \\ \left. \left. + 32 H_{1,0,0,0,0} - 64 H_{1,0,0,0,0,0} - 12 H_{1,0,0,0,0,0,0} + \frac{268}{3} H_{1,0,0,0,0,0,0,0,0} - 22 H_{1,0,0,0,0,0,0,0,0} - \frac{44}{3} H_{1,0,0,0,0,0,0,0,0} \right] \right) \end{math>$$

1

2

$$\begin{aligned} P_{\text{split}}^{(1)}(x) &= 4 C_F C_F \left(p_{\text{W}}(x) \left[\frac{57}{3} - \zeta_3 + \frac{11}{6} H_0 + H_{0,0} \right] + p_{\text{W}}(-x) \left[\zeta_3 + 2 H_{1,0} - H_{0,0} \right] \right. \\ &\quad + \frac{14}{3} (1-x) \delta(1-x) \left[\frac{17}{24} - \frac{11}{3} \zeta_3 - 3 \zeta_5 \right] \left(-4 C_F \gamma_F \left(p_{\text{W}}(x) \left[\frac{5}{6} - \frac{1}{2} \zeta_3 \right] + \frac{2}{3} (1-x) \right. \right. \\ &\quad + \delta(1-x) \left[\frac{1}{12} + \frac{2}{3} \zeta_3 \right] \left. \right) + 4 C_F \gamma_F \left(p_{\text{W}}(x) \left[H_{1,0} - \frac{3}{4} H_0 + H_{2,0} \right] - 2 p_{\text{W}}(-x) \left[\zeta_3 + 2 H_{1,0} \right. \right. \\ &\quad \left. \left. - H_{0,0} \right] - H_0 (1 - \frac{1}{2} \zeta_3) H_0 - (1+x) H_0 + \delta(1-x) \left[\frac{3}{8} - 3 \zeta_3 + 6 \zeta_5 \right] \right), \\ P_{\text{split}}^{(1)}(-x) &= P_{\text{split}}^{(1)}(x) + 16 C_F \left(C_F - \frac{C_F}{2} \right) \left(p_{\text{W}}(-x) \left[\zeta_3 + 2 H_{1,0} - H_{0,0} \right] - 2(1-x) \right. \\ &\quad \left. - (1+x) H_0 \right). \end{aligned} \quad (4.6)$$

3

$$\begin{aligned} &\quad - (1-x) \left[\frac{2}{3} H_{1,0,0} + \frac{1}{2} H_2 \right] + \frac{1}{4} \zeta_3 + H_0 + \frac{1}{6} H_{0,0,0} + 6 H_{1,0,0} + \frac{1}{20} \zeta_3^2 + \frac{25}{18} \zeta_5 \right] \Big) \\ &\quad + 16 C_F C_F \gamma_F^2 \left(p_{\text{W}}(x) \left[\frac{2}{3} \zeta_3 - \frac{69}{20} \zeta_5^2 - H_{1,0,0,0} - 3 H_{1,0,0,0,0} - 4 H_{1,0,0,0,0,0} + \frac{23}{12} H_0,0,0 + 5 H_{1,0,0,0,0,0} \right. \right. \\ &\quad - 4 H_{1,0,0,0,0,0} - \frac{151}{48} H_0,0,0 + \frac{41}{24} H_0,0,0,0 - \frac{17}{2} H_0,0,0,0,0 - \frac{13}{4} H_0,0,0,0,0,0 - \frac{23}{12} H_0,0,0,0,0,0,0 + \frac{21}{4} H_0,0,0,0,0,0,0,0 \\ &\quad - 24 H_1 H_5 - 10 H_{1,0,0,0,0,0} - \frac{67}{2} H_{1,0,0,0,0,0,0} - 2 H_{1,0,0,0,0,0,0,0} + \frac{31}{4} H_{1,0,0,0,0,0,0,0,0,0} - 42 H_{1,0,0,0,0,0,0,0,0,0,0} \\ &\quad + \frac{67}{9} H_2 - 2 H_2 H_5 + \frac{11}{3} H_2 H_6 + 5 H_{1,0,0,0,0,0,0,0,0,0,0} + H_3 H_5 + \frac{11}{6} H_3 H_6 + 5 H_{1,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad - 32 H_{1,0,0,0,0,0,0,0,0,0,0} - 4 H_{1,0,0,0,0,0,0,0,0,0,0,0} - \frac{31}{6} H_{1,0,0,0,0,0,0,0,0,0,0,0,0} + 21 H_{1,0,0,0,0,0,0,0,0,0,0,0,0} + 30 H_{1,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad - 4 H_{1,0,0,0,0,0,0,0,0,0,0,0,0} - 36 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0} - 36 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0} - 5 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad + 67 \zeta_3 - 2 H_2 H_5 - 2 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0} - 3 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 3 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad - 32 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - \frac{31}{6} H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 2 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 2 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad - 4 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad + 32 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - \frac{31}{6} H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 17 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 17 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad - 4 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 36 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 36 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 36 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad + 32 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - \frac{31}{6} H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 17 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 17 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad - 4 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad + 32 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - \frac{31}{6} H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 21 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 21 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad - 4 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad + 32 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - \frac{31}{6} H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 17 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 17 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad - 4 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad + 32 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - \frac{31}{6} H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 21 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 21 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad - 4 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad + 32 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - \frac{31}{6} H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 21 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 21 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad - 4 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad + 32 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - \frac{31}{6} H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 21 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 21 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad - 4 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad + 32 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - \frac{31}{6} H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 21 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 21 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad - 4 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad + 32 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - \frac{31}{6} H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 21 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 21 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad - 4 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad + 32 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - \frac{31}{6} H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 21 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 21 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad - 4 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - 56 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} \\ &\quad + 32 H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} - \frac{31}{6} H_{1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} + 21 H_{$$

QCD auf dem Gitter

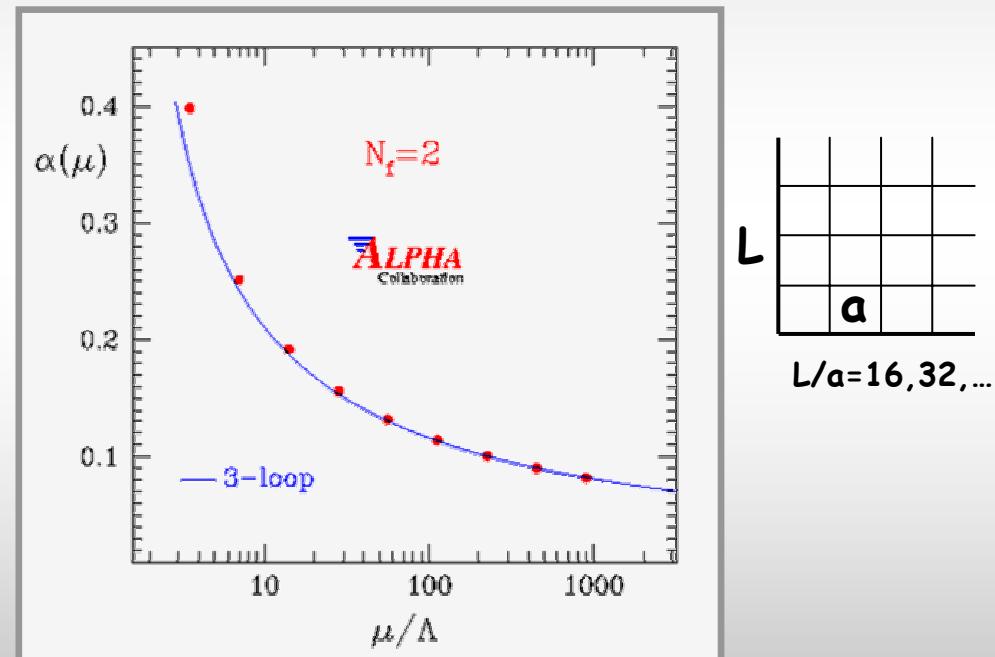
APE Gitter-Rechner



bei DESY Zeuthen
2005/6: 3 TFlop

Alternative:

- diskrete Raum-Zeit
- minimiere QCD Pfad-Integrale
- simuliere QCD auf dem Gitter :



- Freiheitsgrade: Color, Flavor, Quark-Massen

$\delta\alpha_s/\alpha_s = 1\% \quad \text{ok mit Expt.+ 3-Loop}$

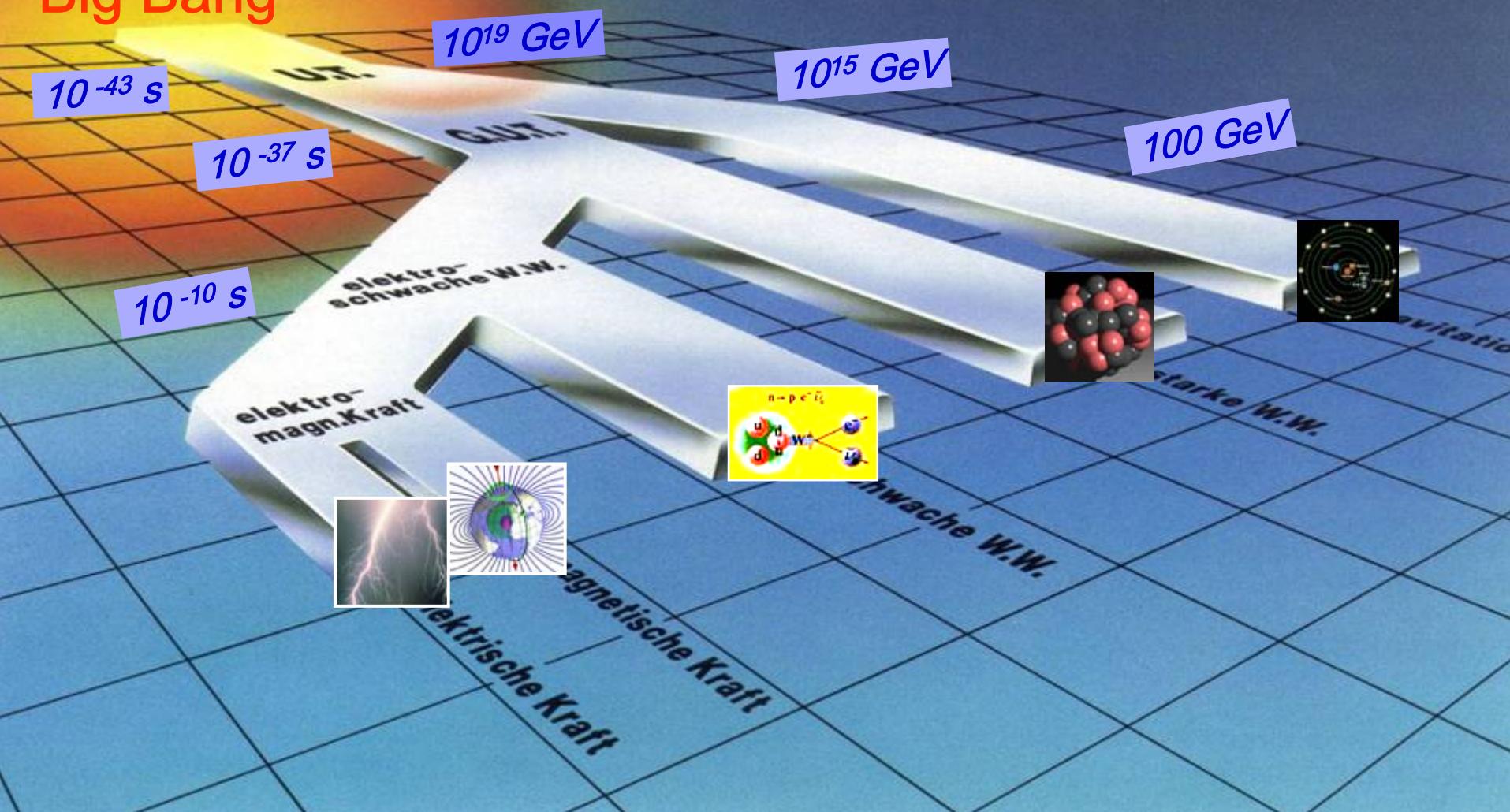
Der Weg zur Urkraft:

**Vereint sind wir
gleich stark !**

Die Vereinigung der Kräfte

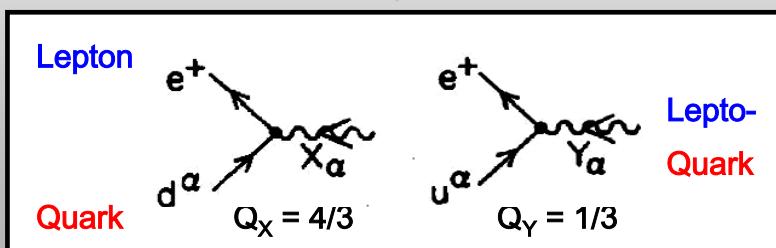
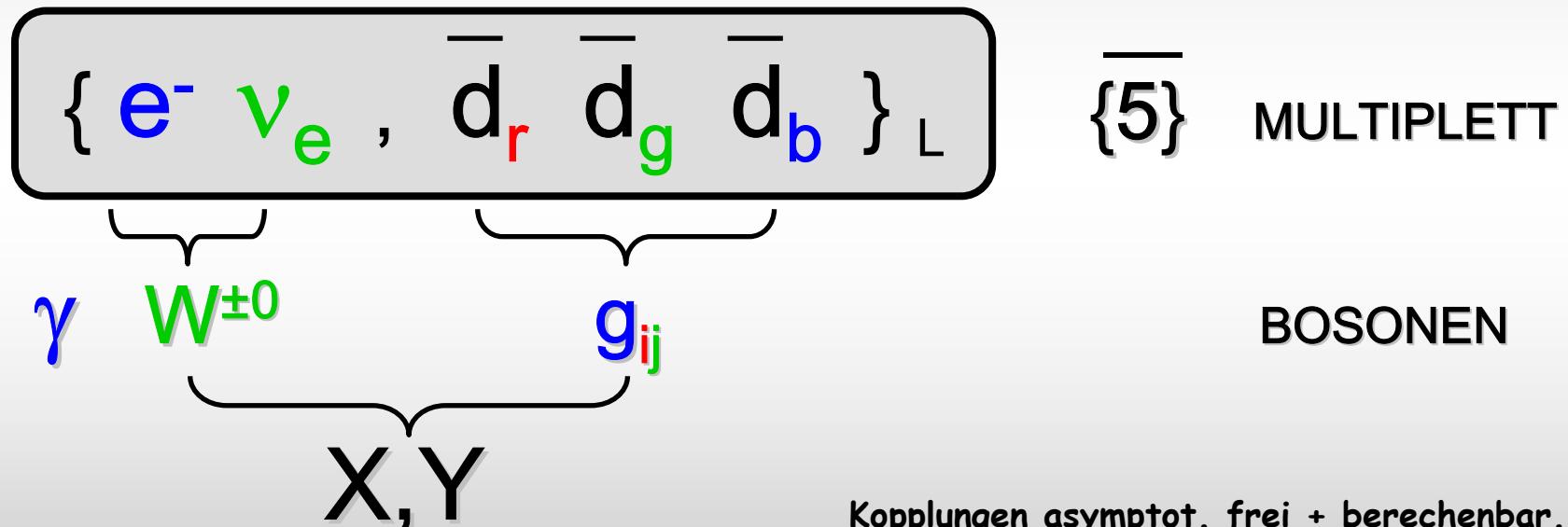


Big Bang



Große Vereinigung

elm. schwach stark vereinigt
 $U(1) \otimes SU(2) \otimes SU(3) \subset SU(5)$ SYMMETRIEN



Kopplungen asymptot. frei + berechenbar.
 Wo wird

$$\alpha_{\text{elm}} = \alpha_{\text{schwach}} = \alpha_{\text{stark}} ?$$

Georgi, Glashow; Georgi, Quinn, Weinberg, 1974.

Große Vereinigung

- Quark-Lepton-Symmetrie:
Quarks + Leptonen in ein Multiplett

- Quantisierung der elektr. Ladung:

$$N_C Q_q - Q_e = 0 = 3 \times 1/3 - 1$$

bzw. $Q_p = Q_e$

- Vorhersage für elektro-schwachen Mischungswinkel:

- $\sin^2 \theta_W(M_X) = g^2 / (g^2 + g'^2) = 3/5 / (1+3/5) = 3/8$
- $\sin^2 \theta_W(M_Z) = 0.20$ GUT
- $\sin^2 \theta_W(M_Z) = 0.22$ Expt.

- Leptonzahl-Verletzung:

Neutrino-Massen u. -Oszillationen:

- Baryonzahl-Verletzung: Proton-Zerfall : $p \rightarrow e^+ \pi^0$

$$\tau_p \sim M_X^{-4} / \alpha^2 m_p^{-5}$$

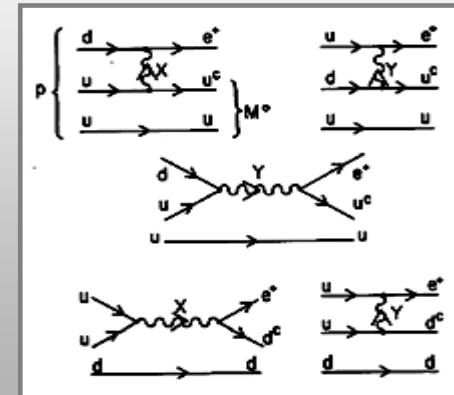
Super-K 1998: $\tau(p \rightarrow e^+ \pi^0) > 5 \cdot 10^{33} \text{ a}$ $\tau(p \rightarrow K^+ \nu) > 1.6 \cdot 10^{33} \text{ a}$

Hyper-K 201x+10: $\tau(p \rightarrow e^+ \pi^0) > 10^{35} \text{ a}$ $\tau(p \rightarrow K^+ \nu) > 3 \cdot 10^{34} \text{ a}$

- SUSY GUT: $M_X \sim 10^{16} \text{ GeV} :$

$$\tau(p \rightarrow e^+ \pi^0) \sim 10^{35 \pm 1} \text{ a} \quad \tau(p \rightarrow K^+ \nu) \sim 10^{32 \pm 3} \text{ a}$$

$$\{ e^- \nu_e , \bar{d}_r \bar{d}_g \bar{d}_b \}_L$$



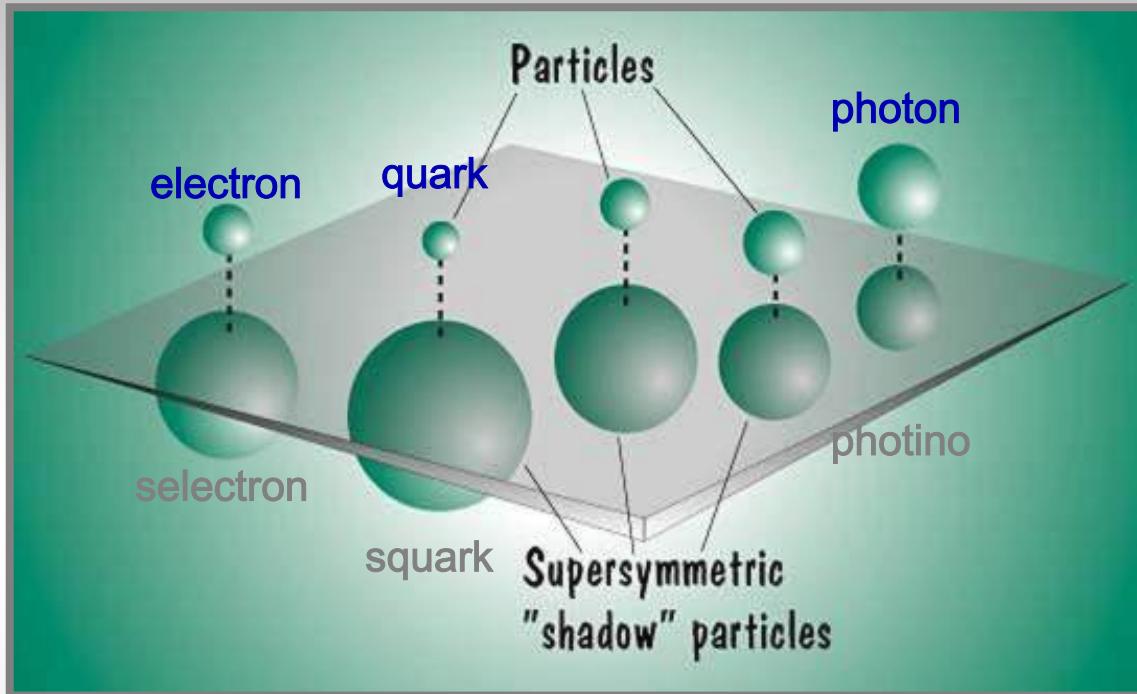
Super-Symmetrie

Fermion

Boson

Boson

Fermion



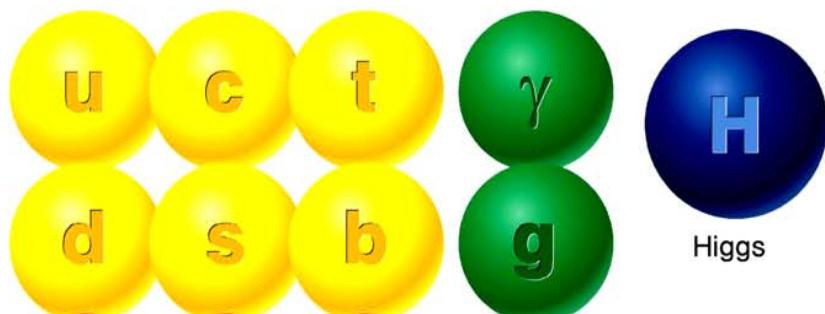
Super-Symmetrie

vereinigt

**Bosonen mit Fermionen
Kraft mit Materie**

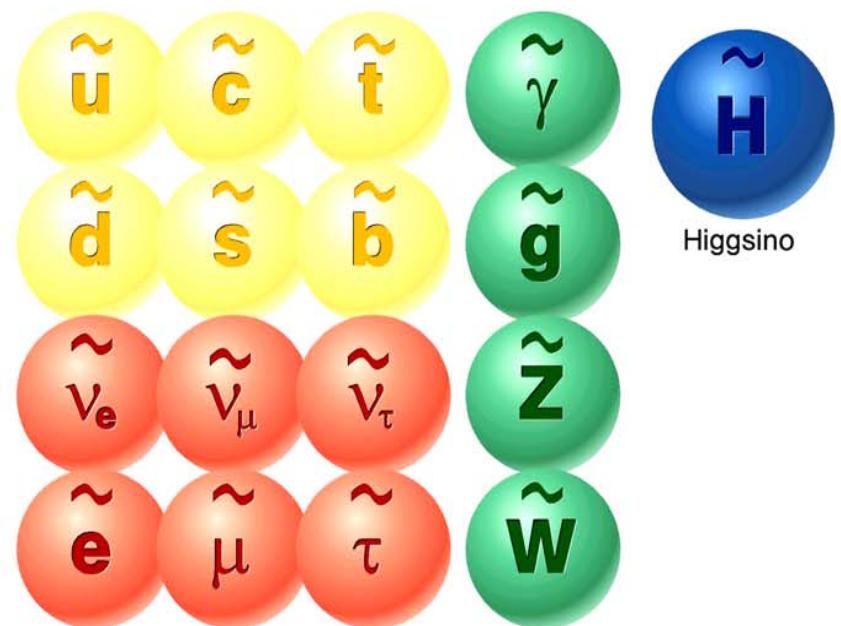
Super-Symmetrie

Standard-Teilchen



Higgs

SUSY-Teilchen



Higgsino

Quarks

Leptonen

Kraftteilchen

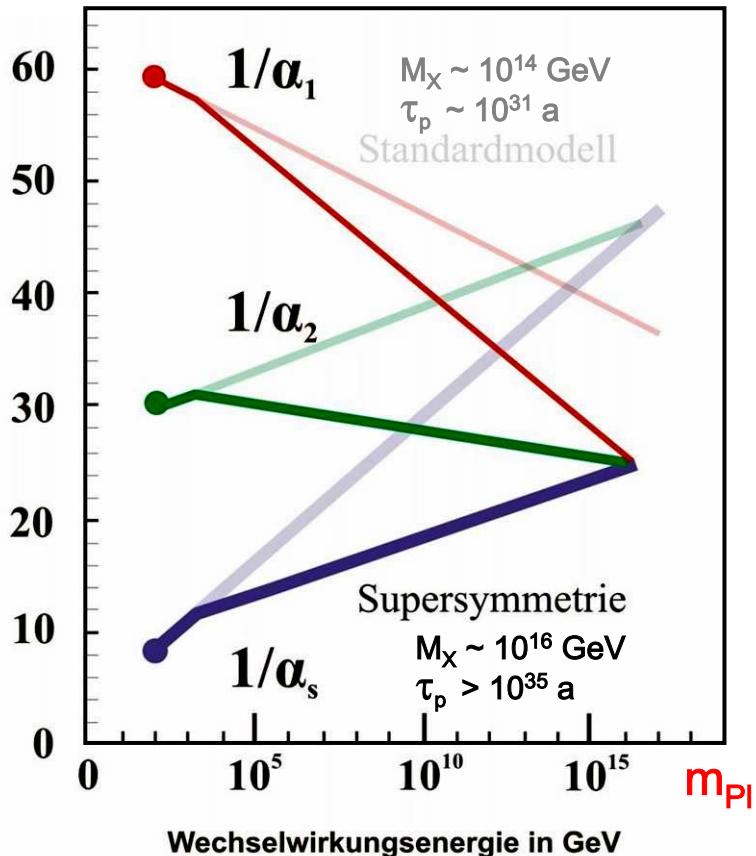
Squarks

Sleptonen

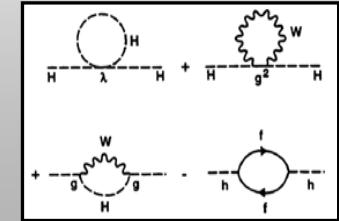
SUSY-Kraftteilchen

Super-Symmetrie

vereinigt Kräfte und Kopplungen

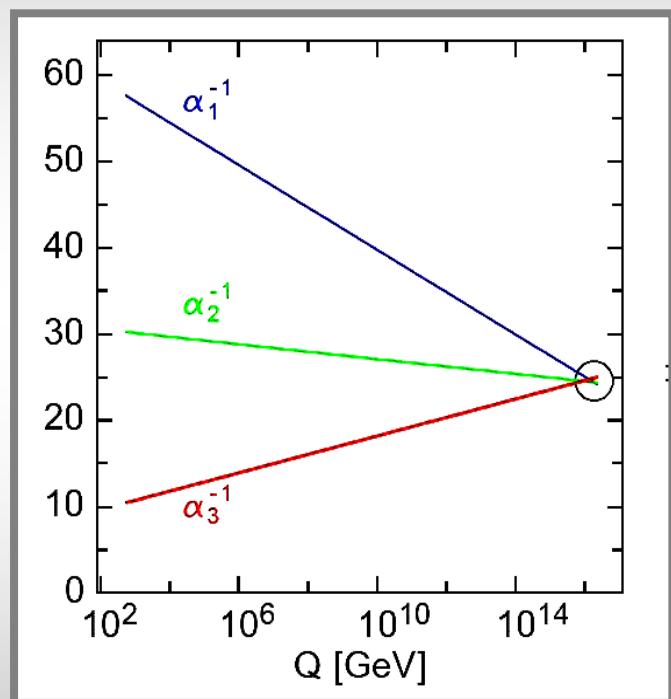


- ändert Energie-Abhängigkeit der Kopplungen:
ein Vereinigungs-Punkt bei $M_X = 2 \cdot 10^{16}$ GeV !
- Proton-Lebensdauer > exptl. Limit
- Neutralino ($\bar{\chi}_1^0, \bar{\chi}_2^0, \bar{H}_1, \bar{H}_2$): leichtestes SUSY-Teilchen
Dunkle Materie im Universum !
- verbindet kontinuierl. Raum-Zeit-Symmetrie mit Spin-Statistik + Symmetrien
 $U(1) \otimes SU(2) \otimes SU(3) \supset SU(5), SO(10), E(6), \dots$
- Eichfelder lokaler SUSY:
Graviton ($J=2$) + Gravitino ($J=3/2$)
Super-Gravitation !
- Higgs-Strahlungskorrekturen stabil:



Super-Symmetrie

Vereinigung: Extrapolation über 10^{14}



Präzision der
Startwerte:

$\delta\alpha / \alpha \sim 10^{-9}$ elektromagnet.
 $\delta G_F / G_F \sim 10^{-5}$ schwach
 $\delta G_N / G_N \sim 10^{-3}$ Gravitation
 $\delta\alpha_s / \alpha_s \sim 10^{-2}$ stark

$\delta\alpha_s / \alpha_s$:

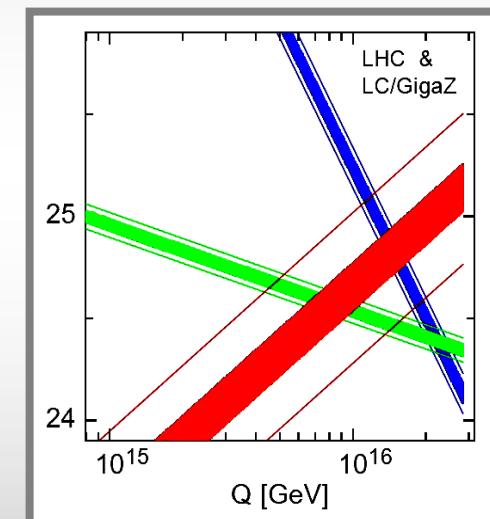
Experiment: ~1%

Theorie:

NLO ~4%

NNLO ~2% ?

Gitter ~1% ?



Was macht die Kernkraft stark ?

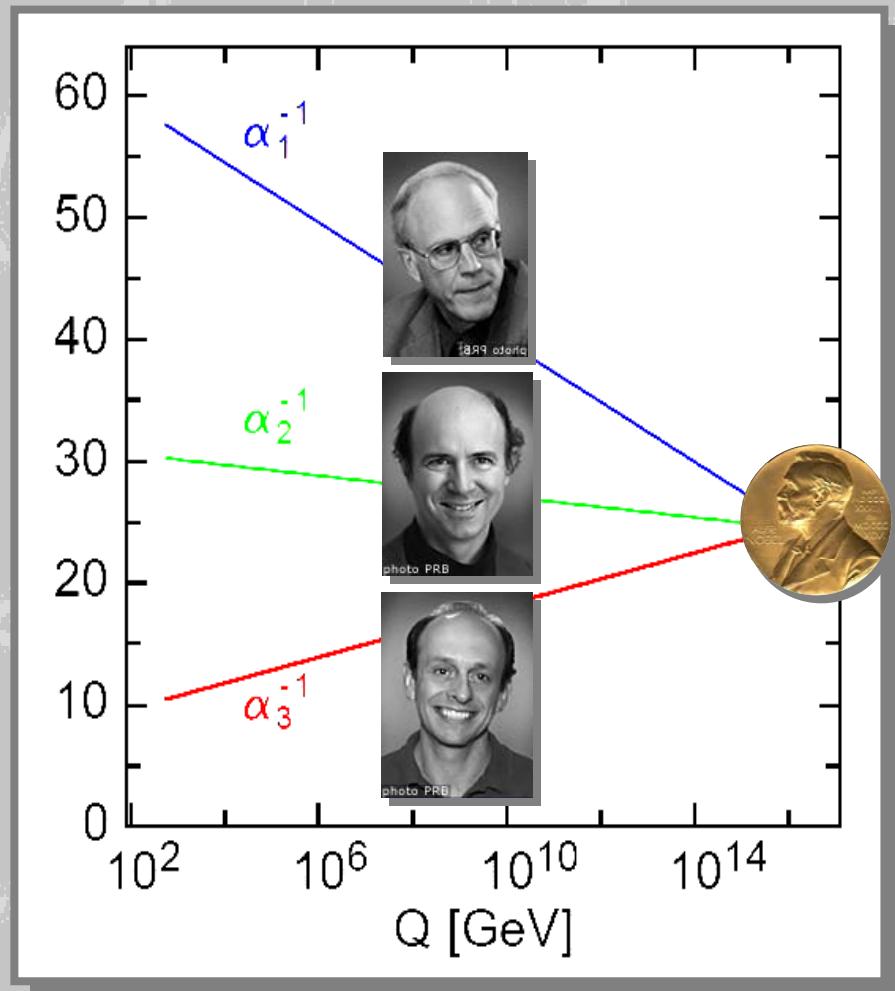
$$G_{\mu\nu}^a G_{\mu\nu}^a + \sum \bar{q}_j (i \gamma^\mu D_\mu + m_j) q_j$$

Was mich schwach macht,
macht mich stark !

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + i f_{abc} A_\mu^b A_\nu^c$$

Selbstkopplung der Eichbosonen
in nicht-abelschen Eichtheorien

Der Weg ist frei ...



zu einer

- **Feldtheorie** mit
 - korrekter Asymptotik +
 - Punkt-Wechselwirkung
- **zurück
zum Urknall**
- **vorwärts
zur Urkraft !**